Hydro-elastoplastic analysis of floating plates in waves

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Highlights:

- A formulation for hydro-elastoplastic analysis is provided to fully couple the time-dependent response of a floating plate with elastoplastic material.
- A numerical procedure based on the incremental finite element analysis is applied to analyze the elastoplastic material behavior.
- The effects of plastic deformations on the response of a floating plate in a regular wave are investigated by comparing the solutions of hydroelastic and hydro-elaistoplastic analyses.

1. Introduction

Hydroelasticity is concerned with interactions between the deformations of floating elastic structures and hydrodynamic responses. Since the deformation effect is more dominant to the dynamic responses as the size of floating structures is getting larger, hydroelastic analysis has been applied to the design of very large floating structures such as pontoon-type offshore structures and bridges, floating airports, and ice floes. In recent years, the analytical and numerical methods have been developed to solve the nonlinear problems of hydroelasticity. Many studies have focused on nonlinear effects related to hydrodynamic forces. However, studies on the nonlinear structural response have been rarely founded.

The recent review [1] mentions the hydroplastic analysis as a way toward future development in hydroelasticity of ships, which may be required to analyze the dynamic collapse response of ship's hull, propagation of crack and so on. In this context, a 2D hydro-elastoplasticity method was proposed by combining strip theory and simplified progressive collapse method for the nonlinear dynamic responses of a ship beam in extreme waves [2]. The aim of the present study is to provide a numerical method that can consider fully coupled behaviors between the deformations of floating elastoplastic plate and linear hydrodynamic loads in time domain.

In this abstract, firstly, the formulation for the 2D hydro-elastoplastic analysis of floating structures is briefly described. Then, the numerical procedure of hydroelastic analysis in time domain and elastoplastic analysis is presented based on finite element method. Finally, the present method is validated by comparing numerical results of previous studies. In addition, the plastic effects on the response of floating structures in a regular wave are investigated by comparing the results of hydroelastic and hydro-elastoplastic analyses.

2. Mathematical model

Let consider a floating plate on water surface under a constant water depth h. The fixed Cartesian coordinate system (x_1, x_2, x_3) is defined on the free surface of the calm water. It is assumed that the floating plate with homogeneous, isotropic and elastoplastic material is infinite in the x_3 direction and is sufficiently thin enough that the draft is ignored. The motion and strain of the floating plate are assumed to be small. The equilibrium equations of the floating plate at time t are

$$\frac{\partial\, {}^t\sigma_{ij}}{\partial\, {}^t x_{ij}} - {}^t\rho_s g \delta_{i2} - {}^t\rho_s\, {}^t\ddot{u}_i \quad \text{in } {}^tV, \qquad {}^t\sigma_{ij}\, {}^t n_j = -{}^t p\, {}^t n_j \quad \text{on } {}^tS_B, \tag{1}$$

where σ_{ij} is the Cauchy stress tenor, u_i is the displacement vector, ρ_s is the density of the structure, p denotes the total water pressure. Note that ${}^t p = -\rho_w g x_2 + {}^t p_d$, in which p_d is the hydrodynamic pressure. Also, n_j is the unit normal vector outward from the structure to the fluid, and δ_{ij} is the Kronecker delta. The overdot means the material time derivative. V denotes the structural domain, and S_B means the wet body surface.

The principle of virtual work for the floating plate at time t can be written as

$$\int_{{}^tV} {}^t\rho_s\,{}^t\ddot{u}_i \delta u_i dV + \int_{{}^tV} {}^t\sigma_{ij}\delta e_{ij} dV = -\int_{{}^tV} {}^t\rho_s g \delta u_2 dV + \int_{{}^tS_B} \rho_w g\,{}^t x_2\,{}^t n_i \delta u_i dS - \int_{{}^tS_B} {}^t p_d\,{}^t n_i \delta u_i dS \,, \tag{2}$$

where δu_i and δe_{ij} refer to the virtual displacement vector and small strain tensor, respectively. Employing the materially-nonlinear-only formulation based on the incremental equilibrium equation [3], Eqs. (2) is linearized as

