Liquid sloshing and impact in a closed container with high filling

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1 Introduction

Violent liquid sloshing is of concern to cargo tank designers due to the problems of safety in extreme loadings. It is an interesting topic with still some disputable problems. For example Abramson, Bass, Faltinsen and Olsen [1] investigated the sloshing and resultant loads in liquid natural gas carriers for different tank geometries and liquid fill depths. Ibrahim [3] provided a comprehensive study with examples of free tank motions. Cooker [2] analysed with experiments a horizontal rectangular wave tank which swings at the lower end of a pendulum. Ten, Malenica and Korobkin [5], presented a semi-analytical approach for fluid-structure interactions inside tanks in different impact situations with high and low fillings. Our work is about the sloshing of standing waves inside a closed highly filled container. Unsteady two-dimensional (2-D), irrotational flow is treated. The liquid-roof interaction is discussed with and without the effect of gravity and a comparison is made. The short-time model of the liquid-roof impact is governed by a Mixed Boundary Value Problem (MBVP), which is solved numerically and using the asymptotic methods.

2 Mathematical formulation

In figure 1, a stationary highly filled rectangular tank, containing an inviscid incompressible liquid is shown. The flow is 2-D and irrotational and surface tension force is neglected. The container has height H and length $2L$ and lies in the region $\tilde{y} \geq 0$. Here $\tilde{y} = 0$ is the bottom and $\tilde{y} = H$ is the roof of the container, $\tilde{y} = H - h$ is the still water level, and $\tilde{x} = \pm L$ are the rigid impermeable walls. The lengths, time, velocity potential and pressure are scaled by H, $\sqrt{H/g}$, $h\sqrt{Hg}$ and $\rho g h$, respectively, where g is the gravitational acceleration and ρ is the constant density. The small parameter $\epsilon = h/H \ll 1$ is responsible for linearisation. In non-dimensional variables (without tilde), the initial shape of the free surface is given by the equation $y = f(x)$, $f(x) = 1 - \epsilon + \frac{2\epsilon}{\lambda} \sum_{n=1}^{\infty} a_n \cos(k_n x)$, a_n and k_n are known constants and $\lambda = \frac{L}{H}$.

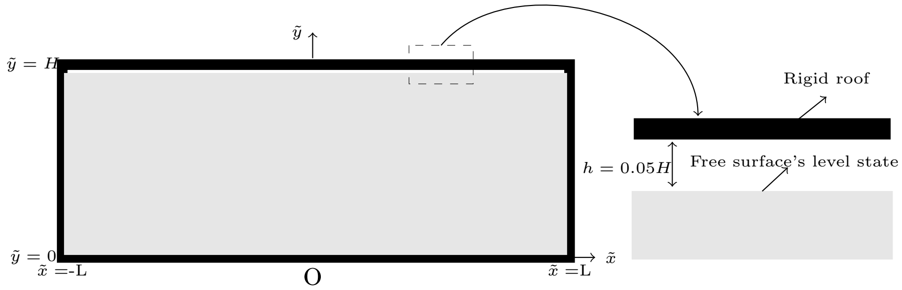

Figure 1: Container description with free surface at its level state.

Considering the container to be without the rigid roof, in non-dimensional linearised initial boundary problem at the leading order the velocity potential $\phi_l(x,y,t)$ and the surface elevation $\eta_l(x,t)$ read

$$\phi_l(x,y,t) = \sum_{n=1}^{\infty} \bar{a}_n \cosh(k_n y)\cos(k_n x)\sin(w_n t), \tag{1}$$

$$\eta_l(x,t) = \frac{2}{\lambda}\sum_{n=1}^{\infty} a_n \cos(k_n x)\cos(w_n t), \qquad a_n = -\bar{a}_n \omega_n \cosh(k_n), \tag{2}$$

where the angular frequencies ω_n are related to the wave numbers $k_n = n\pi/\lambda$ by the dispersion relation $\omega_n = \sqrt{k_n \tanh(k_n)}$, for $n \in \mathbb{N}$. In the next section we introduce the rigid roof to the tank, we decompose the velocity potential $\phi(x,y,t)$ and the surface elevation $\eta(x,t)$ into two parts. This has been done by adding unknown functions: a so called correction velocity potential $\phi_c(x,y,t)$ and a correction surface elevation $\eta_c(x,t)$ resulting in $\phi = \phi_c + \phi_l$ and $\eta = \eta_l + \eta_c$ respectively.

