HYDRODYNAMIC IMPACT ON AN ERODIBLE BODY

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1. INTRODUCTION

Liquid/structure or liquid/liquid impact is widely observed in nature. Examples include wave impact on marine structures and coastline, droplet impact or waterfalls on solid floor, a solid body passing through a liquid surface. While high speed liquid impacts can cause structural damage or failure and many other adverse effects, these processes are also used in applications such as 3-D printing technology, cool jet-cutting and cleaning of metals, coating and painting.

In many water impact studies, the bodies are treated as rigid and impermeable. However, in many other fluid/solid impact processes, the body surface may be non rigid or permeable. There may be surface erosion or other types of material removal that changes the body shape during impacts, such as liquid drop impacting a soil or granular materials [1], metal cutting through jets [2,3], surface penetration through shaped charges [4,5], and cavitation erosion by high-speed jets generated during collapse of vapour bubbles [6,7]. In maritime engineering, liquid may penetrate through a perforated or porous surface, and typical examples can be found in [8 – 11].

In this study we consider the two dimensional self similar velocity potential flow problem for impact between a liquid wedge and a permeable or/and erodible body [12]. Integral hodograph method [13] is used. It enables the original partial differential equation with the nonlinear boundary conditions on the unknown free surface to be converted into a system of intergro-differential equations along straight lines in the parameter plane. The method has been successfully used in variety of impact problems [12 - 14]. However, the application of the method to the present problem has some new difficulties. On the impermeable solid surface, the normal velocity is prescribed, while on the free surface the pressure is provide. On the permeable body surface or the moving wetted surface caused by erosion, neither of these functions is known in advance explicitly. Instead the boundary condition is written in terms of a relationship between the pressure and the normal component of the velocity trough the body surface. This leads to a new singularity on the mathematical formulation. Further in the case of an erodible body, the wetted surface of the solid deforms and body material moves away, and its shape into the body is determined by the local speed of erosion or melting. The formulation of the problem includes an additional equation through a law relating the speed of erosion with other flow parameters, which gives the means to determine the unknown shape of the interface.

Various case studies are considered. The first one is porous body, for which a linear relationship between the pressure and normal velocity through non deforming body boundary is employed. The second case is a perforated body, for which the quadratic relationship between the pressure and the normal velocity trough the non deforming body surface is used. These two cases are related to the problems in coastal and offshore engineering [8, 9, 15] where the porous/perforated bodies are used to reduce the hydrodynamic impact loading on a structure. The third case considered here is associate with jet-cutting or penetration of the shaped charge, in which the solid material is removed by the hydrodynamic pressure and shear stress. There are some previous studies related to this case, in particular, that done by Pool [5], which are based on a further development of the classical Birkhoff-problem of steady impinging jets.

2. FORMULATION OF THE PROBLEM

We consider the impact problem between a liquid wedge of half-angle α and a permeable and/or erodible body. A sketch of the problem and the definitions of the geometric parameters are shown in Fig. 1*a*. The flow is self similar and will be studied in the frame of reference with its origin attached to the stagnation point *A* which may move during the erosion of the body surface.

The liquid wedge has uniform velocity at infinity, which is indicated as *V* in figure 1*a* and it is relative to point *A*. The symbol v_{ni} in the figure is the normal velocity of surface *OA*, which is zero at point *A*, as the origin of the coordinate system is fixed there. Within this surface, AD is the wetted surface of the body after erosion and forms an angle $\alpha_A < \pi/2$ with the y-axis at point A. The symbol v_{np} in the figure is the velocity due to the body surface permeability. Thus, the total normal component of the velocity of the liquid along OA can be expressed as $v_n = v_{ni} + v_{np}$, in which $v_{np} = 0$ on the free surface OD.

For a constant impact velocity of the liquid wedge, the time-dependent problem in the physical complex plane $Z = X + iY$ can be written in the stationary similarity plane $z = x + iy$ in terms of the self-similar variables $x = X/(Vt)$, $y = Y/(Vt)$ where t is the time. The complex velocity potential $W(Z,t)$ for the self-similar flow can be written as

$$W(Z,t) = V^2 t w(z) = V^2 t\left[\phi(x,y) + i\psi(x,y)\right]. \qquad (1)$$

The problem is to determine the function $w(z)$ which conformally maps the similarity plane z onto the complex-velocity potential region w. We choose the first quadrant of the $\zeta-$plane in figure 1b as the parameter region to derive expressions for the nondimensional complex velocity, dw/dz, and for the derivative of the complex potential, $dw/d\zeta$, both as functions of the variable ζ. Once these functions are found, the velocity field and the relation between the parameter region and the physical flow region can be determined as follows:

$$v_x - iv_y = \frac{dw}{dz}(\varsigma), \quad z(\varsigma) = z(0) + \int_0^{\varsigma} \frac{dw}{d\varsigma} \bigg/ \frac{dw}{dz} d\varsigma. \quad (2)$$

