The Raked-Wedge Hull: A Severe Test of Linear Wave-Making Theory

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Summary

The idealized raked-wedge hull has a triangular waterplane with its base of finite beam at the transom stern and its apex of zero beam at the stem of the bow. The sections are rectangular and the draft increases linearly from zero at the stern to the maximum draft at the bow. This hullform provides a challenge for the traditional linear wave-making theory. Nevertheless, there is good agreement between the theory and the towing-tank data for the total resistance, provided that a reasonable value of the frictional form factor of 1.24 is chosen. It is also demonstrated that almost identical predictions are provided by simpler equivalent hulls — including distributions of line sources — which possess the same prismatic coefficient and the same local vertical centroid of the section area. Lastly, the calculations indicate that this new hull has a wave resistance which exceeds that of traditional vessels by generally more than 30%.

1 Introduction

A novel hull design has been recently considered as a basis for a patrol vessel, by Epstein, Heisler & Yancy (2013), Heisler, Franks & Montemarano (2014), Kelso, Pipkin, Rodriguez, Sawyer & Stover (2014) and Nichols, Holbert, Rubin & Yancy (2014). The waterplane area is triangular with its finite-width base at the transom. The draft varies from zero at the stern to the maximum draft at the stem in a linear fashion with respect to the distance from the stern. The sections are essentially rectangular but they possess a small degree of rounding at the otherwise sharp bilges. A split view of the towing-tank model is depicted in Figure 1(a) and the idealized hull is shown in Figure 1(b).

This unusual hullform is a challenge to the traditional thin-ship theory first developed by Michell (1898), because of its rectangular sections and its large beam at the stern — although the latter difficulty might be mitigated by the fact that the local draft approaches zero in that region.

2 Restricted-Water Wave Resistance

As the model was towed in a tank with a finite width w and filled to a finite depth d, we consider the modifications to the wave-resistance theory to include these two generalizations. Sretensky (1936) was the first to include channel effects for a displacement hull. Newman & Poole (1962) provided the earliest and most convenient formula for the wave resistance R_W associated with a traveling pressure distribution. This is easily modified to apply to a displacement hull with local beam $b(x,z)$. Their formula is re-expressed here as

$$R_W = \frac{\rho g}{\pi} \sum_{i=0}^{\infty}{}' \epsilon\, \Delta k_y\, k k_x^2 (\mathcal{U}^2 + \mathcal{V}^2) \Big/ \frac{\mathrm{d}f}{\mathrm{d}k}\,, \tag{1}$$

$$\epsilon = \begin{cases} 1/2 & \text{for } i = 0 \\ 1 & \text{for } i \geq 1 \end{cases}\,, \tag{2}$$

$$\mathcal{U} = \frac{\mathcal{P}^+ + \exp(-2kd)\mathcal{P}^-}{1 + \exp(-2kd)}\,, \tag{3}$$

$$\mathcal{V} = \frac{\mathcal{Q}^+ + \exp(-2kd)\mathcal{Q}^-}{1 + \exp(-2kd)}\,, \tag{4}$$

in which the deep-water Kochin functions are

$$\mathcal{P}^{\pm} + \mathrm{i}\mathcal{Q}^{\pm} = \int\limits_{S_0} b(x,z) \exp(\mathrm{i}k_x x \pm kz)\, \mathrm{d}S\,. \tag{5}$$

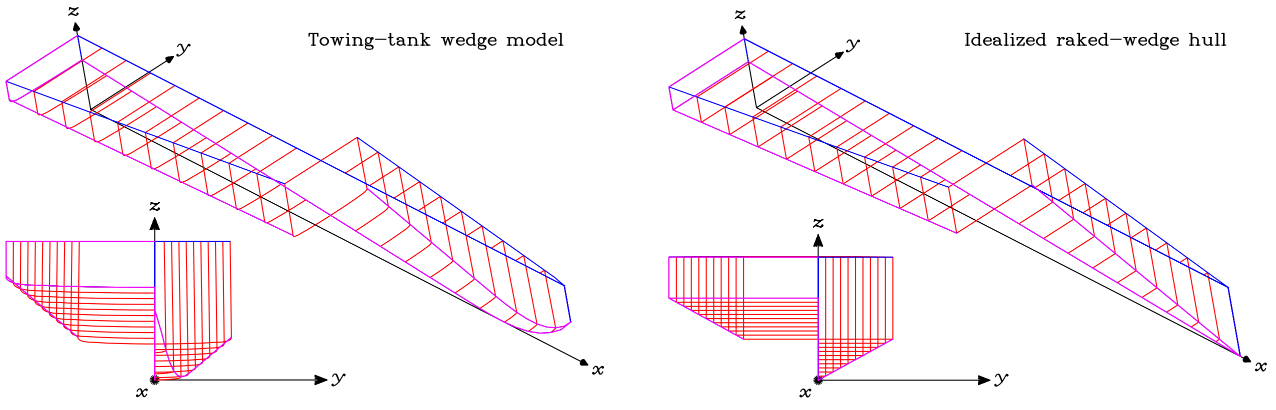
Here, ρ is the water density, g is the acceleration due to gravity, k_x is the longitudinal wave number, k_y is the transverse wave number, k is the circular wave number and ϵ is the Fourier series summation weight.

