Towards a higher-order spectral force model for vertical circular cylinders

H. Bredmose and S. J. Andersen

DTU Wind Energy, Denmark, hbre@dtu.dk

1 Introduction

Accurate and efficient computation of diffraction loads is a classical topic in Engineering design of offshore structures. In the linear case diffraction generally reduces the wave forces. Nonlinear diffraction, however, can be significant for slender structures, where higher-harmonic forcing can lead to ringing.

Figure 1(left) shows an example of such a flow from experiments at DHI within the Wave Loads project (Bredmose et al., 2013). The incident wave travels towards left. Small diffracted waves are seen at the free surface around the cylinder. The flow is gentle enough to allow a potential flow approach, but in this and steeper cases, linear methods may be questionable for a detailed flow and force description. In the present paper we pursue a force model that includes diffraction and (almost) full nonlinearity by application of the higher-order spectral method in a small domain around the cylinder. Inspired by the work of Ducrozet et al. (2014), we pursue a decoupled treatment of the incident and diffracted wave fields. This enables accurate pre-computation of the incident wave transformation. Paulsen et al. (2014) demonstrated that the size of the interaction domain between the incident and diffracted wave field can be very small.

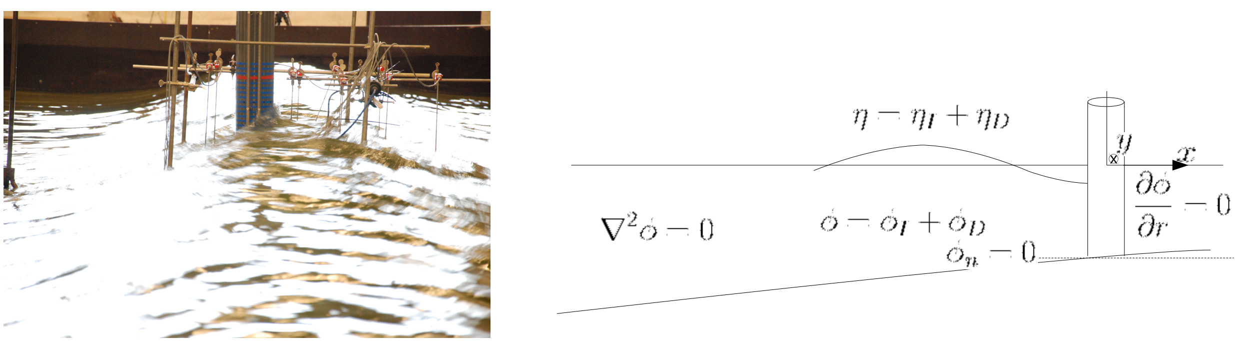

Figure 1: Left: Wave incident at a vertical cylinder. Right definition sketch for the diffraction problem with a large domain for incident wave computation and a smaller flat-bed domain for the diffraction computation.

2 Governing equations and choice of spectral basis

We consider the domain sketched in figure 1(right) where a Cartesian coordinate system is placed with the z-axis pointing upwards from the still water level and the x-axis pointing in the direction of the waves. The theory is general enough to allow incident waves from any or many simultaneous directions. While the incident wave field may exist at varying depth, we approximate the diffracted wave field by its constant depth solution. The free surface elevation η and the velocity potential at the free surface $\tilde{\phi}$ satisfies the kinematic and dynamic free surface conditions

$$\eta_t = \left[1 + \epsilon^2 \left(\nabla\eta\right)^2\right] \tilde{\phi}_z - \epsilon \nabla\tilde{\phi} \cdot \nabla\eta \qquad z = \eta \tag{1}$$

$$\tilde{\phi}_t = -g\eta - \tfrac{1}{2}\epsilon \left[\left(\nabla\tilde{\phi}\right)^2 - \tilde{\phi}_z^2 \left(1 + \epsilon^2 \left(\nabla\eta\right)^2\right)\right] \qquad z = \eta \tag{2}$$

where g is the acceleration of gravity and ϵ is a small ordering parameter. The velocity potential satisfies the Laplace equation in the fluid domain and impermeability conditions on the sea bed and cylinder wall. We further decompose the wave field into an incident and diffracted part

$$\begin{pmatrix}\eta\\ \phi\end{pmatrix} = \begin{pmatrix}\eta\\ \phi\end{pmatrix}_I + \begin{pmatrix}\eta\\ \phi\end{pmatrix}_{D1} + \begin{pmatrix}\eta\\ \phi\end{pmatrix}_{D2} \tag{3}$$

where 'D1' denotes the linear diffraction solution and 'D2' the nonlinear diffraction solution. The sum of these diffracted fields must satisfy the Somerfeld radiation condition.

