Coupled Motion Equations for Two Interconnected Floating Bodies in an Auxiliary Function Approach

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Highlights

- Formulation of a coupled auxiliary function approach is derived for indirectly calculating hydrodynamic wave forces in case of multiple floating bodies.
- Interconnection between two floating bodies is mathematically modeled by a constraint matrix method.

1 Introduction

This abstract presents an indirect method for calculating hydrodynamic forces on multiple floating bodies in a nonlinear potential flow model. This is motivated by the fact that a direct method in calculating the forces in time-domain simulations, i.e., $F_i = -\rho \int_{S_B} (\phi_t + \frac{1}{2}\nabla\phi\cdot\nabla\phi + gz) n_i \mathrm{d}S$, is prone to numerical instability for cases of floating bodies, which is caused by the inaccurate evaluation of the time derivative $\phi_t = \frac{\partial \phi}{\partial t}$ by a simple backward difference scheme (Tanizawa, 1995).

Several methods have been developed to compute ϕ_t in the literature. An acceleration potential method was presented by (Tanizawa, 1995), who defined an acceleration potential Φ as $\Phi = \phi_t + \frac{1}{2}\nabla\phi\cdot\nabla\phi$. This method involves the evaluation of surface local curvature, which is extremely difficult in 3D cases where the body geometry is rather complicated. Vinje and Brevig (1981) introduced a mode decomposition method which decomposes ϕ_t into modes corresponding to each degree of freedom of body motions, plus a mode due to velocity potential. Each mode component can be obtained by calculating its respective boundary integral equation. This method was adopted by Cointe et al. (1990) in their 2D numerical wave tank. Nonetheless, much additional computational effort is required while evaluating each mode component. Notably, an auxiliary function approach for indirect calculation of wave forces was proposed by Wu and Eatock Taylor (1996). They defined a set of auxiliary functions ψ_i which correspond to each degree of freedom of the body motions. Instead of calculating the time derivative ϕ_t itself, they evaluated its integral over the wetted body surface as a whole term, $\int_{S_B} \phi_t n_i \mathrm{d}S$. This abstract extends the application of auxiliary function approach to the case of multiple bodies, which results in a coupled motion equation system.

To mathematically model an interconnection between two floating bodies, a constraint matrix method is incorporated into the existing time-domain potential flow model, where different types of interconnections are represented by proper constraint matrices depending on the system configuration.

2 Formulation

The schematic diagram in Fig. 1 defines a circular wave tank, with two bodies floating in the tank. A space-fixed global coordinate system $Oxyz$ and a body-fixed local coordinate system $O'x'y'z'$ are defined. The global origin O is placed on the undisturbed water surface at the center of the

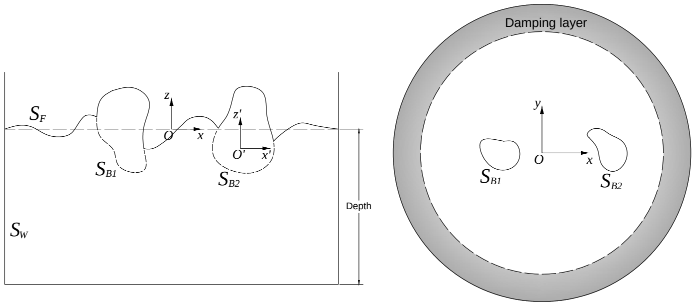


Figure 1: Definition of a circular numerical wave tank

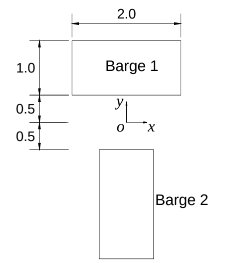


Figure 2: Two floating barges

circular tank with its z-axis pointing upward; the local origin O' is fixed at the center of gravity $\mathbf{X}_g = (x_g, y_g, z_g)$ of the corresponding body.

Within the framework of fully nonlinear potential flow theory, a time-domain numerical model was presented by Feng and Bai (2015). In order to incorporate the auxiliary function approach into that model, we define two sets of auxiliary functions, i.e. ψ_i $(i = 1, 2, \cdots, 6)$ for Body 1 and ψ_i $(i = 7, 8, \cdots, 12)$ for Body 2. Naturally we let the auxiliary functions satisfy the Laplace equation $\nabla^2\psi_i = 0$ in the fluid domain. The following boundary conditions are imposed.
On the free surface S_F: $\psi_i = 0$ $(i = 1, 2, \cdots, 12)$.
On the tank side wall S_W: $\frac{\partial\psi_i}{\partial n} = 0$ $(i = 1, 2, \cdots, 12)$.
On the surface of Body 1 S_{B1}: $\frac{\partial\psi_i}{\partial n} = n_{1,i}$ $(i = 1, 2, \cdots, 6)$, $\quad \frac{\partial\psi_i}{\partial n} = 0$ $(i = 7, 8, \cdots, 12)$.
On the surface of Body 2 S_{B2}: $\frac{\partial\psi_i}{\partial n} = 0$ $(i = 1, 2, \cdots, 6)$, $\quad \frac{\partial\psi_i}{\partial n} = n_{2,i-6}$ $(i = 7, 8, \cdots, 12)$,
where $n_{1,i}$ and $n_{2,i}$ are the components of the normal unit vector on Body 1 and Body 2 respectively.