Equation 1 involves the dispersion relationship, defined by

$$\begin{aligned} f &= k^2 - k_0 k \tanh(kd) - k_y^2 \\ &= 0\,, \end{aligned} \tag{6}$$

together with its derivative

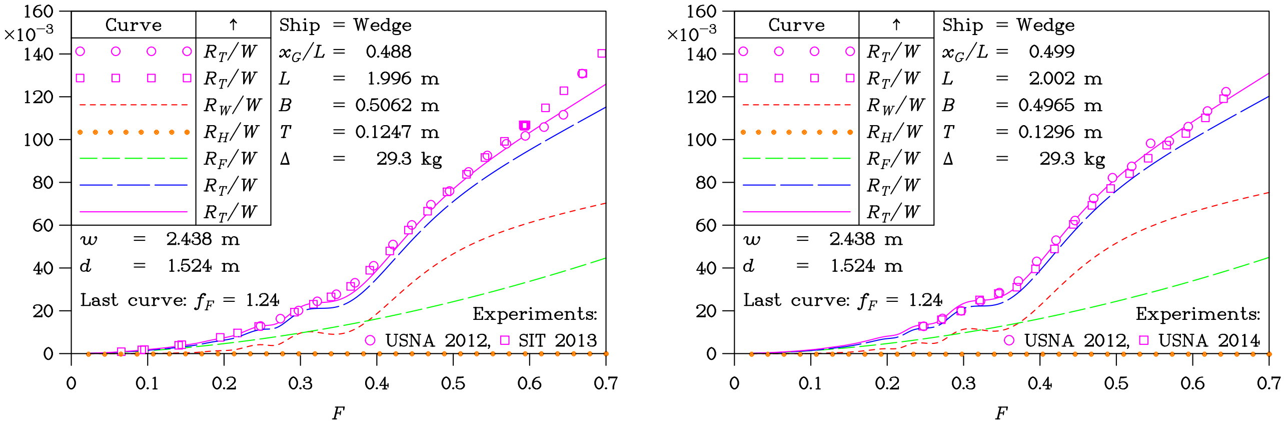
$$\frac{\mathrm{d}f}{\mathrm{d}k} = 2k - k_0 \tanh(kd) - k_0 kd\, \mathrm{sech}^2(kd)\,. \tag{7}$$



(a) Towing-Tank Wedge Model

(b) Idealized Raked-Wedge Hull

Figure 1: Raked-Wedge Hulls



(a) $x_G/L = 0.488$

(b) $x_G/L = 0.499$

Figure 2: Resistance Components

The transverse wave number k_y is given by

$$\Delta k_y = 2\pi/w \,, \tag{8}$$

$$k_y = i\Delta k_y \,, \tag{9}$$

in which i is the index for the wave component; this subscript, which should nominally appear on many of the variables in these equations, has been omitted for the sake of brevity. The prime $'$ on the summation in Equation 1 is used to indicate that the zeroth term (the transverse wave) is to be ignored when the depth Froude number $F_d = U/\sqrt{gd}$ exceeds unity. The circular wave number k is provided by the solution of Equation 6 and the longitudinal wave number is

$$k_x = \sqrt{k^2 - k_y^2} \,. \tag{10}$$

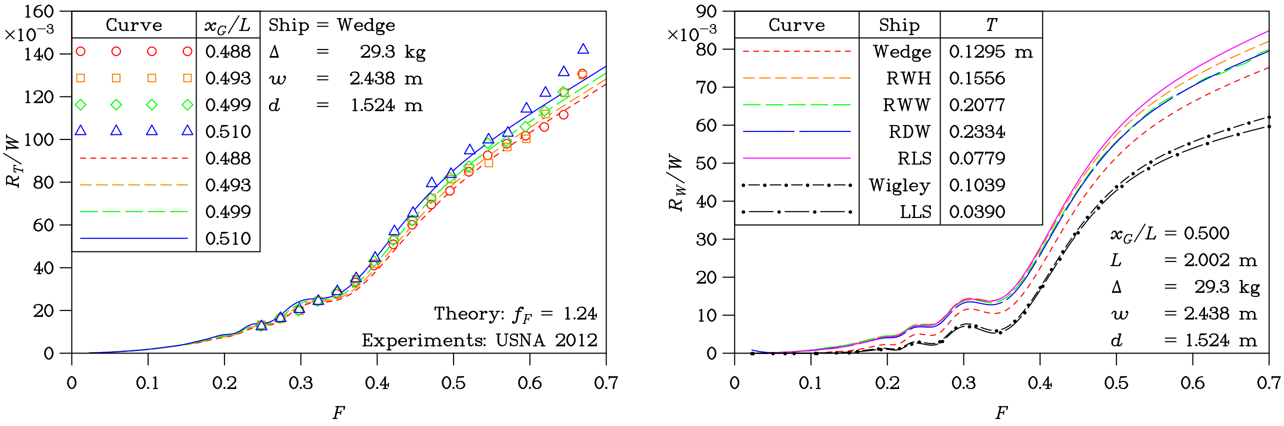
Lastly, the fundamental circular wave number is