Following Bonnefoy et al. (2006)(IWWWFB), we apply the higher-order spectral method in a cylindrical domain. Our aim, however is the calculation of the 'D2' field only, since the 'I' field is known and the 'D1' field can be constructed directly from it. We thus expand η and ϕ in cylindrical coordinates (r, θ, z) as follows

$$\eta(\mathbf{x}, t) = \sum_{p=0}^{P}\sum_{n=1}^{N} (A_{pn}(t) + B_{pn}(t)) \left[J_p(k_n r) - \frac{J_p'(k_n r_0)}{H_p^{(1)'}(k_n r_0)} H_p(k_n r)\right] \exp(ip\theta) + \text{c.c.} \tag{4}$$

$$\phi(\mathbf{x}, z, t) = \sum_{p=0}^{P}\sum_{n=1}^{N} (C_{pn}(t) + D_{pn}(t)) \left[J_p(k_n r) - \frac{J_p'(k_n r_0)}{H_p^{(1)'}(k_n r_0)} H_p(k_n r)\right] F_{pn}(k_{pn} z) \exp(ip\theta) + \text{c.c.} \tag{5}$$

Here $F_{pn}(k_{pn}z) = \cosh(k_{pn}h(z + h))/\cosh(k_{pn}h)$, $H_p^1(k_n r)$ is the Hankel function of the first kind and the primes denote differentiation with respect to r. This expansion satisfies (**??**)–(**??**). Further, the incident wave field and its linear diffraction solution $(\eta, \phi)_I + (\eta, \phi)_{D1}$ are represented by A_{pn}, C_{pn}, while the remaining nonlinear fields $(\eta, \phi)_{D2}$ are represented by B_{pn}, D_{pn}.

3 Splitting, boundary conditions and the higher-order spectral method

Due to the nonlinearity in the free surface conditions, the solution for the D2 field cannot be computed separately. We adopt the approach of Ducrozet et al. (2014), extended to inclusion of the lateral boundary conditions through application of a linear operator.

We introduce the notation $\mathbf{v} = (\eta, \phi)$ and write (1)–(2) as $\mathbf{v}_t = \mathcal{RHS}(\mathbf{v})$. Next we observe that expansion of the incident wave field in the J_p basis of (4)–(5) yields the (A_{pn}, B_{pn}) coefficients at any time. Addition of the H_{pn} terms and back-transformation to the physical space establishes the linear diffraction solution. This full operation can be represented by a linear operator, such that $\mathbf{v}_{I+D1} = \mathcal{L}_{BC}\mathbf{v}_I$. This relation also applies to the time derivative of $\mathbf{v}$. Hereby (1)–(2) become

$$\mathbf{v}_{D2,t} = \mathcal{RHS}\left(\mathbf{v}_{D2} + \mathcal{L}_{BC}\mathbf{v}_I\right) - \mathcal{L}_{BC}\mathbf{v}_{I,t}. \tag{6}$$

In this approach, the D2 solution can thus be time stepped with application of the known incident wave field as driving source term. The key here is that the nonlinear interaction terms are calculated with the full wave field and therefore leaves no need for explicit splitting into separate products of the incident and diffracted fields.