The above mixed boundary value problems for the 12 auxiliary functions are solved simultaneously with the boundary value problem for the velocity potential. Green's identity constructs the relationship between ψ_i and ϕ_t, which reads

$$\iint_S \left(\phi_t \frac{\partial\psi_i}{\partial n} - \psi_i \frac{\partial\phi_t}{\partial n}\right) \mathrm{d}S = 0. \tag{1}$$

Boundary conditions for ϕ_t were presented in Wu and Eatock Taylor (1996). Substituting the boundary conditions for ψ_i and ϕ_t into Eq. (1) yields

$$\int_{S_{B1}} \phi_t n_{1,i} \mathrm{d}S = \mathit{\Gamma}_i^F + C_{ij}^{(1)} \cdot A_i^{(1)} + E_i^{(1)} + D_{ij}^{(1)} \cdot A_i^{(2)} + E_i^{(2)} \quad (i = 1, 2, \cdots, 6) \tag{2}$$

$$\int_{S_{B2}} \phi_t n_{2,i-6} \mathrm{d}S = \mathit{\Gamma}_i^F + C_{ij}^{(2)} \cdot A_{i-6}^{(2)} + E_i^{(1)} + D_{ij}^{(2)} \cdot A_{i-6}^{(1)} + E_i^{(2)} \quad (i = 7, 8, \cdots, 12) \tag{3}$$

where $A_i^{(1)}$ and $A_i^{(2)}$ are accelerations of Body 1 and Body 2, and

$$\mathit{\Gamma}_i^F = \int_{S_F} (gz + \frac{1}{2}\nabla\phi \cdot \nabla\phi) \cdot \frac{\partial\psi_i}{\partial n} \mathrm{d}S \quad (i = 1, 2, \cdots, 12) \tag{4}$$

$$C_{ij}^{(1)} = \int_{S_{B1}} \psi_i n_j \mathrm{d}S; \qquad D_{ij}^{(1)} = \int_{S_{B2}} \psi_i n_j \mathrm{d}S \quad (i, j = 1, 2, \cdots, 6) \tag{5}$$

$$C_{ij}^{(2)} = \int_{S_{B2}} \psi_i n_j \mathrm{d}S; \qquad D_{ij}^{(2)} = \int_{S_{B1}} \psi_i n_j \mathrm{d}S \quad (i = 7, 8, \cdots, 12; j = 1, 2, \cdots, 6) \tag{6}$$

$$E_i^{(1)} = \int_{S_{B1}} \psi_i \Theta^{(1)} \mathrm{d}S; \qquad E_i^{(2)} = \int_{S_{B2}} \psi_i \Theta^{(2)} \mathrm{d}S \quad (i = 1, 2, \cdots, 12) \tag{7}$$

$$\boldsymbol{\Theta}^{(k)} = -\dot{\boldsymbol{\xi}}^{(k)} \cdot \frac{\partial \nabla \phi}{\partial n} + \dot{\boldsymbol{\alpha}}^{(k)} \cdot \frac{\partial}{\partial n} \left[(\mathbf{X} - \mathbf{X}_g^{(k)}) \times (\dot{\boldsymbol{\xi}}^{(k)} - \nabla \phi) \right] \quad (k = 1, 2) \tag{8}$$

Note that the accelerations remain unknown. We now consider the motion equation for Body 1

$$M_{ij}^{(1)} A_i^{(1)} = -\rho \int_{S_{B1}} \phi_t n_{1,i} \mathrm{d}S - \rho \int_{S_{B1}} (gz + \frac{1}{2} \nabla \phi \cdot \nabla \phi) \cdot n_{1,i} \mathrm{d}S \tag{9}$$

where $M_{ij}^{(1)}$ is the mass matrix of Body 1. Substituting Eq. (2) into the motion equation Eq. (9) leads to the following in matrix form as

$$[\mathbf{M_1} + \mathbf{C_1}]\, \mathbf{A_1} + \mathbf{D_1} \mathbf{A_2} = \mathbf{Q_1}. \tag{10}$$

A similar motion equation for Body 2 can be obtained. Combining these two results in the following