$$k_0 = g/U^2 \,. \tag{11}$$

3 Resistance Experiments

The resistance experiments were conducted in the 36.6 m towing tank in the Hydromechanics Laboratory at the US Naval Academy in 2012 and 2014 and in the Davidson Laboratory at the Stevens Institute of Technology in 2013. Four different locations of the longitudinal center of gravity x_G ahead of the stern were studied by means of shifting the ballast weights in the model. Some of the results of the analysis are presented in Figure 2, in which the theoretical resistance components are plotted as well as the total resistance. The wave resistance R_W was computed using Equation 1 with the implementation of tent-function hull elements, as described by Doctors (2012, Equations 19 and 20).

The hydrostatic resistance R_H, due to the absence of water pressure on the partly or fully vented transom, is negligible because the model was closely ballasted to its ideal waterline. Recent relevant studies of transom



(a) Total Resistance

(b) Wave Resistance

Figure 3: Comparison of Resistance

flow include that of Maki, Doctors, Beck & Troesch (2005). The frictional resistance R_F was based on the 1957 International Towing Tank Committee (ITTC) formulation. The total resistance R_T was first obtained through a simple sum of these components, whereas the modified result was derived from the formula

$$R_T \quad = \quad R_W + R_H + f_F R_F \; , \tag{12}$$

in which a frictional-form factor of 1.24 was found to give a good fit to the experimental data. The weight W of the model has been employed to nondimensionalize all the resistance components.

The results for the total resistance for all four locations of the center of gravity are replotted in Figure 3(a), in which the theory reasonably predicts the small increase in total resistance as the center of gravity is shifted forward.

4 Equivalent Hullforms

Further light is shed on the hydrodynamics of the raked-wedge hull by considering the seven related hulls listed in Table 1. The models share common values of the displacement Δ, the length L, the prismatic coefficient C_P, the longitudinal center of gravity x_G and the vertical center of buoyancy z_B. The seven models also possess the same parabolic variation of the sectional-area curve.

The first five hulls additionally share a common geometric feature in that their local sectional vertical centroid is identical and it varies linearly from zero at the stern to the value −0.0900 m at the bow. The fifth "hull" is actually the slender-body line-source approximation. The results of the calculations plotted for these five hulls in Figure 3(b) illustrate that the wave resistance is essentially independent of the section shape, provided that the global hull descriptors are identical. The third and fourth of these hulls are illustrated in Figure 4.

The last two hulls listed in Table 1 represent a second numerical experiment in which the keels are level. The last "hull" is the slender-body line-source approximation. So their sectional vertical centroid does not vary with location.

The corresponding last two curves in Figure 3(b) are almost identical, indicating the reasonableness of the slender-body approximation. Furthermore, the wave resistance of these two level hulls is substantially lower than that of the five raked hulls, despite the fact that they possess the same vertical center of buoyancy.

5 Concluding Comments

The robustness and the effectiveness of the linear theory is proven for this unusual raked-wedge hull. The calculations indicate that a frictional-form factor of 1.24 is required, whereas predictions of the total resistance of a Wigley (1934) hull quoted by Doctors (2015, Section 14.3.7) show that a value of 1.12 is applicable. Hence the raked-wedge hull has a wave resistance which is typically higher by 30% and a frictional resistance which is higher by 11%.