While the linear solution of (1)–(2) is explicitly given as $\mathcal{L}_{BC}\mathbf{v}_I$, solution of the full equations requires evaluation of the vertical velocity $\tilde{w} = \tilde{\phi}_z$ at the free surface. The straight-forward calculation of ϕ_z at $z = 0$ through (5), however, can be utilized to achieve an almost fully nonlinear evaluation of $\tilde{\phi}_z$. The approach was pioneered by West et al. (1987); Dommermuth & Yue (1987) among others. From a formal Stokes expansion of the velocity potential, $\epsilon\phi = \sum_{m=1}^{M} \epsilon^m \phi^{(m)}$, and subsequent Taylor expansion from $z = 0$, the surface potential can be written

$$\epsilon\tilde{\phi} = \sum_{m=1}^{M} \sum_{n=0}^{m-1} \epsilon^m \frac{\eta^n}{n!} \left(\frac{\partial}{\partial z}\right)^n \phi_0^{m-n} \tag{7}$$

where $\phi_0 = \phi|_{z=0}$. Thus by repeated solution of (7) through increased orders of ϵ, $\phi_0^{(1...M)}$ can be computed to yield $\tilde{\phi}_z$ at any desired order of accuracy. Hereby (6) can be solved with almost full nonlinearity to yield the nonlinear part of the diffraction solution.

4 First results and next steps

Results from the model and its development is shown in figure 2. The top row shows spatial snapshots of linear diffraction obtained with the solver for $h = 25$ m, $r_0 = 3$ m, $k_{\text{inci}} = 5.3 \cdot 10^{-2}$ m^{-1} and $H = 1$ m. The total free surface elevation field is shown in the left panel, while the diffracted 'D1' field is shown in the right panel. This is compared at $\theta = 0$ to the MacCamy-Fuchs solution in the middle right panel. We see that the spectrally constructed linear diffraction field matches the analytical solution well except for some high-frequency oscillations that appear to emerge from the outer boundary. The middle left panel shows a comparison of $\phi_{I,z}$ from the spectral solver and the analytical solution for the total wave field of incident and diffracted waves. An overall good match is seen, although with some visible overlaid oscillations. To overcome these oscillations and any effect from them on the cylinder force, a spectral absorption scheme has been developed (bottom left). This is compared to absorption by simple damping for a case of an axisymmetric wave group interacting with the cylinder (bottom right). The absorption scheme is seen to succesfully absorb the outward propagating waves with no reflection in the in-ward direction from the outer boundary. Elimination of the high-frequency oscillations with the spectral absorption scheme is part of present work. Further, the nonlinear solver for the 'D2' field has been implemented and is subject to ongoing tests. Results of this and comparison to known nonlinear solutions will be presented at the workshop.

The work was partly carried out in the DIMSELO project, which is a Knowledge-building Project for Industry funded by the Norwegian Research Council (NRC) under the ENERGIX program. The project is also funded by its industry partners Statoil and Statkraft. The projects research partners are IFE, NTNU and DTU.

References

Bonnefoy, F., Taylor, R. E., Taylor, P. H. & Ferrant, P. (2006), A high order spectral model for wave interaction with a bottom mounted cylinder, *in* 'Proc. 21st Int. Workshop Water Waves Floating Bodies, Loughborough, UK'.

Bredmose, H., Mariegaard, J., Paulsen, B. T., Jensen, B., Schlöer, S., Larsen, T. J., Kim, T. & Hansen, A. M. (2013), The wave loads project, Technical Report Report E-45, ISBN: 978-87 92896-77-3, DTU Wind Energy. Final report for the Danish ForskEL Wave Loads project.

