Forces on vertical cylinders due to steep asymmetric and breaking waves based on the Froude Krylov approximation x2

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Highlights

- Novel implementation of state-of-the-art incompressible SPH with Froude-Krylov forcing to determine 3D loads on cylinders in breaking wave conditions, and with good experimental agreement.
- Determination of the asymmetric wave profile likely to exert maximum loading during wave breaking.

1 Introduction

Studies of wave forces on cylinders is a fundamental problem in fluid mechanics and is of particular interest in offshore and coastal engineering given the many structures with semi-submerged cylindrical supports. Interest has been renewed in recent years with the increased deployment of offshore wind turbines in ever challenging offshore environments and high sea-states. The prediction of forces exerted on these wind turbine columns is key to designing and deploying structures with maximal functionality and survivability. Breaking waves are likely, and, despite being the subject of sustained research for seven decades, there is still debate over the loads and forces exerted due to wave breaking. In this paper, we present the novel numerical approach of using state-of-the-art incompressible SPH with Froude-Krylov forcing to determine 3D loads on cylinders due to 2D plane incident waves - both breaking and non-breaking. The effectiveness of this approach is demonstrated through thorough comparisons with recent experimental work on breaking and non-breaking regular [5] and focused waves [2, 7]. The applicability of this approach is remarkable, and it benefits from relatively low computational cost and ease of implementation. Following validation, the method is also used to gain insight into the maximum forces exerted due to wave breaking, and considers at what point in the breaking process maximum loading occurs.

2 Model and Numerical Method

2.1 The Problem Set-Up and Governing Equations

A two dimensional numerical wave basin of length L and maximum water depth, D, is constructed with a piston wave paddle positioned at the left hand side of the domain, centred at x=0. A cylinder of diameter d_c is centred at a distance x_c from the origin. To enable breaking in the regular wave studies, as in the experiments of [5], a 2.5% gradient ramp is inserted such that the local water depth at the cylinder is then D_{loc}. The governing equations of a low viscosity Newtonian fluid are to be solved: namely, the conservation of momentum,

$$\frac{d\mathbf{u}}{dt}=-\frac{1}{\varrho}\nabla p+\nu\nabla^2\mathbf{u}+\mathbf{f},$$

and the conservation of mass,

$$\nabla\cdot\mathbf{u}=0$$

The symbols $\mathbf{u}$, p, ϱ, ν, and $\mathbf{f}$ denote the fluid velocity, pressure, density, constant kinematic viscosity, and constant gravity body force, respectively.

2.2 Incompressible Smoothed Particle Hydrodynamics

The Lagrangian particle method, incompressible SPH (ISPH) is used to solve the governing equations [4]. In SPH a variable A at a point $\mathbf{r}$ is approximated by a convolution product of the variable A with a smoothing kernel function $\omega_h(|\mathbf{r}-\mathbf{r}'|)$, with a smoothing length h, and is written as

$$A(\mathbf{r})\approx\int_{\Omega} A(\mathbf{r}')\omega_h(|\mathbf{r}-\mathbf{r}'|)\,d\mathbf{r}',$$

where Ω is the supporting domain. When discretised over surrounding Lagrangian fluid particles the interpolation can be written as

$$A(\mathbf{r_i}) \approx \sum_j V_j A(\mathbf{r_j}) \omega_h(r_{ij}),$$

where V_j is the particle volume, $\mathbf{r}_{ij}$ is a distance vector between particle i and j. Incompressibility is imposed through a projection method, which provides a pressure Poisson equation from which the pressure is obtained [4].

2.3 Wave Generation and Froude-Krylov Forcing

A piston-type wavemaker is used to generate all waves, with its motion prescribed using linear wave theory. In the case of focused waves, a JONSWAP spectrum provides the amplitude distribution. In both the experimental studies of interest, the Keulegan-Carpenter numbers are moderate to small and the Reynolds numbers are large, meaning that inertial forces dominate the wave-cylinder interaction. Horizontal forces on the cylinder are calculated using the Froude-Krylov approximation, with an added mass multiplier based on uniform potential flow:

$$F_x = -2 \int_C p n_x dS.$$

3 Results

3.1 Regular Wave Loading

The first wave type to be considered and compared with experiment is that of a plane regular wave incident on a cylinder placed on a slope. The slope has a gradient of 2.5% and the cylinder (diameter 0.2m) is positioned at x_c=10m. A water depth at the cylinder of D_{loc}=0.4m is considered here. The test case considers a regular wave of period T=1.6 s interacting with the cylinder for a range of (local) wave heights. Figure 1 presents the horizontal forces on the cylinder due to experiments of [5] (circles), ISPH (joined black squares), and semi-analytical results (thick black line) from non-linear stream function theory for periodic water waves used in combination with the Morison equation (calculated using the SAWW software [1]). For all wave heights, the ISPH results lie centrally within the experimental measurements recorded by [5], and begin to deviate significantly from SAWW predictions for wave heights above 0.15m. Indeed, for heights above approximately 0.15m, the waves begin to break.