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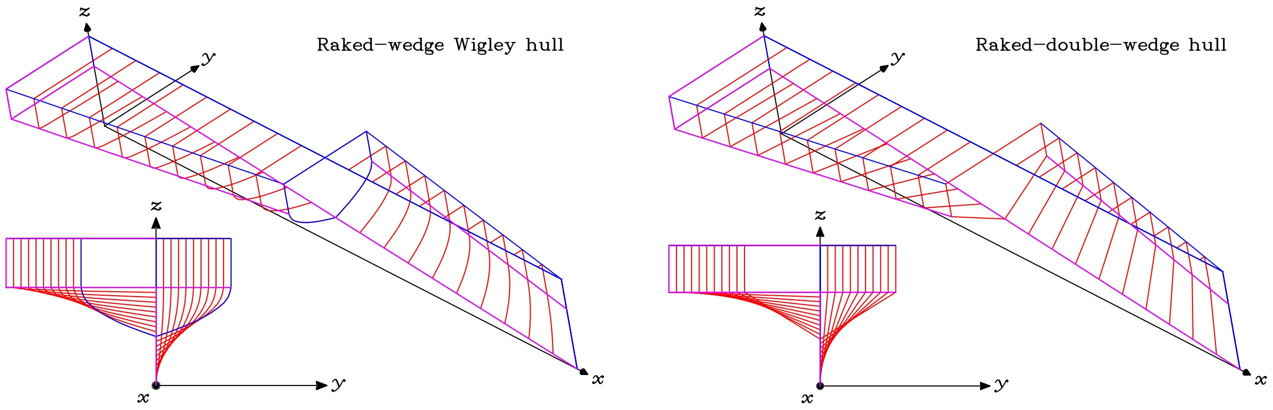

(a) Raked-Wedge Wigley Hull (b) Raked-Double-Wedge Hull

Figure 4: Equivalent Raked Hullforms

Table 1: Seven Hullforms

Full Name	Designation	Beam B (m)	Draft T (m)
Towing-tank wedge model	Wedge	0.4967	0.1295
Idealized raked-wedge hull	RWH	0.5651	0.1556
Raked-wedge Wigley hull	RWW	0.6345	0.2077
Raked-double-wedge hull	RDW	0.7522	0.2334
Raked-line-source theory	RLS	∞	0.0779
Level Wigley hull	Wigley	0.3179	0.1039
Level-line-source theory	LLS	∞	0.0390

Common values: $\Delta = 29.30$ kg, $L = 2.002$ m, $C_P = 2/3$
$x_G = 1.001$ m, $z_B = -0.0389$ m

7 References

Doctors, L. J. (2012). “Near-Field Hydrodynamics of a Surface-Effect Ship”, *J. Ship Research*, Vol. 56, No. 3, pp 183–196, September.

Doctors, L. J. (2015). *Hydrodynamics of High-Performance Marine Vessels*, Printed by CreateSpace, an Amazon.com Company, Charleston, South Carolina, Vol. 2, pp 401–836+ii, July.

Epstein, J., Heisler, E., & Yancy, T. (2013). “The Wedge Hull Patrol Vessel”, Naval Surface Warfare Center, Carderock Division, Center for Innovation in Ship Design, Technical Report NSWCCD-CISD-2013/009, 61+v pp, December.

Heisler, E., Franks, N., & Montemarano, D. (2014). “Wedge Hull: Feasibility of a Wedge Hull as a Coastal Patrol Vessel”, Stevens Institute of Technology, Davidson Laboratory, Senior Design Project, 50 pp, May.

Kelso, J., Pipkin, C., Rodriguez, J., Sawyer, A., & Stover, M. (2014). “Wedge Hull Tank Testing”, Naval Surface Warfare Center, Carderock Division, Center for Innovation in Ship Design, Technical Report NSWCCD-CISD-2013/006, 33+iv pp, September.

Maki, K. J., Doctors, L. J., Beck, R. F., & Troesch, A. (2005). “Transom-Stern Flow for High-Speed Craft”, *Proc. Eighth International Conference on Fast Sea Transportation (FAST ’05)*, Saint Petersburg, Russia, 8 pp, June.

Michell, J. H. (1898). “The Wave Resistance of a Ship”, *Philosophical Magazine*, Series 5, Vol. 45, No. 272, pp 106–123, January.

Newman, J. N. & Poole, F. A. P. (1962). “The Wave Resistance of a Moving Pressure Distribution in a Canal”, *Schiffstechnik*, Vol. 9, No. 45, pp 21–26.

Nichols, N., Holbert, J., Rubin, R., & Yancy, T. (2014). “Wedge Hull Air Lubrication Testing”, Naval Surface Warfare Center, Carderock Division, Center for Innovation in Ship Design, Technical Report NSWCCD-CISD-2014/002, 49+iv pp, October.

Sretensky, L. N. (1936). “On the Wave-Making Resistance of a Ship Moving along in a Canal”, *Philosophical Magazine*, Series 7, Supplement, Vol. 22, No. 150, pp 1005–1013, November.

Wigley, W. C. S. (1934). “A Comparison of Experiment and Calculated Wave-Profiles and Wave-Resistances for a Form Having Parabolic Waterlines”, *Proc. Royal Society of London*, Series A, Vol. 144, No. 851, pp 144–159 + 4 plates, March.