Wave-interference and wave-breaking effects on the Kelvin wakes of high-speed monohull ships and catamarans

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Highlight

Wave-interference and wave-breaking effects on the wave pattern of a ship that advances at constant speed along a straight path in calm water are considered. Realistic numerical computations, based on the Neumann-Michell theory or the related Hogner approximation, for seven ship hulls that correspond to broad ranges of main hull-shape parameters (beam/length, draft/length, beam/draft, waterline entrance angle), show that the apparent wake angle ψ_{max} where the largest waves created by a ship (monohull or catamaran) are found (at high Froude numbers) is only weakly influenced by the hull shape, and moreover can be well approximated by simple analytical relations. These relations provide useful relatively-accurate practical estimates (without computations) of the apparent wake angle ψ_{max} for *general* monohull ships and catamarans at any Froude number. Furthermore, elementary considerations suggest that wave-breaking effects are significant and result in a *lower* bound $\psi_{min} < |\psi|$. This lower bound complements Kelvin's classical upper bound $|\psi| \le \psi_K \approx 19°28'$ and the more precise high-Froude-number upper bound $|\psi| \le \psi_{max} \le \psi_K$ related to interference between divergent waves.

1. Simple analytical models of far-field ship wave patterns

• 1.1 The 1-point wavemaker approximation and Kelvin's analysis

The simplest analysis of the wave pattern of a ship was given by Kelvin in 1887. A ship is approximated as a 1-point wavemaker in this classical analysis. Although particularly crude, the 1-point approximation is sufficient to determine essential features of the far-field waves created by a ship. In particular, Kelvin showed that ship waves can only be found inside a wedge $|\psi| \le \psi_K$ with $\psi_K \approx 19°28'$. Kelvin's analysis also shows that the pattern of transverse and divergent waves created by a ship does not depend on the length L or the shape of the ship and only depends on the ship speed V, specifically on $(X, Y)g/V^2$.

• 1.2 Wave-interference effects

Within the context of a linear potential-flow analysis, considered by Kelvin and here, the flow around a ship hull can be represented by a continuous distribution of sources over the ship hull surface. The 1-point wavemaker approximation used in Kelvin's analysis evidently cannot account for interference effects that occur between the waves created by the sources distributed over a ship hull surface. However, wave-interference effects are very important. Indeed, interference between *transverve* waves is an essential element of the design of common displacement ships, and [1] shows that observations of narrow wave patterns at high Froude numbers are explained by interference effects between *divergent* waves.

• 1.3 The 2-point wavemaker approximation

[1] shows how interference effects can be approximately taken into account via a trivial refinement of Kelvin's analysis. Specifically, a monohull ship is approximated in [1] as a 2-point wavemaker, in accordance with the well-known property that a steadily-advancing ship creates two dominant waves that originate at the ship bow and stern (where the hull geometry varies most rapidly). The superposition of two basic Kelvin wakes with origins at the bow and the stern of a ship considered in [1] introduces an important additional parameter, the ship length L, that determines the occurrence of constructive or destructive interference between the basic Kelvin bow and stern wakes. Thus, the superposition of two Kelvin wakes associated with the dominant waves created by the bow and the stern of a monohull ship, or by the bows (or sterns) of the twin hulls of a catamaran, introduces the Froude numbers

$$F \equiv V/\sqrt{gL} \quad \text{or} \quad F_s \equiv V/\sqrt{gS} \equiv F\sqrt{L/S} \tag{1}$$

where S denotes the lateral separation distance between the two hulls of a catamaran. The elementary analysis of interference effects given in [1] does not involve the *amplitudes* of the dominant (bow and/or stern) waves created by a ship, and is then a particularly simple 'geometrical' analysis.

• 1.4 Longitudinal (x) interference effects for a 2-point wavemaker

[1] shows that longitudinal (x) interference between two basic Kelvin wakes associated with the dominant bow and stern waves created by a monohull ship yields largest waves along rays $\psi = \pm\psi^x_{max}$ that are inside the cusp lines $\psi = \pm\psi_K$ of the Kelvin wake for Froude numbers $F^x < F$. The apparent wake angle ψ^x_{max} and the related Froude number F^x are given by the analytical relations

$$\psi_{max}^x \approx \arctan\left(\frac{\sqrt{\pi^2F^4/\ell^2-1}}{2\pi^2F^4/\ell^2-1}\right) \approx \arctan\left(\frac{0.16\,\ell}{F^2}\right) \text{ and } F^x = \frac{\sqrt{\ell/\pi}}{(2/3)^{1/4}} \approx 0.62\sqrt{\ell} \tag{2}$$

where ℓ denotes the nondimensional distance (related to the dimensional distance ℓL) between the effective origins of the bow and stern waves, assumed to be located slightly aft of the bow or slightly ahead of the stern. For lack of better knowledge, ℓ is taken as $\ell = 0.9$ in [1], as commonly used by naval architects in the analysis of interference between the transverse waves created by a ship bow and stern and the selection of a ship length L that avoids unfavorable interference effects (and related humps of the wave resistance curve). The choice $\ell = 0.9$ in (2) yields $\psi_{max} \approx \arctan(0.14/F^2)$ and $F^x \approx 0.59$.