3 Semi-analytical solution

At the leading order, we derive the linearised non-dimensional MBVP, with gravity and the rigid roof included. During the early stage of the impact, with some asymptotic analysis, we have found that gravity has a small influence. We define $\delta \ll 1$ to stretch the time $t = t^* + \delta\hat{t}$, $t = t^*$ instant of impact, consequently the other variables, $x = \delta^{1/2}\hat{x}$, $y = \delta^{1/2}\hat{y}$, $\phi_c(x,y,t) = \delta^{1/2}\hat{\phi}_c(\hat{x},\hat{y},\hat{t})$, $x_{c0}(t) = \delta^{1/2}\hat{x}_{c0}(\hat{t})$ and $\eta_l(x,t) = \delta\hat{\eta}_l(\hat{x},\hat{t})$. By combining the dynamic and kinematic boundary conditions we arrive at (without hat)

$$\frac{\partial^2\phi_c}{\partial t^2} + \delta^{\frac{3}{2}}\frac{\partial\phi_c}{\partial y} = 0 \qquad |x| > x_c(t), \qquad y = 0, \tag{3}$$

where $x = x_c(t)$ is where the free surface meets the roof. The corresponding limiting problem as $\delta \to 0$ is depicted in Figure 2. This problem does not account for gravity.

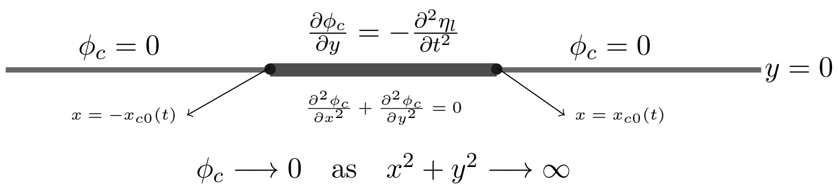

Figure 2: The MBVP at the leading order.

Working with stretched variables, we can approximately replace the wall and the bottom conditions with the far-field condition given in Figure 2. With the condition introduced by Wagner [6] at the contact points and the displacement potential $\Phi(x,y,t) = \int_0^t \phi_c(x,y,\tau)\mathrm{d}\tau$ introduced by Korobkin [4], we are able to calculate semi-analytically the leading order position of the contact point, $x = x_{c0}(t)$, during impact. Consequently the hydrodynamic pressure distribution can be found (see Figure 3).

4 Numerical solution

A collocation method was used for the symmetric flow considered in the original coordinates x and y. The unknowns, velocity potential $\phi(x,y,t)$, surface elevation $\eta(x,t)$ and the pressure on the roof $p(x,0,t)$ are respectively presented by Fourier series. The domain is discretized into N regularly spaced nodes ($N = 350$ in our calculations).

The combined kinematic and dynamic boundary conditions (with gravity included) lead to a system of the form

$$\vec{\eta} = \vec{\eta_l} - A^{-1}B\vec{P} - A^{-1}\sum_{m=1}^{M-1} G^{t_m}\vec{p}^{\,t_m}, \tag{4}$$

where the initial pressure $\vec{p}^{\,t_1}$, found in the previous section, with respect to this algorithm is given. The tri-diagonal matrix A is associated with unknown and known surface elevation vectors, $\vec{\eta}$ and $\vec{\eta_l}$ respectively. The matrix G is calculated at every time step while the matrix B is independent of time and depends only on the time step length. The pressure $\vec{P}$ and surface elevation $\vec{\eta}$ are to be determined at the instant $t = t_M$. However, the fact that on the free surface, $x_{c0}(t_M) < x < 1$, we have $p(x, t_M) = 0$ and on the impact region, $0 < x < x_{c0}(t_M)$, we have $\eta(x, t_M) = 1$, with some rearrangements makes the system (4) solvable on its own. The free surface and the wetted region are distinguished and updated at each time step by calculating the position of $x = x_{c0}(t_M)$ as part of the solution. The pressure vector $\vec{p}^{\,t_m}$ is known from the previous time steps $t = t_m$, for $1 \leq m \leq M-1$.

Continuing with the stretched variables introduced from Section 3, we study the gravity influence on the length of the wetted region during the impact stage, that is $x_{c0}(t) + \delta \cdot x_{c1}(t)$. The correction due to gravity $x_{c1}(t)$ in Figure 4b, is found to be almost completely insignificant at the early stage of impact. As time goes on the effect of gravity is that it decreases the length of the wetted region and even at the very late period of this stage, its effect is found to be small. Figure 4a shows the wetted length with and without gravity. More results of the numerical simulation will be presented at the workshop.