The boundary-value problems for the complex velocity function, dw/dz, and for the derivative of the complex potential, $dw/d\zeta$, can be formulated in the parameter plane. Then, applying the integral formulae [13] determining an analytical function from its modulus and argument, and from its argument on the boundary of the first quadrant, respectively, we obtain the following expression for the complex velocity and for the derivative of the complex potential [16]

$$\frac{dw}{dz} = v_0 \exp\left[\begin{array}{l} \dfrac{1}{\pi}\displaystyle\int_0^1 \frac{d\beta}{d\xi} \ln\left(\frac{\xi-\zeta}{\xi+\zeta}\right) d\xi \\ -\dfrac{i}{\pi}\displaystyle\int_0^\infty \frac{d\ln v}{d\eta} \ln\left(\frac{i\eta-\zeta}{i\eta+\zeta}\right) d\eta - i\beta_0 \end{array}\right], \quad (3)$$

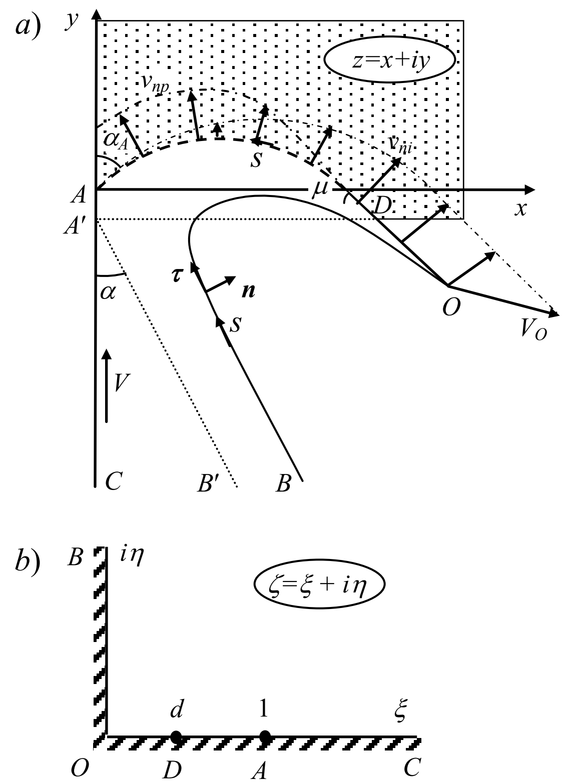


Figure 1. Sketch of the problem for impact between a liquid wedge (dotted line at the time of impact) and an erodible/permeable wall: (a) similarity plane and (b) parameter plane.

$$\frac{dw}{d\zeta} = K\frac{\zeta^{2\mu/\pi-1}}{(1-\zeta^2)^{\alpha_A/\pi}} \exp\left[\begin{array}{l} \dfrac{1}{\pi}\displaystyle\int_0^1 \frac{d\gamma}{d\xi}\ln(\xi^2-\zeta^2)d\xi \\ +\dfrac{1}{\pi}\displaystyle\int_0^\infty \frac{d\theta}{d\eta}\ln(\eta^2+\zeta^2)d\eta \end{array}\right]. \quad (4)$$

where K is a real scale factor, $v_0 = v(\eta)_{\eta=0}$ is the velocity magnitude at point O, $\theta(\eta) = \tan^{-1}(v_n/v_s)$ is the angle between the velocity vector and the free surface, and $\gamma(\xi) = \tan^{-1}(v_n/v_s)$ is the angle between the velocity vector and the interface.

The functions $v(\eta)$ and $\theta(\eta)$ are determined from dynamic and kinematic boundary conditions. In contrast to the impact between the liquid and impermeable/ no eroding solid wedges [14], the functions $\beta(\xi)$ and $\gamma(\xi)$ now become unknown on AD, as well as OD. According to the definitions, these functions can be found if the normal component of the velocity, $v_n = v_{ni} + v_{np}$ on the body surface is known.

The normal component of the velocity, v_{ni}, determines by the shape of the interface, i.e. $v_{ni} = \mathrm{Im}(\bar{z}e^{i\delta})$, where $\bar{z}$ is complex conjugate coordinate and δ is the slope to the interface. The normal component of the velocity due to the permeable interface, v_{np}, depends on the pressure on the body. For porous or/and permeable surfaces, the following equations were proposed in [9, 10]

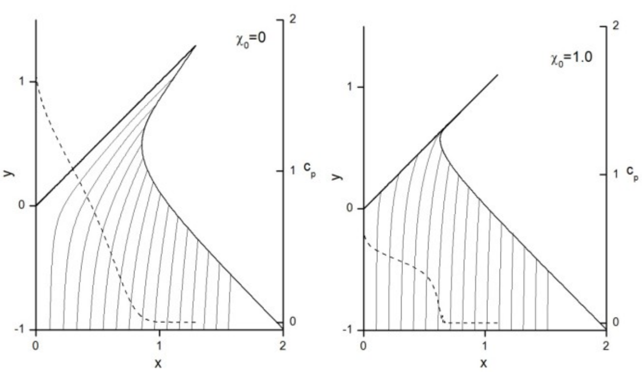
$$v_{np} = \alpha_0 c_p, \qquad v_{np} = \chi_0 \sqrt{c_p}\,, \tag{5}$$

where c_p is the pressure coefficient, α_0 and χ_0 are the non-dimensional coefficients characterizing the porosity and perforation of the thin wall, respectively.