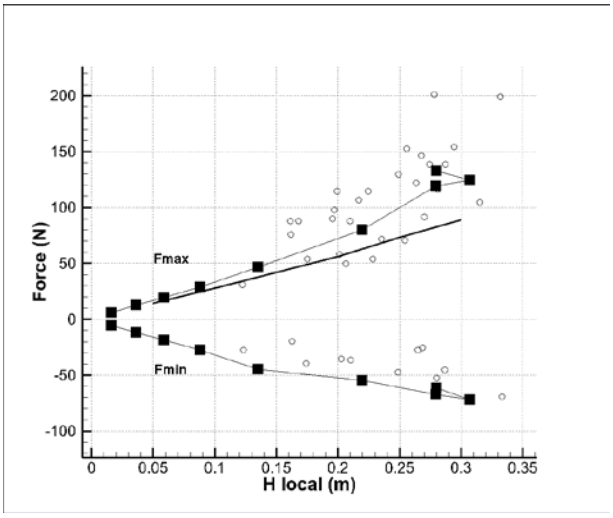
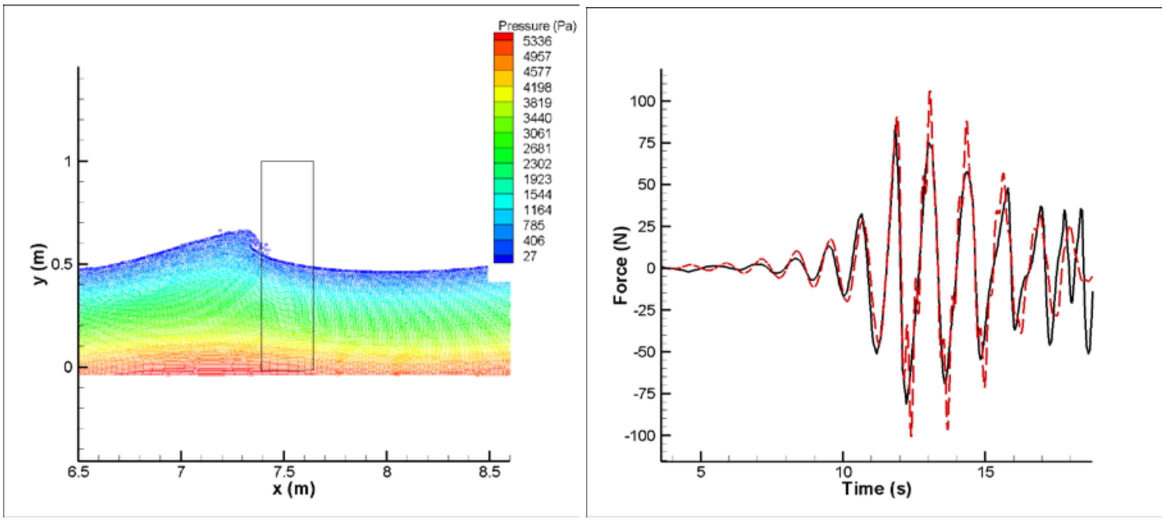


Figure 1: Comparison with experimental results [5] (circles) for loading on cylinder due to regular waves on a slope. Black squares are ISPH, and the thick black line denotes predictions from SAWW.

3.2 Focused Wave Loading

The second type of wave-cylinder interaction considered is that due to focused wave groups as in the experimental study detailed in [7, 2]. There is no slope in this case, as breaking waves may be formed by the focusing of the wave group into an unstable wave form at the front of the cylinder. The numerical wave tank is taken to be of length L=12m and still water depth D=0.505m (as in the experiment). The cylinder (diameter 0.25m) is centred at x_c=7.52m.

