A linearized exit model for prediction of forces on a body within the 2D+T framework

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The present study is motivated by hydrodynamics of high-speed vessels and aircraft ditching on the water surface, where the wetted part of the hull is streamlined and the hull is elongated in the direction of the motion. Hydrodynamic loads over the wetted part of the elongated hull can be estimated by using the 2D+T approximation [1]. In this approximation, the three-dimensional nonlinear stationary problem is reduced to a two-dimensional transient problem of water entry and exit. This two-dimensional problem can be linearized if both the draft of the body and the deadrise angles of the body cross sections are small. For the stationary three-dimensional problem of a smooth body moving at a constant speed along the water surface, it is convenient to introduce a vertical plane perpendicular to the direction of the body motion and consider the unsteady two-dimensional flow in this plane caused by the body passing through the plane. The intersection of the body surface with this control plane provides a two-dimensional contour which changes its shape in time and interacts with the water surface. For a three-dimensional body with smooth surface the penetration stage ends when the two-dimensional contour stops expanding. During the next stage, which is referred below as the exit stage, the contour contracts and exits from the water. The entry stage was investigated in [2] by the modified Logvinovich model [3]. It was found that the theoretical results are very close to the CFD results obtained by numerical simulations of the Navier- Stokes equations (see [4-6] for details of the simulations). However, the theoretical results from [2] for the exit stage were not as good as for entry stage. During the exit stage, the von Karman model was used in [2]. Recently a linearized exit model was developed in [7]. The exit model is formulated in terms of the linearized pressure with the condition that the speed of the contact points is proportional to the local speed of the flow. This model does not account for the shape of the body but still corresponds quite well to the CFD results from [4]. The model was developed further in [8] to account for a varying in time acceleration of the body. The bodies in [7] and [8] were rigid and only vertical motions were allowed. The two-dimensional problem of a body whose shape varies in time was studied in [2] for an expanding and contracting circular cylinder. The numerical and theoretical forces were very close to each other during the expansion (entry) stage but rather different during the contraction (exit) stage.

In the present paper, we apply the exit model from [7] to the bodies of varying shapes together with corrections accounting for the shape of the body (see [2]) and nonlinear effects. The entry stage is considered below within the original Wagner theory of water impact. It is known (see [2]) that the MLM provides better prediction of the hydrodynamic loads during the entry but here we are concentrated on the negative loads during the exit stage. In the next sections, we provide the solution of the linearized exit model for a body of varying shape, introduce the correction terms and compare the theoretical forces with the CFD results from [2]. Then we apply the model to the problem of an ellipsoid which moves horizontally at a fixed penetration depth and compare the distributions of the sectional forces and pressures with the CFD results. Finally we summarize our findings and draw conclusions.

Exit model for a body shape of which varies in time

The linearized exit model [7] for a body with a shape that is described by the equation $y = y_b(x,t)$, is formulated in terms of the acceleration potential $\varphi_t(x,y,t)$:

$$\nabla^2\varphi_t = 0 \quad (y<0), \qquad \varphi_t = 0 \quad (y=0,\ |x|>c(t)),$$
$$\partial\varphi_t/\partial y = y_{b,tt}(x,t) \quad (y=0,\ |x|<c(t)), \qquad \varphi_t \to 0 \quad (x^2+y^2\to\infty), \tag{1}$$

where $y=0$ corresponds to the level at which the problem is linearized, the function $y_b(x,t)$ is given and the function $c(t)$ is calculated by using the condition that the velocity of the contact points $c'(t)$ is proportional to the local velocity of the flow at these points

$$dc/dt = \gamma\varphi_x[c(t),0,t], \qquad c(0)=c_0, \tag{2}$$

where c_0 is the solution of the equation $y_b(c_0,0)=0$. The coefficient γ is equal to two in the present analysis as in all previous calculations (see [7,8]). The pressure is given by the linearized Bernoulli equation $p(x,y,t) = -\rho\varphi_t(x,y,t)$, where ρ is the water density, and the hydrodynamic force $F_L(t)$ is given by (the subscript L stand for the linearized exit model)

$$F_L(t) = \int\limits_{-c(t)}^{c(t)} p(x,0,t)\,dx. \tag{3}$$

The solution of the boundary problem (1) with the acceleration potential being continuous at the contact points $x=\pm c(t)$, $y=0$ is given by

$$\varphi_{xt}(x,0,t) = \frac{1}{\pi\sqrt{c^2-x^2}}\ \text{p.v.}\int\limits_{-c(t)}^{c(t)} y_{b,tt}(\xi,t)\frac{\sqrt{c^2-\xi^2}}{\xi-x}\,d\xi, \qquad \varphi_x(x,0,t) = \int\limits_0^t \varphi_{xt}(x,0,\tau)\,d\tau. \tag{4}$$