$$\int_V \rho_s\,{}^{t+\Delta t}\ddot{u}_i \delta u_i dV + \int_V {}^tC^{EP}_{ijkl}\,{}^{t+\Delta t}_{\;\;t}e_{ij}\delta e_{ij} dV - \int_{S_B} \rho_w g\,{}^{t+\Delta t}u_2 n_i \delta u_i dS = -\int_{S_B} {}^{t+\Delta t}p_d n_i \delta u_i dS - \int_V {}^t\sigma\,{}^t e_{ij}\delta e_{ij} dV \,, \tag{3}$$

where C^{EP}_{ijkl} is the elastoplastic stress-strain relation tensor. Note that the hydrostatic analysis is not performed in this problem since we assume that the static equilibrium state is known [4].

The incompressible, inviscid and irrotational fluid flow is assumed and surface tension is neglected. The incident wave is coming from right side to plate and its amplitude is small enough to use the linear wave theory. This implies that the hydrodynamic pressure can be described in form of the convolution integral of the arbitrary time-dependent motion with the radiation ϕ^R and diffraction ϕ^D potentials corresponding to the impulsive velocity of the plate or impulsive wave elevation, respectively [5].The hydrodynamic pressure is expressed by the linearized Bernoulli equation as follows

$$\begin{aligned} p_d &= -\rho_w \left[\int_{-\infty}^{\infty} \frac{\partial}{\partial t}\phi^R\left(x_1,x_2;t-\tau\right)\dot{u}\left(\tau\right)d\tau + \int_{-\infty}^{\infty} \frac{\partial}{\partial t}\phi^D\left(x_1,x_2;t-\tau\right)\eta\left(\tau\right)d\tau \right] \\ &= -\rho_w \left[\psi\left(x_1,x_2\right)\ddot{u} + \int_{-\infty}^{t} \frac{\partial}{\partial t}\varphi\left(x_1,x_2;t-\tau\right)\dot{u}\left(\tau\right)d\tau + \int_{-\infty}^{\infty} \frac{\partial}{\partial t}\phi^D\left(x_1,x_2;t-\tau\right)\eta\left(\tau\right)d\tau \right], \end{aligned} \tag{4}$$

where η is the incident wave elevation. Note that $\phi^R = \psi\left(x_1,x_2\right)\delta\left(t\right) + \varphi\left(x_1,x_2;t\right)h\left(t\right)$, in which $h\left(t\right)$ is the Heaviside function.

Substituting the Eqs. (4) into Eqs. (3), we finally obtain the equation coupled between an elastoplastic plate and fluid:

$$\begin{aligned} &\int_V \rho_s\,{}^{t+\Delta t}\ddot{u}_i \delta u_i dV - \int_{S_B} \rho_w \psi\,{}^{t+\Delta t}\ddot{u}_i n_i \delta u_i dS - \int_{-\infty}^{t+\Delta t}\int_{S_B} \rho_w \frac{\partial}{\partial t}\varphi \dot{u}_i n_i \delta u_i dS d\tau + \int_V {}^tC^{EP}_{ijkl}\,{}^{t+\Delta t}_{\;\;t}e_{ij}\delta e_{ij} dV - \int_{S_B} \rho_w g\,{}^{t+\Delta t}u_2 n_i \delta u_i dS \\ &= \int_{-\infty}^{\infty}\int_{S_B} \rho_w \frac{\partial}{\partial t}\phi^D \eta n_i \delta u_i dS d\tau - \int_V {}^t\sigma\,{}^t e_{ij}\delta e_{ij} dV \,. \end{aligned} \tag{5}$$