• 1.5 Lateral (y) interference effects for a 2-point wavemaker

[1] also shows that lateral (y) interference between two Kelvin wakes associated with the dominant waves created by the twin bows (or the twin sterns) of the two hulls of a catamaran similarly yields largest waves along rays $\psi = \pm\psi_{max}^y$ that are inside the cusp lines $\psi = \pm\psi_K$ of the Kelvin wake for Froude numbers $F_s^y < F_s$ with F_s defined by (1). The wake angle ψ_{max}^y and the Froude number F_s^y are given by

$$\psi_{max}^y \approx \arctan\left(\sqrt{\frac{\sqrt{1+16\pi^2F_s^4}-1}{2\,(1+16\pi^2F_s^4)}}\right) \approx \arctan\left(\frac{0.2}{F_s}\right) \text{ and } F_s^y \equiv \frac{3^{1/4}}{2\sqrt{\pi}} \approx 0.37 \tag{3}$$

The Froude numbers F that correspond to $F_s = 0.37$ are $F \approx 0.17,\ 0.26,\ 0.33$ for $S/L = 0.2,\ 0.5,\ 0.8$.

• 1.6 Merits and limitations of the 2-point wavemaker approximation

The basic relations (2) and (3) show that the apparent wake angle ψ_{max} decreases like $1/F^2$ for longitudinal (x) interference or like $1/F$ for lateral (y) interference. These relations are based on a highly-simplified analysis that essentially approximates a continuous distribution of sources over a ship hull surface by means of a point source and a point sink (for a monohull ship) or two point sources (for a catamaran), i.e. as a 2-point wavemaker. This relatively crude approximation has the merit of providing useful basic insight into wave interference effects, ignored in Kelvin's classical analysis. Another merit of the elementary analysis given in [1] is that it yields the simple analytical relations (2) and (3), which provide a realistic practical estimate of the apparent wake angle of a ship without computations. However, the analytical estimates (2) and (3) are based on a relatively crude 2-point wavemaker approximation, and therefore cannot be expected to be very accurate (obviously).

2. Numerical analysis of wave-interference effects

• 2.1 Practical numerical determination of apparent wake angle

A more precise estimate of the apparent wake angle ψ_{max} related to the largest waves created by a ship requires numerical computations. A realistic and practical method for determining ψ_{max} for arbitrary ship hulls (and/or distributions of pressure at the free surface) is used in [2]. The method, which closely follows [3], is based on the numerical determination of the highest peak of the amplitude function associated with the Fourier-Kochin representation of far-field ship waves [4]. The amplitude function in the Fourier-Kochin representation of far-field waves is evaluated in [2] via the classical Hogner approximation [4,5]. Numerical predictions (of the sinkage, trim, and drag experienced by several ship hulls, and of wave profiles along the hulls, for a range of Froude numbers) based on the Hogner approximation are found in [4,5] to be consistent with experimental measurements as well as numerical predictions given by the more accurate Neumann-Michell theory. The Hogner approximation is explicitly defined in terms of the speed and the length of a ship (the Froude number) and the hull shape via a distribution of sources with density n^x equal to the x-component of the unit vector $\mathbf{n} \equiv (n^x, n^y, n^z)$ normal to the hull surface. An important major consequence of this feature is that the *far-field waves* created by a ship (and the related wave drag of the ship) can be determined without having to compute the *near-field flow* around the ship hull, i.e. very simply, as is well known [4]. Indeed, the method considered in [2] can be applied to realistic ship hulls (including multihulls) of arbitrary shape (as well as general pressure distributions over the free surface), and moreover only involves elementary numerical computations that can be performed simply and very efficiently.