The method of successive approximations is used to solve the total system of integral equations through the iteration procedure.

3. NUMERICAL RESULTS

Results for the angles of the liquid and solid wedges $\alpha = \alpha_A = 45°$ are shown in figure 2. For the impermeable surface in case (*a*), the pressure decreases almost linearly from the wedge apex to the root of the tip jet, while in case (*b*) the pressure decreases more mildly and then faster near the root of the tip jet. This is caused by larger pressure reduction near the apex of the wedge due to the larger flowrate into the wedge side there.



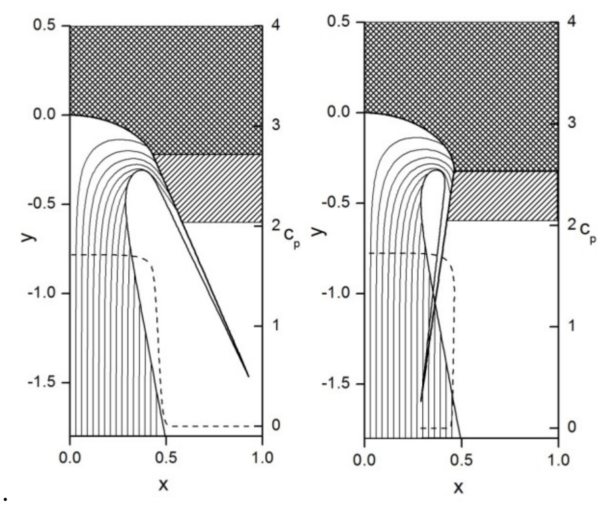
a) *b*)

Figure 2. Streamline patterns, free surfaces (solid lines), and the pressure distribution for the liquid and solid wedges $\alpha_A = 45°$ and $\alpha = 45°$, increment in the stream function $\Delta\psi = 0.1$, and (*a*) χ_0 =0 and (*b*) χ_0 =1.0.

For the part *AD* of the interface, the body surface erosion considered in the present study is assumed to be due to an extremely high normal or/and shear stresses during impact by liquid. This behaviour occurs when the stress has exceeded the yield stress of the material. The problem of flow/structure interaction of eroding bodies has some analogy to the classical Stefan problem for bodies undergoing melting, dissolution, or other similar processes of phase change. The choice of an appropriate model of erosion significantly depends on a particular problem and the cause of the erosion. For the present impact problem, the local normal velocity V^* of the eroding surface is assumed to be linearly related to the pressure *P* and the shear stress τ. In non-dimensional form the constitute equation of erosion takes the form

$$v^* = -K_p c_p - K_\tau v_s^2\,, \tag{6}$$

where v_s is dimensionless tangential component of the velocity, K_p and K_τ are the material-dependent constants. In physical reality, the erosion of the body surface is due to both the pressure and the shear stress, as shown in Eq. (6). However, if $K_\tau = 0$ and $K_p \neq 0$ in Eq. (6), it gives $v^* = 0$ at point *D*, since c_p at point *D* equals to zero. Thus, we introduce a minimal velocity $v^*_{\min}$. When the v^* obtained from Eq.(6) is smaller than $v^*_{\min}$, we set $v^* = v^*_{\min}$. The obtained results are shown in figure 3. More results will be provided in the workshop and can be found in Semenov and Wu [12] (submitted for publication).



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(*a*) (*b*)

Figure 3. Impact between a liquid wedge of half-angle $\alpha = 10°$ and an initially flat solid wall with $v^*_A = 0.6$, (*a*) $v^*_{\min} / v^*_A$ =0.9, (*b*) $v^*_{\min} / v^*_A$ =0.7. The lower dashed region corresponds to the restriction $v^*(s) \geq v^*_{\min}$ along the interface.

4. CONCLUSIONS

The present work gives an extended summary to the work of Semenov & Wu [12] submitted for publication. Its calculations confirmed the expected reduction of the hydrodynamic pressure on a porous or perforated wedge.

For an eroding wall, the interface between the liquid and the body is assumed to change according to the constitute law relating the rate of erosion and the normal pressure. The result shows that in this case the erosion shapes the cavity in such way to provide nearly constant pressure on most part of the cavity surface. The cavity shape is composed of an arc of a near circle, where the pressure is almost constant, and an almost straight line where the restriction $v^*_{\min} \le v^*(s)$ is applied.

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