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Wave interaction with multiple flexible-porous barriers near a rigid wall

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Highlights

- The study deals with oblique wave interaction with multiple bottom-standing flexible porous barriers near a rigid wall in the presence of slopping step.
- The physical problem is solved using the least squares approximation method along with the multi-mode approximation method associated with the modified mild-slope equation.
- Effects of slopping step, number of barriers, structural rigidity, compressive force, angle of incidence, barrier length, porosity and height of the step are examined.
- The study reveals that for various combinations of bed profile, wave and structural parameters, nearly zero and full reflection occurs.
- The present study is likely to be helpful for ocean engineers to design of perforated breakwater to create clam region near sea wall, port and harbor wall.

1. Introduction

Recently, perforated breakwaters have been introduced into the art of reducing wave reflection and wave run-up in front of the structures to protect ports and operation areas for ship loading and unloading. Jarnal (1961) was initially proposed a perforated wall breakwater consisting of a front porous wall, a rigid back wall and a wave absorbing chamber between them. Since then, Jarlan type breakwater has been received considerable attentions due to its significant effect on the reduction of the wave reflection and wave force. A through review of the developments on wave interaction with various perforated breakwater can be found in Huang et al. (2011) and the literature cited therein. Due to light in weight, economical, reusable and environmental friendly, vertical fully/partial flexible porous barriers are preferred in many situations as a perforated breakwater. Behera et al. (2013) investigated the problem of wave trapping by a flexible porous barrier near a rigid wall. Although wave interaction with single/multiple fully/partial porous barriers in uniform bottom bed has been well studied in the literature, wave interaction with perforated breakwater consists of single/multiple fully/partial flexible porous barriers in the presence of step type bottom bed has not received attention to the best of the knowledge of the authors. Therefore, in the present study, wave interaction with multiple bottom-standing flexible barriers near a rigid wall is studied in the presence of slopping step.

2. Mathematical formulation

Oblique wave interaction with multiple bottom-standing flexible porous barriers is studied in the presence of slopping step under the assumptions of linearized water wave theory and small amplitude structural response. The problem is considered in the three-dimensional Cartesian co-ordinate system with xy being the horizontal plane and the z-axis being vertically upward. It is considered that the slopping step occupies the region $0 < x < L$ with variable depth $h(x)$, and the uniform open water regions $-\infty < x < 0$ and $L < x < \infty$ with water depth h_1 and h_2 respectively. It is assumed that N number of bottom-standing barriers are placed at $x = \alpha_j$, where $\alpha_j = L + L_1 + (j-3)L_2$ for $j = 3, 4, ..., (N+2)$. First barrier is located at a distance L_1 from end edge of the slopping step, L_2 is the spacing between each barrier and D is the distance between last barrier and rigid wall as shown in Figure 1. Assuming that the fluid is inviscid, incompressible, and the motion is irrotational and simple harmonic in time with angular frequency ω. The fluid is assumed to be extended horizontally along the $y-$axis over $y < \infty$. Obliquely incident wave over slopping bed is a quasi-3D problem. If the the bottom changes only in the x-direction, the wave component does not change its wave number in the y-direction. Thus, the form of the velocity potential for $j = 1, 2, 3, ..., (N+3)$ is given by

$\Phi_j(x,y,z,t) = \text{Re}\{\phi_j(x,z)\text{e}^{-\text{i}(k_y y+\omega t)}\}$, where θ is the incidence angle with x-axis and $k_y = k_{10}\sin\theta$ with k_{10} being the wave number of the incident wave in region 1. Along the vertical $z-$direction, $H_b = (-h_2, -h_2+b)$ and $H_g = (-h_2+b, 0)$ are denoted as the barrier and gap regions respectively with b is the length of the barriers. The spatial velocity potential $\phi_j(x,z)$ for $j = 1,2,3,...,(N+3)$

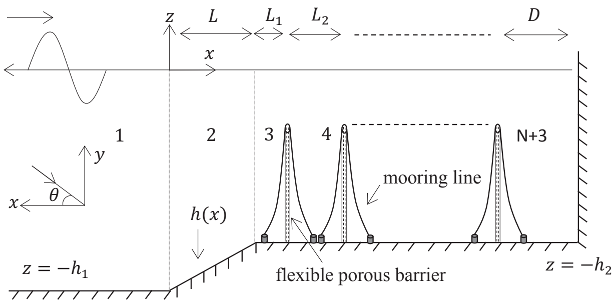


Figure 1: Wave interaction with multiple flexible porous barriers near a rigid wall.