• 2.2 Apparent wake angle $\psi_{max}(F)$ for general monohull ships

The method considered in [2] is applied to seven simple (analytically-defined) hull forms at ten Froude numbers $F \approx 0.65$, $F = 0.7, 0.8, ..., 1.5$. The seven hull forms correspond to a broad range of main hull-form parameters; specifically, to beam/length ratio B/L, draft/length ratio D/L, beam/draft ratio B/D and waterline entrance angle $2\,\alpha$ within the ranges $0.1 \le B/L \le 0.25$, $0.025 \le D/L \le 0.1$,

$1 \leq B/D \leq 10$, $33° \leq 2\,\alpha \leq 90°$. A notable interesting finding of the numerical computations reported in [2] is that the main parameters related to the shape of a ship hull only have a modest influence on the wake angle ψ_{max}. A useful practical consequence of this finding is that ψ_{max} can be estimated (without computations) for *general* monohulls (of any shape), specifically via the simple analytical relations

$$\psi_{max} \approx \psi_K \approx 19°28' \;\text{ for }\; F \leq 0.573 \tag{4a}$$

$$\psi_{max} \approx \arctan(0.116/F^2) \;\text{ for }\; 0.573 \leq F \leq 0.85 \tag{4b}$$

$$\psi_{max} \approx \arctan[0.08\,(1+0.6/F)/F] \;\text{ for }\; 0.85 \leq F \tag{4c}$$

These relations account for both *longitudinal* interference between the waves created by the fore and aft regions of a monohull ship and *lateral* interference between the waves created by the port and starboard sides of the hull, whereas the relation (2) only accounts for longitudinal interference between the dominant bow and stern waves. As expected, the relations (4) yield a practical estimate of the wake angle $\psi_{max}(F)$ of a general monohull ship that is more precise than the analytical estimate (2), although differences are not very large.

• 2.3 Apparent wake angle $\psi_{max}(F, s)$ for general catamarans

Interference effects are significantly more complicated for catamarans than for monohull ships because catamarans essentially are 4-point wavemakers and involve the additional parameter $s \equiv S/L$ that defines the lateral separation distance between the two hulls of the catamaran. Furthermore, interference effects between the divergent waves created by a monohull ship and a catamaran differ in a major way because a peak, called *outer peak* in [6], of the amplitude function in the Fourier-Kochin-Hogner representation of far-field waves [2] can occur for $\psi^y_{max} < |\psi| < \psi_K$ for catamarans (but not for monohulls). The method given in [2] is applied to catamarans in [6], where seven simple (analytically-defined) hulls are considered for lateral separation distances $s \equiv S/L$ and corresponding Froude numbers F_s within the ranges $0.2 \leq s \leq 0.8$ and $0.4 \leq F_s \leq 3.5$. The seven hulls correspond to a broad range of main hull-form parameters; specifically, to $0.05 \leq B/L \leq 0.1$, $0.0375 \leq D/L \leq 0.075$, $1 \leq B/D \leq 2$, $17° \leq 2\,\alpha \leq 48°$. The parametric study considered in [6] shows that the main parameters related to the shape of a ship hull only have a weak influence on the wake angle ψ_{max}, as also found in [2] for monohull ships. Moreover, the numerical computations considered in [6] show that the relation (3) that defines the *inner peak* of the amplitude function in the Fourier-Kochin-Hogner representation of far-field waves can be refined as

$$\psi_{max} \approx \psi_K \approx 19°28' \;\text{ for }\; F \leq 0.46 - 0.02/s \tag{5a}$$

$$\psi_{max} \approx \psi^y_{max} + \frac{1.4}{s}\left(\frac{0.39 + 0.13\,s - \sqrt{s}\,F}{0.39 - 0.77s + 0.55\,s^2}\right)^2 \;\text{ for }\; 0.46 - \frac{0.02}{s} \leq F \leq \frac{0.39 + 0.13\,s}{\sqrt{s}} \tag{5b}$$

$$\psi_{max} \approx \psi^y_{max} \;\text{ for }\; (0.39 + 0.13\,s)/\sqrt{s} \leq F \qquad \text{where }\; s \equiv S/L \tag{5c}$$

Here, ψ^y_{max} is given by (3) and the angles ψ_{max} and ψ^y_{max} are assumed to be expressed in degrees. The relation (5c) shows that the apparent wake angle ψ_{max} is equal to the angle ψ^y_{max} if F is large and/or if s is large, i.e. for fast and/or wide catamarans. The Froude number $F = (0.39 + 0.13\,s)/\sqrt{s}$ in (5c) varies between 0.93 and 0.55 for $0.2 \leq s \leq 0.8$. Thus, the systematic numerical study considered in [6] shows that *lateral* interference effects between the two hulls of a catamaran are dominant for fast and/or wide catamarans. However, *longitudinal* interference effects are important and cannot be ignored for slow narrow catamarans. Indeed, the outer peak can be higher than the inner peak in a region of the 'Froude-number and separation-distance plane' (F, s) that corresponds to small values of F and s. This relatively small region of the (F, s) plane where the outer peak is dominant and the relations (5a)-(5c) are not valid is given in [6].