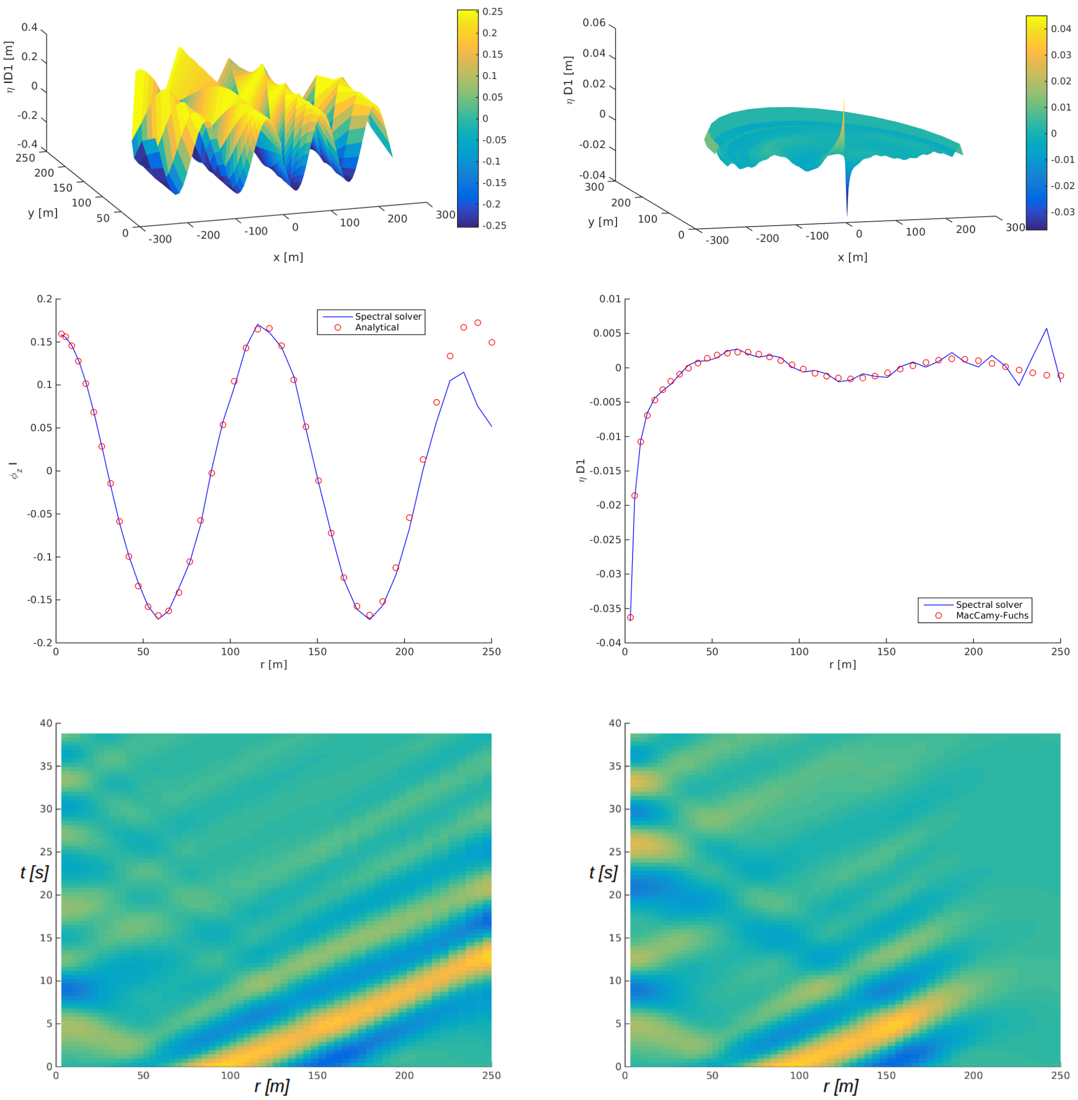

Figure 2: Top row: Comparison to MacCamy-Fuchs theory for free surface elevation of total wave field (left) and linear diffraction field (right). Middle row: Comparison at $\theta = 0$ of $\partial\phi/\partial z$ for the total wave field (left) and the linear diffracted field free surface elevation (right). Bottom row: Test of spectral absorption technique (left) and standard damping of the total wave field (right).

Dommermuth, D. G. & Yue, D. K. P. (1987), 'A high-order spectral method for the study of nonlinear gravity waves', *J. Fluid Mech.* **184**, 267–288.

Ducrozet, G., Engsig-Karup, A. P. & Bingham, H. B. (2014), 'A nonlinear wave decomposition model for efficient wave-structure interaction. Part A: Formulation, validation and analysis', *J. Comp. Physics* **257**, 863–883.

Paulsen, B. T., Bredmose, H. & Bingham, H. B. (2014), 'An efficient domain decomposition strategy for wave loads on surface piercing circular cylinders', *Coastal Engng.* **86**, 57–76.

West, B. J., Brueckner, K. A., Janada, R. S., Milder, D. M. & Milton, R. L. (1987), 'A new numerical method for surface hydrodynamics', *J. Geophys. Res.* **92**(C11), 11803–11824.