satisfies the Helmholtz equation given by

$$\left(\nabla^2_{xz} - k_y^2\right)\phi_j = 0, \tag{1}$$

where $\nabla^2_{xz} = (\partial^2/\partial x^2 + \partial^2/\partial z^2)$. The linearized free surface boundary condition is given by

$$\partial\phi_j/\partial z - K\phi_j = 0 \quad \text{on} \;\; z = 0, \;\text{for}\; j = 1,2,3,...,(N+3), \tag{2}$$

where $K = \omega^2/g$ and g is the acceleration due to gravity. Further, the uniform rigid bottom boundary condition is given by

$$\partial\phi_j/\partial z = 0 \quad \text{on} \quad z = -h_i, \tag{3}$$

where $i = 1$ for $j = 1$ whilst, $i = 2$ for $j = 3,4,...,(N+3)$. On the other hand, bottom boundary condition for slopping step region 2 on $z = -h(x)$ is given by

$$\partial\phi_2/\partial z + (dh/dx)(\partial\phi_2/\partial x) = 0. \tag{4}$$

The continuity of pressure and normal velocity along the gap at $x = \alpha_j$ is given by

$$\phi_j = \phi_{j+1}, \quad \partial\phi_j/\partial x = \partial\phi_{j+1}/\partial x, \quad z \in H_g, \tag{5}$$

where $j = 3,4,...,(N+2)$. The flexible barriers are assumed to be oscillating in the horizontal direction with displacement of the form $\zeta_{(j-2)}(y,z,t) = \text{Re}\{\xi_{(j-2)}(z)e^{-\text{i}(k_y y-\omega t)}\}$, where $\xi_{(j-2)}(z)$ for $j = 3,4,...,(N+2)$ are the complex deflection amplitudes of the flexible barriers. Thus, the boundary condition on the flexible porous barriers at $x = \alpha_j$ for $j = 3,4,...,(N+2)$ is given by

$$\partial\phi_j/\partial x = \text{i}k_{10}G(\phi_j - \phi_{j+1}) - \text{i}\omega\xi_{(j-2)}, \quad z \in H_b, \tag{6}$$

where G being the complex porous-effect parameter as in Yip et al. (2002). The equation of motion of the barrier at $x = \alpha_j$, $j = 3,4,...,(N+2)$ acted upon by fluid pressure yields

$$EI(D^2 - k_y^2)^2\xi_{(j-2)} + Q(D^2 - k_y^2)\xi_{(j-2)} - m_s\omega^2\xi_{(j-2)} = \text{i}\rho\omega(\phi_j - \phi_{j+1}), \quad z \in H_b, \tag{7}$$

where $EI = E^3 d_s/12(1-\nu^2)$ is the rigidity of the barriers, E is the Young's modulus, d_s is the thickness of the barrier, ν is the Poisson's ratio, Q is the uniform compressive force acting on the barrier, $m_s = \rho_s d_s$ is the uniform mass per unit length with ρ_s being barrier density and ρ_s is the density of water. To keep the barrier in position and for the unique solution of the BVP, the barriers are assumed to be clamped near sea bed at $(\alpha_j, -h_2)$ and moored near the submerged end at $(\alpha_j, -h_2+b)$ for $j = 3, 4, .., (N+2)$. Thus, clamped-moored edge conditions are given by

$$\xi_{(j-2)}(u) = 0,\ \xi'_{(j-2)}(u) = 0, \tag{8}$$

$$(D^2 - \nu k_y^2)\xi_{(j-2)}(u) = 0, \quad [EI\{D^2 - (2-\nu)k_y^2\}D + QD]\xi_{(j-2)}(u) = 2K_m \sin^2\sigma_m \xi_{(j-2)}(u), \tag{9}$$

where K_m = mooring line stiffness and σ_m = the mooring line angle in the static position, u being $-h_2 + b, -h_2$ as appropriate.