• 2.4 Neumann-Michell computations of Kelvin wakes

The numerical results obtained in [2] for monohulls and in [6] for catamarans are based on the Hogner approximation, used to evaluate the amplitude function in the Fourier-Kochin representation of far-field ship waves. The relations (4) and (5) are further considered in [7] via the Neumann-Michell theory given in [4,5]. Specifically, computations of far-field waves are reported in [7] for seven monohull ships that correspond to broad ranges of main hull-shape parameters at four Froude numbers $F = 0.58$, 0.68, 0.86, 1.58, for which the relations (4b) and (4c) yield $\psi_{max} \approx 19°$, $14°$, $9°$, $4°$. These numerical computations confirm that the apparent wake angle ψ_{max} related to the largest waves created by a ship is only weakly influenced by the hull shape and thus mostly depends on the Froude number, in accordance with the relations (4). The Neumann-Michell theory is also used in [6] to supplement and confirm the 'Hogner-approximation-based' parametric study of interference effects for catamarans.

3. Wave-interference effects in shallow water

The elementary analysis of longitudinal (x) or lateral (y) interference between the dominant waves created by the bow and the stern of a monohull ship, or by the bows of the twin hulls of a catamaran, given in [1] for deep water is extended in [8,9] to the more general, and considerably more complicated, case of uniform finite water depth. This analysis shows that the largest waves due to constructive interference are found at an 'apparent wake angle' ψ_{max} that can differ greatly from the cusp or asymptote angles associated with the wave pattern of a ship when interference effects are ignored. Thus, wave-interference effects on the wave signature of a ship in shallow water are very large and cannot be ignored. The analysis given in [8,9] also yields practical relations that determine when water-depth effects on the apparent wake angle ψ_{max} are small and can be neglected.

4. Wave-breaking effects on the Kelvin wake

The foregoing linear potential-flow analysis of wave-interference effects ignores important effects related to wave-breaking, notably the breaking of bow waves. Indeed, ship bow waves typically are higher and shorter, and therefore steeper as well as far more influenced by nonlinear effects, than waves aft of the bow wave. Two main types of ship bow waves exist. Specifically, a slow ship with a blunt bow typically creates a highly unsteady and turbulent bow wave, whereas the bow wave created by a fast ship with a fine bow consists of a detached thin sheet of water that is mostly steady, until it hits the main free surface and undergoes turbulent breaking up and diffusion [10]. Both these two bow-wave regimes result in the dissipation of a portion of the wave energy of a ship bow wave, as well as the partial transformation of the wave drag of a ship into a wave-breaking drag component [11]. A reasonable *conjecture* is that the wave-breaking that commonly occurs at a ship bow destroys short waves more effectively than long waves. This *assumption* means that wave-breaking may result in the effective elimination of short waves with wavelengths $\lambda < \lambda^{min}$ from the spectrum of farfield ship waves. Moreover, the wavelength λ^{min} may be taken as a fraction ϵ of the longest wave $\lambda^{max} \equiv 2\pi F^2$ created by a ship, i.e. as

$$\lambda^{min} = \epsilon\,\lambda^{max} \equiv \epsilon\,2\pi F^2 \tag{6}$$

The assumption that wavelengths $\lambda < \lambda^{min}$ are eliminated as a result of wavebreaking is mathematically equivalent to the restriction $\lambda^{min} < \lambda$, which is mathematically equivalent to the relation $\psi_{min} < |\psi|$ as shown in [12] if $\epsilon \le 2/3$. Specifically, expression (15) in [12] yields the approximation

$$\psi_{min} \approx \arctan(\sqrt{\epsilon}/2) \quad \text{where} \quad 0 < \epsilon \equiv \lambda^{min}/\lambda^{max} \le 2/3 \tag{7}$$

The special case $\epsilon = 2/3$ corresponds to $\psi_{min} = \psi_K \approx 19°28'$, and means that all divergent waves are eliminated. The relation (7) yields

$$\psi_{min} \approx 6°23' \approx \psi_K/3 \;\text{ for }\; \epsilon = 5\% \quad \text{and} \quad \psi_{min} \approx 12°32' \approx 2\,\psi_K/3 \;\text{ for }\; \epsilon = 20\% \tag{8}$$

Thus, the angle ψ_{min} of the 'no-divergent-wave wake' that is obtained if waves with wavelengths smaller than 5% or 20% of the dominant wavelength $\lambda^{\max} \equiv 2\pi F^2$ are assumed to be eliminated due to wave-breaking is approximately equal to $\psi_K/3$ or $2\,\psi_K/3$, i.e. is not small, and much larger than the angle ψ_{min} related to surface-tension effects [1]. The relations (8) suggest that wave-breaking, commonly found at a ship bow, may be assumed to have a very large influence on the Kelvin wake of a ship.

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