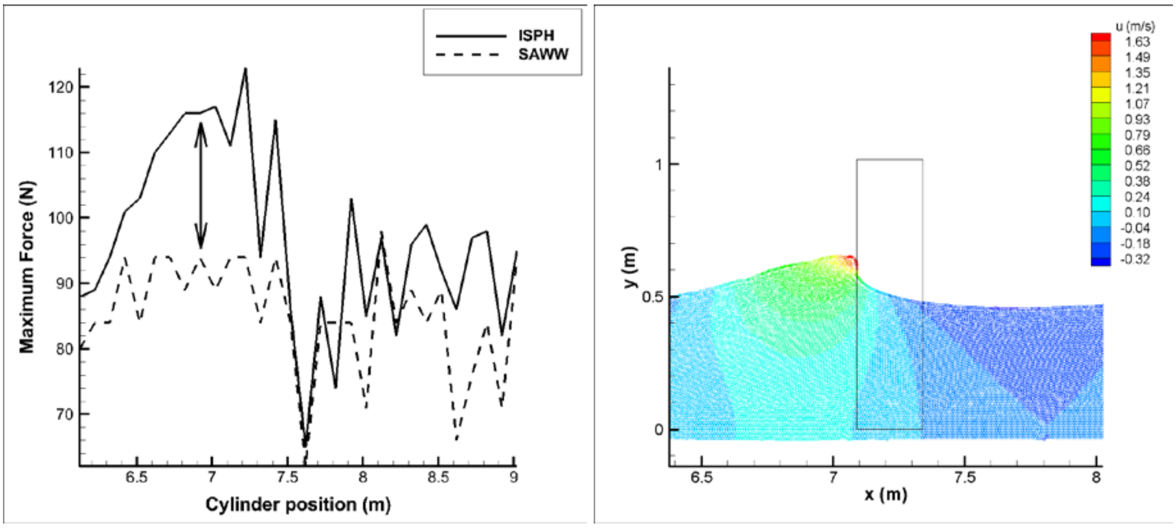
The focused wave case considered is that which results in a plunging breaker with jet impact direct on the cylinder (H=0.22m, f=0.82Hz). Figure 2a shows the breaking wave (with pressure contours) at time t≈11.8s. Quite remarkably, subsequent force measurements up to and including the plunging jet impact at t≈11.8s are well predicted by the FK modelling (Fig. 2b). At later times agreement worsens, but this is to be expected as full cylinder interaction would be required for accurate modelling after such an impact. It seems that in this case a consideration of the undisturbed flow field alone is sufficient to get reasonable agreement in the *total loading* on the cylinder - including at jet impact.



(a) (b)

Figure 2: (a) Wave profile and pressure contours at select times near the focal point at the cylinder. (b) Comparison between experimental and numerical results for the total horizontal force on the cylinder for a focused wave case. The dashed lines denote experimental measurements, the black line is ISPH.

For this case, the cylinder position is moved incrementally about the original experimental position x_c=7.52m, in order to vary the form of the breaking wave at impact. For each cylinder location, the local wave height at the cylinder is determined and input into the SAWW program to determine the loading at that wave height for the equivalent fully non-linear but symmetric (non-breaking) wave profile. Figure 3a plots the maximum horizontal force determined from ISPH and SAWW against various cylinder positions. There is clearly an amplification region (6.5≲x≲7.5m) as highlighted approximately by the arrow, that produces an increased load for ISPH resulting purely from asymmetry in the impacting wave crest. From Figure 3a the globally maximum force occurs at a cylinder location x_c=7.22m, which corresponds to an impact as displayed in Figure 3b, where the wave is at the point of overturning and the wave front vertical. This supports the numerical study of [3], and the experimental findings of [6], where maximum forces were observed with impact at the point of overturning when newly-formed jets remain horizontal.



(a) (b)

Figure 3: (a) Maximum predicted loads on the cylinder at various cylinder locations. (b) The wave profile that exerts the largest load on the cylinder during the breaking process as determined through ISPH predictions. Contours denote the horizontal velocity values.

4 Conclusions

This paper presents a novel and efficient numerical approach to the calculation of three dimensional loads on cylinders due to breaking and non-breaking waves. Fully non-linear wave profiles and dynamics are determined accurately using a state-of-the-art incompressible SPH method. Forces on the cylinder are then determined from the undisturbed flow field using the Froude-Krylov force with theoretical added mass for a uniform flow. Two wave types are studied (regular and focused) and thorough comparisons are made with experimental data [5, 7]. The Froude-Krylov approximation with theoretical added mass is remarkably accurate, and is able to predict loads on cylinders due to waves in various stages of breaking. The numerical simulations remain two dimensional and of moderate resolution, meaning that computations are comparatively fast.

Acknowledgement

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