3. Method of Solution

Using the expansion formulae, the form of the special velocity potentials $\phi_j(x, z)$ for $j = 1, 2, 3, ..., (N+3)$ in each regions are expressed as

$$\phi_j = \begin{cases} A_{10}e^{\mathrm{i}p_{10}x} f_{10}(k_{10}, z) + \sum_{n=0}^{\infty} B_{1n}e^{-\mathrm{i}p_{1n}x} f_{1n}(k_{1n}, z), & x < (\alpha_1 = 0), j = 1, \\ \sum_{n=0}^{\infty} \psi_n(x)\ W_n(h(x), z), & \alpha_1 < x < (\alpha_2 = L), j = 2, \\ \sum_{n=0}^{\infty} (A_{jn}e^{\mathrm{i}p_{2n}x} + B_{jn}e^{-\mathrm{i}p_{2n}x}) f_{2n}(k_{2n}, z), & \alpha_{(j-1)} < x < \alpha_j, j = 3, 4, ..., (N+2), \\ \sum_{n=0}^{\infty} B_{jn} \cos p_{2n}(x - M_3) f_{2n}(k_{2n}, z), & \alpha_{(N+2)} < x < \alpha_{(N+3)}, j = N+3, \end{cases} \tag{10}$$

where $f_{in}(k_{in}, z) = \cosh k_{in}(z + h_i)/\cosh k_{in}h_i$ and $p_{in} = \sqrt{k_{in}^2 - k_y^2}$ for $i = 1, 2$ with k_{10} and k_{20} are the positive real roots and k_{in} for $n = 1, 2, 3, \cdots$ are the purely imaginary roots of the dispersion equation $\omega^2 = gk_{in} \tanh k_{in}h_i$ in k_{in} for $i = 1, 2$. Further, in the sloping step region, $\psi_n(x)$s are unknown functions and $W_n = \cosh \tilde{k}_n(z + h)/\cosh \tilde{k}_n h$ with the wave number $\tilde{k}_0$ is a positive real root and $\tilde{k}_1, \tilde{k}_2, \tilde{k}_3, \cdots$, are purely imaginary roots of the dispersion equation $\omega^2 = g\tilde{k} \tanh \tilde{k}h$ in $\tilde{k}$. It may be noted that the roots $\tilde{k}_0, \tilde{k}_1, \tilde{k}_2, \tilde{k}_3$... are functions of bottom profile $h(x)$. A_{jn} and B_{jn} are the unknown constants to be determined. Using the procedure for extended modified mild-slope equation (MMSE) as in Porter and Staziker (1995) to obtain $\psi_n(x)$ in undulated region, it is derived that

$$\frac{d}{dx}\left(a_n \frac{d\psi_n}{dx}\right) + \sum_{m=0}^{\mathbb{N}} \left[\left(b_{mn} - b_{nm}\right)\frac{dh}{dx}\frac{d\psi_m}{dx} + \left\{b_{mn}\frac{d^2h}{dx^2} + c_{mn}\left(\frac{dh}{dx}\right)^2 + d_{mn} - k_y^2 a_n\right\}\psi_m\right] = 0, \tag{11}$$

where the form of $a_n(h)$, $b_{mn}(h)$ $c_{mn}(h)$ and $d_{mn}(h)$ for $n = 0, 1, 2, \cdots, \mathbb{N}$ being same as given in Porter and Staziker (1995). Using the velocity potential as in Eq. (10) and continuity of pressure across the interfaces $x = 0$ and $x = L$ in the jump conditions as in Porter and Staziker (1995) yield

$$\left.\begin{aligned} &a_0\frac{d\psi_0}{dx} + \mathrm{i}p_{10}a_0\psi_0 + h' \sum_{m=0}^{\mathbb{N}} b_{m0}\psi_m - 2\mathrm{i}p_{10}a_0A_{10} = 0, \\ &a_n\frac{d\psi_n}{dx} + \mathrm{i}p_{1n}a_n\psi_n + h' \sum_{m=0}^{\mathbb{N}} b_{mn}\psi_m = 0, \end{aligned}\right\} \text{at } x = 0+,\ n = 1, 2, \cdots, \mathbb{N}, \tag{12}$$

$$a_n\frac{d\psi_n}{dx} - \mathrm{i}p_{2n}a_n\psi_n + h' \sum_{m=0}^{\mathbb{N}} b_{mn}\psi_m - 2\mathrm{i}a_n p_{2n}B_{3n}e^{-\mathrm{i}p_{2n}x} = 0 \ \text{ at } x = L-,\ n = 0, 1, 2, \cdots, \mathbb{N}. \tag{13}$$