Equations (3) and (4) yield the formula for the hydrodynamic force in term of the functions $c(t)$ and $y_{b,tt}(x,t)$

$$F_L(t) = -2\rho c^2(t)\int\limits_0^{\pi/2} y_{b,tt}(c\sin\theta,t)\cos^2\theta\,d\theta. \tag{5}$$

Equations (2) and (4) provide the equation for the function $c(t)$

$$\frac{dc}{dt} = \frac{\gamma}{\pi}\int\limits_0^t\left(\int\limits_{-c(\tau)}^{c(\tau)}\frac{y_{b,tt}(\xi,\tau)\sqrt{c^2(\tau)-\xi^2}}{\xi-c(t)}\,d\xi\right)\frac{d\tau}{\sqrt{c^2(\tau)-c^2(t)}}. \tag{6}$$

To transform equation (6) to a form suitable for numerical integration, we introduce a function

$$H(t) = \frac{2}{\pi}\int\limits_0^{c(t)}\frac{y_{b,tt}(\xi,t)}{\sqrt{c^2(t)-\xi^2}}\,d\xi,$$

which plays a role of an averaged acceleration of the body. Then we introduce new unknown functions $\sigma(t)$ by $c^2(t) = c_0^2(1-\sigma(t))$ and $f(\sigma)$ by

$$H(t) = f(\sigma)\frac{dc^2(t)}{dt} = -c_0^2 f(\sigma)\frac{d\sigma}{dt} \tag{7}$$

(see [7] for details). Then equation (6) can be written as

$$H\Big(\sigma,t(\sigma)\Big) = 2\gamma c_0^3(1-\sigma)f(\sigma)\int\limits_0^{\sigma}\frac{f(\sigma)}{\sqrt{\sigma-\alpha}}R\Big(\sigma,\alpha,t(\alpha)\Big)\,d\alpha, \tag{8}$$

$$H\Big(\sigma,t(\sigma)\Big) = \frac{2}{\pi}\int\limits_0^{\pi/2} y_{b,tt}\Big(c_0\sqrt{1-\sigma}\sin\theta,t(\sigma)\Big)\,d\theta, \qquad R\Big(\sigma,\alpha,t(\alpha)\Big) = 1-2(\sigma-\alpha)\,\frac{L\Big(\sigma,\alpha,t(\alpha)\Big)}{H\Big(\alpha,t(\alpha)\Big)},$$

$$L\Big(\sigma,\alpha,t(\alpha)\Big) = \int\limits_0^{\pi/2}\frac{y_{b,tt}\Big(c_0\sqrt{1-\alpha}\sin\theta,t(\alpha)\Big) - y_{b,tt}\Big(c_0\sqrt{1-\sigma}\sin\theta,t(\sigma)\Big)}{(1-\alpha)\sin^2\theta-(1-\sigma)}\,d\theta.$$

For parabolic shapes with time-dependent curvature we have

$$y_b(x,t) = B(t)x^2 + h(t), \qquad H(\sigma,t) = \frac{1}{2}B''(t)c_0^2(1-\sigma)+h''(t), \qquad L(\sigma,\alpha,t) = \frac{\pi}{2}c_0^2 B''(t). \tag{9}$$

Equations (7) and (8) serve to determine the functions $f(\sigma)$ and $t(\sigma)$, where $0 \leq \sigma < 1$. Equations (7) and (8) are solved numerically by the generalized version of the algorithm from [8]. Note that the hydrodynamic force (5) and the size of the wetted area predicted by (8) depend on the acceleration of the body but not on its shape within the linearized exit model. In order to account for the shape of the body and, at least partly, for the nonlinear terms in the Bernoulli equation, we use the ideas from the modified Logvinovich model [3], where the pressure distribution along the wetted part of the entering water contour is given by

$$p(x, y_b(x,t), t) = -\rho\Big(\phi_t - \phi_x y_{b,t} y_{b,x}/(1+y_{b,x}^2) + (\phi_x^2 - y_{b,t}^2)/(2(1+y_{b,x}^2))\Big).$$

In this formula, we neglect ϕ_x, $y_{b,x}$ and approximate

$$\phi_t(x,t) \approx \varphi_t(x,0,t) + +vp_{ty}(x,0,t)(y_b(x,t) - y_b(c,t)) = \varphi_t(x,0,t) + y_{b,tt}(x,t)(y_b(x,t) - y_b(c,t)).$$

The term $y_b(c,t)$, the splash-up height, indicates that the problem (1) is obtained by the linearization on the splash-up level as in the generalized Wagner model. In the parabolic approximation (9), the corrected pressure is given by

$$p(x, y_b(x,t), t) = -\rho\phi_t + \rho(x^2 B'' + h'')(c^2 - x^2)B(t) + \rho(B'x^2 + h')^2/2, \tag{10}$$

where $\varphi_t(x,0,t)$ is given by Wagner theory at the entry stage and by the linearized exit model at the exit stage. Correspondingly the force is decomposed as

$$F(t) = F_L(t) + F_b(t), \tag{11}$$

where $F_L(t)$ is given by the linearized models of entry and exit and the corrections term $F_b(t)$ is obtained by integration of the second and third terms in (11) over the wetted interval, $-c(t) < x < c(t)$, both during the entry and exit stages.