3. Numerical Procedure

The formulation in Eqs. (5) can be transformed into the matrix form using the standard finite element discretization as

$$\left(\mathbf{M}+\mathbf{A}\right){}^{t+\Delta t}\ddot{\mathbf{U}} + \int_{-\infty}^{t+\Delta t} \mathbf{B}\left(t+\Delta t-\tau\right)\dot{\mathbf{U}}\left(\tau\right)d\tau + \mathbf{C}\,{}^{t+\Delta t}\mathbf{U} + {}^t\mathbf{K}\mathbf{U} = \int_{-\infty}^{\infty} \mathbf{X}\left(t+\Delta t-\tau\right)\boldsymbol{\eta}\left(\tau\right)d\tau - {}^t\mathbf{F} \,, \tag{6}$$

in which the matrices and vectors are defined as follows:

$$\int_V \rho_s\,{}^{t+\Delta t}\ddot{u}_i \delta u_i dV = \delta\mathbf{U}^T\mathbf{M}\,{}^{t+\Delta t}\mathbf{U} \,, \quad -\int_{S_B} \rho_w \psi\,{}^{t+\Delta t}\ddot{u}_i n_i \delta u_i dS = \delta\mathbf{U}^T\mathbf{A}\,{}^{t+\Delta t}\ddot{\mathbf{U}} \,,$$

$$-\int_{-\infty}^{t+\Delta t}\int_{S_B} \rho_w \frac{\partial}{\partial t}\varphi \dot{u}_i n_i \delta u_i dS d\tau = \delta\mathbf{U}^T \int_{-\infty}^{t+\Delta t} \mathbf{B}\left(t+\Delta t-\tau\right)\dot{\mathbf{U}}\left(\tau\right)d\tau \,, \quad -\int_{S_B} \rho_w g\,{}^{t+\Delta t}u_2 n_i \delta u_i dS = \delta\mathbf{U}^T\mathbf{C}\,{}^{t+\Delta t}\mathbf{U} \,,$$

$$\int_V {}^tC^{EP}_{ijkl}\,{}^{t+\Delta t}_{\;\;t}e_{ij}\delta e_{ij} dV = \delta\mathbf{U}^T\,{}^t\mathbf{K}\mathbf{U} \,, \quad \int_{-\infty}^{\infty}\int_{S_B} \rho_w \frac{\partial}{\partial t}\phi^D \eta n_i \delta u_i dS d\tau = \delta\mathbf{U}^T \int_{-\infty}^{\infty} \mathbf{X}\left(t+\Delta t-\tau\right)\boldsymbol{\eta}\left(\tau\right)d\tau \,, \quad \int_V {}^t\sigma\,{}^t e_{ij}\delta e_{ij} dV = \delta\mathbf{U}^T\mathbf{F} \,. \tag{7}$$

We here employ the 2-node Euler-Bernoulli beam element for the finite element model of plate structures in 2D and the three-dimensional von Mises plasticity model with the associated flow rule and isotropic hardening for the elastoplastic material. At each integration point in beam cross-sections, the unknown stress and plastic strain are implicitly evaluated by solving a single nonlinear equation in accordance with the governing parameter method [6, 7]. In order to solve the nonlinear Eqs. (6), the full Newton-Raphson iterative scheme and Newmark method are employed.

Since the impulse response functions such as $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{X}$ are related to the corresponding coefficients in the frequency domain by Fourier transformation, we evaluate the frequency domain results using the direct coupling analysis of floating structures, in which we solve the coupled equations discretized by finite and boundary elements for structure and fluid, respectively [4, 8, 9]. A 2-node boundary element is used for fluid modeling.

4. Numerical Experiments

To validate the proposed formulation, the numerical results are compared with reference solutions. Since there are no available results for hydro-elastoplastic analysis of floating plates, we solve two basic problems. The first problem deals with the transient response of a floating elastic plate and the second problem considers the dynamic response of an elastoplastic beam.

Meylan and Sturova (2009) provided the benchmark solutions for the time-dependent motion of a floating elastic plate released from rest [10]. As shown in Fig. 1, our numerical solutions were in good agreement with the reference solutions for a symmetric initial displacement case ($h / L = 0.02$).