5 Acknowledgements

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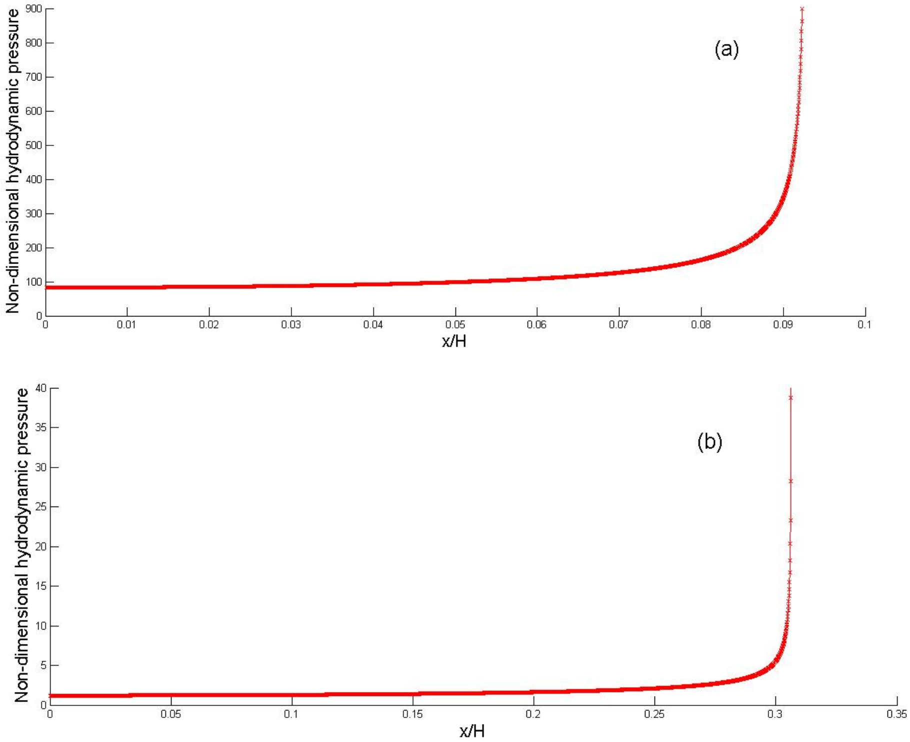

Figure 3: Semi-analytical pressure distribution without gravity at: (a) $t = 1.0790$; (b) $t = 1.5760$.

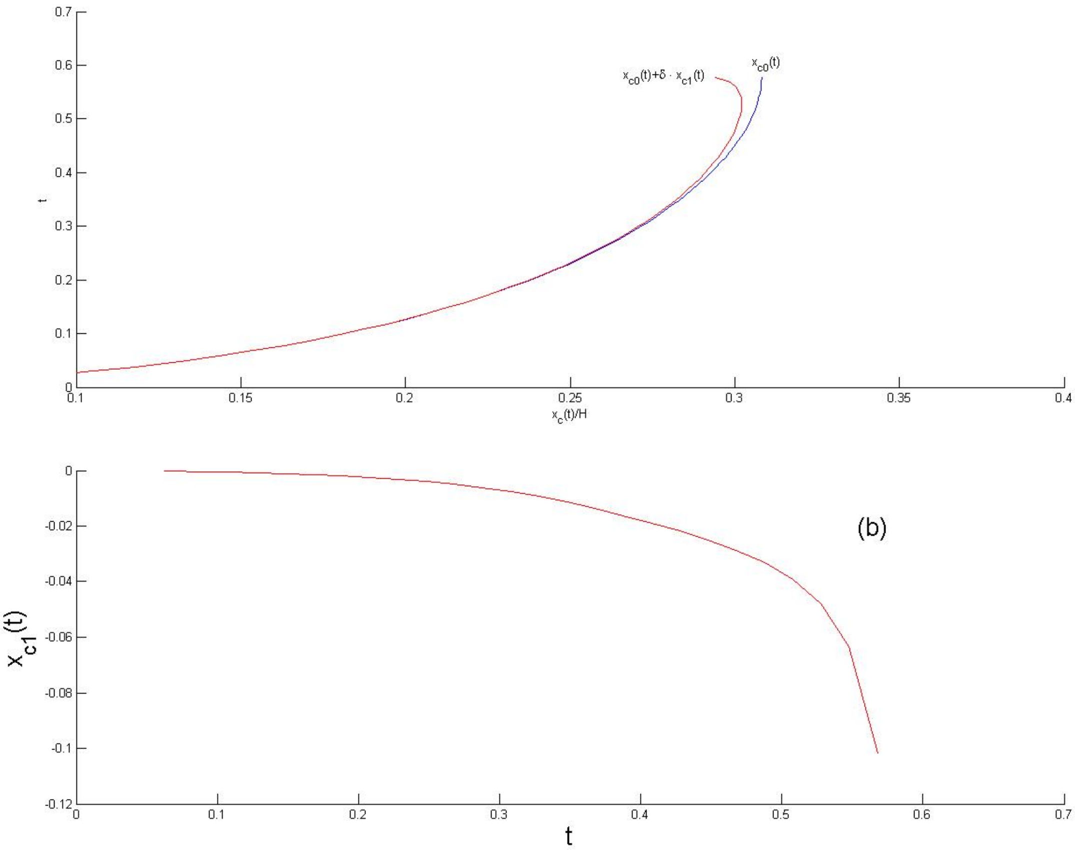

Figure 4: Contact point position with correction due to gravity, here $t = 0$ is the impact time: (a) The leading order contact point, blue line,and the contact point with correction due to gravity, red line; (b) details of correction $x_{c1}(t)$ due to gravity,for the contact point position.