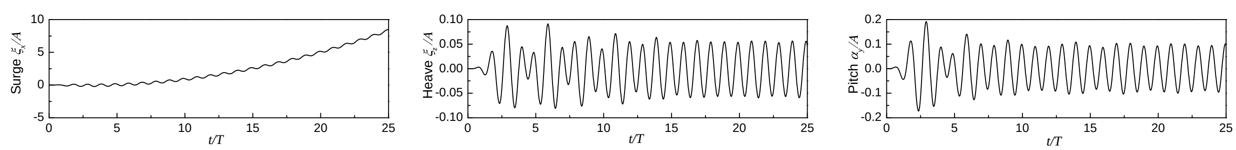
$$\begin{bmatrix} \mathbf{M_1} + \mathbf{C_1} & \mathbf{D_1} \\ \mathbf{D_2} & \mathbf{M_2} + \mathbf{C_2} \end{bmatrix} \begin{bmatrix} \mathbf{A_1} \\ \mathbf{A_2} \end{bmatrix} = \begin{bmatrix} \mathbf{Q_1} \\ \mathbf{Q_2} \end{bmatrix}. \tag{11}$$

In this way, the accelerations of Body 1 and Body 2 are solved, and the hydrodynamic forces are computed by substituting back the accelerations.

With interconnections, the bodies are constrained and additional unknown constraint forces at the connections appear. The motion equations need to be modified to include the constraint forces.

3 Numerical results

For demonstration and validation, we first consider two freely floating barges asymmetrically arranged with a relatively large spacing between them, as shown in Fig. 2. The two barges are 2.0 m long and 1.0 m wide, separated by a distance of 1 m. They have a draft of 0.3 m and the center of gravity of each barge is assumed to locate coincided with its center of buoyancy in still water, i.e. $z_g = -0.15$ m. The water depth is 3 m and the tank radius is four times the incident wave length. The barges are assumed to have a uniform mass distribution.



(a) Surge (b) Heave (c) Pitch

Figure 3: Motion responses of Barge 1 with a 0 degree incident wave of frequency $\omega = 6.0$ rad/s

Figure 3 plots typical time histories of motion responses of Barge 1 in a 0 degree incident wave of frequency $\omega = 6.0$ rad/s and amplitude $A = 0.005$ m. It is clear that there exists a drift component in the motions of surge while heave and pitch oscillate around zero. This is because there is no restoring force, resulting in drift effect in the directions of surge, sway and yaw, due to the mean wave forces. In order to compare the first order response with a linear model, FFT is performed to obtain each harmonic component. With the motion responses of heave, roll and pitch (no drift effect), we can directly perform the FFT to obtain their response spectra. As for surge, sway and yaw, we eliminate the drift effect by subtracting the displacement due to the mean acceleration and velocity, to recover the single-harmonic dominated response. Fig. 4 shows the comparisons of the first order motions of Barge 1 between the present model and the commercial software package HydroStar® over the frequency range 5-8 rad/s. The forces are normalized by the incident wave amplitude. The overall agreements are generally satisfactory. It has to been borne in mind that higher harmonics can also be captured using the present nonlinear model.

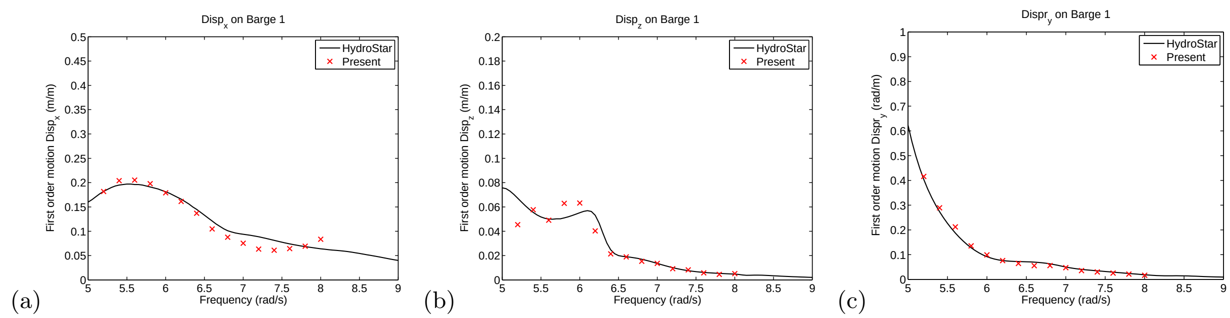


Figure 4: Comparisons of first order motions of Barge 1: (a) Surge, (b) Heave and (c) Pitch

A second case considered here is two tandem arranged barges connected by a rigid bar subject to a 0 degree wave of low steepness, as studied in Newman (1994). Fig. 5 shows the comparison of heave response with the results of Newman (1994). Our results are essentially very close to those of Newman (1994), which well validates the present model. More results will be presented in the workshop.

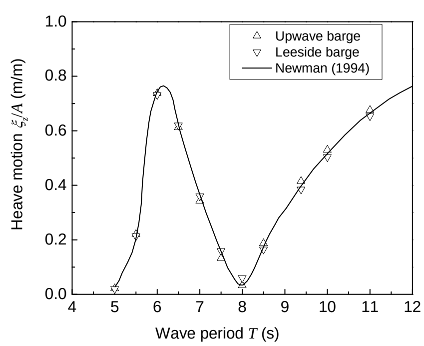


Figure 5: Comparison of heave response of rigidly connected barges

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