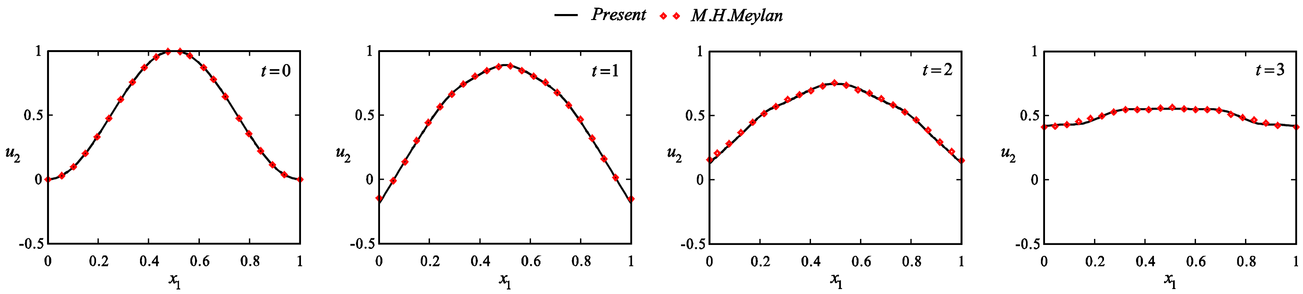

Fig. 1. Time history of the floating elastic plate deflection.

We consider a cantilever beam with bilinear elastoplastic material model subject to an impact load and compare the results with the reference solutions obtained using ADINA [11] as shown in Fig 2. The results show that the proposed formulation is suitable for hydro-elastoplastic analysis.

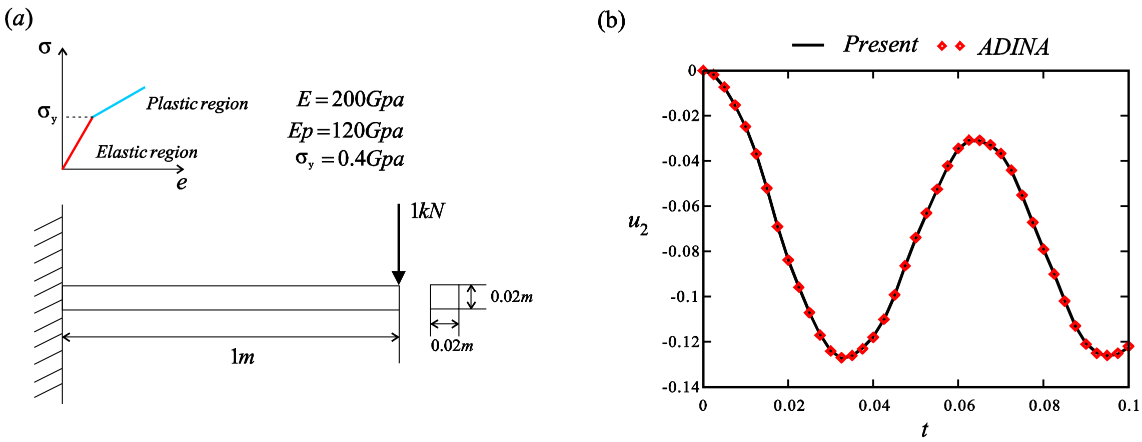

Fig. 2. (a) Elastoplastic cantilever beam problem and (b) deflection at the free tip.

To investigate the effect of plastic deformations on the response of floating plate in a regular wave, we consider a floating plate subjected to an impact load at the middle and compare the numerical results of hydroelastic and hydro-elastoplastic analyses. The structural properties are based on the weight-drop experiment carried out by Endo and Yago [12] and the regular wave length is $\lambda = 3.25m$. As shown in Fig. 3, the response of the floating plate is influenced by the plastic deformation. It is observed that the deflection of the elastoplastic plate is smaller than that of the elastic plate since the energy is dissipated due to plastic mechanism.

5. Conclusions

In this study, we proposed a formulation for hydro-elastoplastic analysis of a floating plate in regular waves and compared our solutions with available numerical results for hydroelastic and elastoplastic analyses. However, it is necessary to carry out related experimental tests and to validate the proposed formulation. In order to extend the research scope, we will investigate elastoplastic behaviors of floating structures under various conditions and extend to the formulation for 3D hydro-elastoplastic analysis.