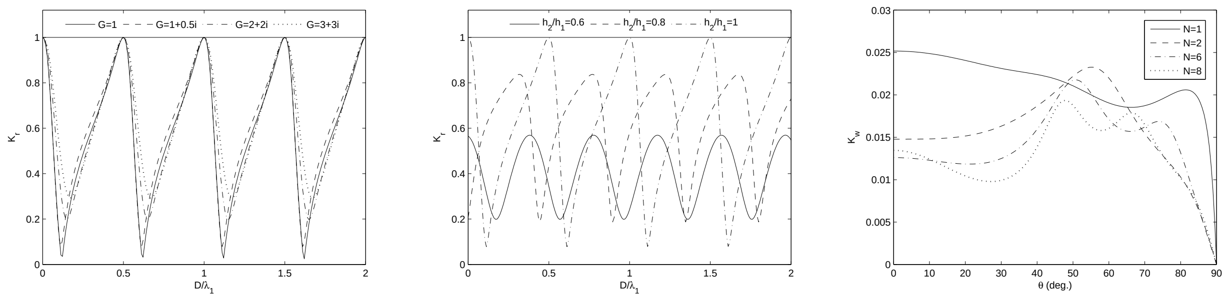
Using Eqs. (5) and (10) in Eq. (7) the plate deflections $\xi_{(j-2)}$ for $j = 3, 4, ..., (N+2)$ are obtained as

$$\xi_{(j-2)}(z) = \sum_{m=1}^{4} \tilde{C}_{jm}\tilde{f}_m(z) + \sum_{n=0}^{\infty}(U_{jn}\tilde{A}_{jn} + V_{jn}\tilde{B}_{jn}) f_{2n}(k_{2n}, z), \quad z \in H_b, \tag{14}$$

where $\tilde{A}_{jn}$, $\tilde{B}_{jn}$, U_{jn}, V_{jm}, e_n and s_n are the known constants obtained after using Eqs. (5) and (10) in Eq. (7). Using the form of $\tilde{f}_m(z)$ and suitable application of the least square approximation method as given in Behera et al. (2013), yields a system of equations. Further, to find ψ_n, modified mild-slope Eq. (11) is solved numerically by using Runge-Kutta method. Finally, using the required edge conditions as in Eq. (8) along with the computed all system of equations are solved.

4. Numerical Results and Discussion

In the present study, time period $T = 8$ sec, acceleration due to gravity $g = 9.81$ m/sec^2, depth ratio $h_2/h_1 = 0.6$, $L/h_1 = 0.25$, $L_1/\lambda_1 = 0.5$, $L_2/\lambda_1 = 0.5$, $D/\lambda_1 = 0.25$, $\lambda_1 = 2\pi/k_{10}$, $\gamma = EI/(\rho g h_2^4) = 0.1$, $\tau = Q/(\rho g h_2^2) = 0.1$, $\upsilon = m_s/(\rho h_2) = 0.1$, $\nu = 0.3$, $a/h_2 = b/h_2 = 0.5$, $\sigma_m = 45°$, $K_m = 10^3$ N m^{-1}, number of barrier $N = 8$ and $\theta = 30°$ are kept fixed unless it is mentioned. The reflection coefficient is denoted by $K_r = |B_{10}/A_{10}|$ and the slopping step bed profile is considered using the bed function $h(x) = h_1 - \tilde{b}x/L$ with $\tilde{b} = h_1 - h_2$.



(a) (b) (c)

Figure 2: Variation of (a) K_r versus D/λ_1 for different values of G with $h_2/h_1 = 1$ and $\theta = 0°$ (b) K_r versus D/λ_1 for different values of h_2/h_1 with $G = 1 + 0.5$i and $\theta = 0°$ (c) K_w versus θ for different values of N with $h_2/h_1 = 0.6$.

From Figures 2(a) and (b), it is found that full and nearly zero reflection occur periodically as D/λ_1 increases for $h_2/h_1 = 1$. The nearly zero reflection is referred as nearly full wave trapping in the confined region as in Yip et al. (2002). Figure 2(a) depicts that the wave reflection increases with an increase in the absolute value of the porous-effect parameter G. This is due to the transmission of more wave energy by the flexible porous barrier with an increase in the absolute value of G. Figure 2(b) shows that the amplitude of the oscillatory patterns of the reflection coefficient decreases with increase in h_2/h_1. Further, from Figure 2(c), it is observed that non-dimensional horizontal force K_w on the rigid wall decreases with an increase in number of barriers N which is due to the dissipation of more wave energy. Various other results will be presented in the workshop.

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