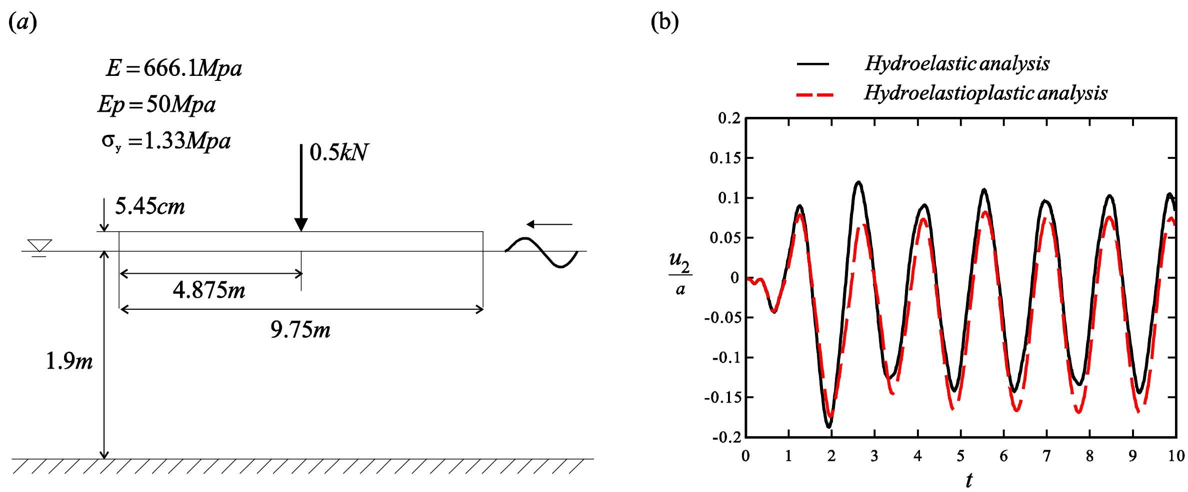

Fig. 3. (a) A floating plate subjected to an impact load in a regular wave and (b) deflections at the middle of plate calculated using hydroelastic and hydro-elastoplastic analyses.

REFERENCES

[1] Hirdaris, S.E. and Temarel, P., Hydroelasticity of ships: Recent advances and future trends, *Proc. IMechE part M*, J Eng Maritime Environ, 2009;223;305-30.

[2] Liu, W., Suzuki, K., Shibanuma, K. and Pei, Z., Nonlinear dynamic response and strength evaluation of a container ship in extreme waves based on hydroelastoplasticity method, *Proc. of the 24th Int Ocean and Polar Eng. Conf.*, Busan, Korea, IOSPE, 2014;652-57.

[3] Bathe, K.J., Finite element procedure, Prentice Hall. Englewood Cliffs, NJ, 1996.

[4] Kim, K.T., Lee, P.S. and Park, K.C., A direct coupling method for 3D hydroelastic analysis of floating structures, Int J Numer Meth Eng, 2013;96;842-66.

[5] Korsmeyer, F.T., Bingham, H.B. and Newma, J.N., TiMIT – a panel method for transient wave-body interactions, Research Laboratory of Electronis, MIT, 1999.

[6] Kojic, M. and Bathe, K.J., Inelastic analysis of solids and structures, Springer, Verlag Berlin Heidelberg, 2005.

[7] Yoon, K. and Lee, P.S., Nonlinear performance of continuum mechanics based beam elements focusing on large twisting behaviors, Comput Methods Appl Mech Engrg, 2014;281;106-30.

[8] Kim, J.G., Cho, S.P, Kim, K.T. and Lee, P.S., Hydroelastic design contour for the preliminary design of very large floating structures, Ocean Eng 2014;78l;112-23.

[9] Yoon, J.S., Cho, S.P., Jiwinangun, R.G. and Lee, P.S., Hydroelastic analysis of floating plates with multiple hinge connections in regular waves, Mar struct, 2014;36;65-87.

[10] Meylan, M.H. and Sturova, I.V., Time-dependent motion of a two-dimensional floating elastic plate, J Fluid Struct, 2009:25:445-60.

[11] ADINA R&D, ADINA theory and modeling guide, Watertown, MA: ADINA R&D, 2013.

[12] Endo, H. and Yago., Time history response of a large floating structure subjected to dynamic load, J Soc Nav Archit Jpn, 1999:186:36-76.