Numerical results

The introduced model is applied to the water entry and exit of an expanding and contracting circular cylinder. This problem was studied numerically and by using the MLM during the expansion stage and von Karman model during the contraction stage in [2]. The forces calculated numerically and by the MLM during the expansion stage are very closed to each other, but the numerical and theoretical predictions of the forces during the contraction stage are rather different. Here we are concentrated on the contraction stage describing the expansion stage by the simplified model (10), (11), where the cylinder was approximated by the parabolic contour (9). The conditions of calculations are the same as in [2]. The non-dimensional forces for different ratios $k = R_{max}/R_0$, where R_{max} is the maximum radius of the cylinder and R_0 is its initial radius, are presented in Fig. 1 as functions of the non-dimensional time t^*, where $F^* = Ft_0^2/(4\rho R_0^3(k-1)^2)$, $t^* = t/t_0$ and t_0 is the duration of the impact stage. Here line 1 corresponds to $F_L^*(t^*)$ without any corrections of the linearized model, line 2 shows the total force $F^*(t^*)$ by (10) and (11), line 3 is for the total force $F^*(t^*)$ but the linearization is performed at the equilibrium water level $y = 0$, and line 4 is for the CFD resulting force from [2]. Star stands for non-dimensional variables. It is seen that the present model provides the force closest to the CFD results. Calculations were also performed for the actual shape of the cylinder without the parabolic approximation to demonstrate the accuracy of the approximation (9).

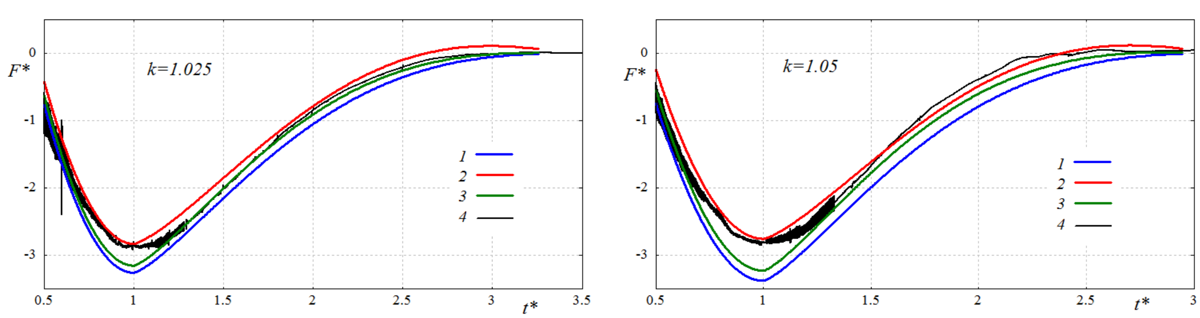

Fig. 1 Non-dimensional forces acting on the expanding/contracting circular cylinder as functions of the non-dimensional time for different values of the parameters k.

Finally we apply the developed model to a three-dimensional steady problem of a rigid ellipsoid

$$(x - Ut)^2/a^2 + y^2/b^2 + (z - h)^2/c^2 = 1$$

with semi-axes a, b and c, which is slightly submerged at $c - h$ and moves along the water surface in the x-direction with constant speed V within the 2D+T approximation. The hydrodynamic loads are determined for each section of the body by using the Wagner theory if the section penetrates water, and by the linearized exit model if the section exits from the water. The control plane is introduced at $x = a$. The forces are calculated for sections of the body

$$z = z_b(y, t) = h - c\sqrt{\tau(2 - \tau) - y^2/b^2}, \qquad \tau = Ut/a,$$

which are approximated by parabolic shapes (9). The distributions of the pressure at $y = 0$ and the sectional forces are shown in Fig. 2 together with CFD results obtained within the Navier-Stokes three-dimensional model without gravity and surface tension. Calculations were performed for $a = 10$m, $b = c = 1$m and $V = 50$m/s. The present linearized model with corrections (10) and (11) over-predicts the loads for the sections in entry but well corresponds to the CFD predictions for the sections in exit. The loads for the sections in entry can be potentially improved by using the MLM and a local three-dimensional model close to the jet overturning region.

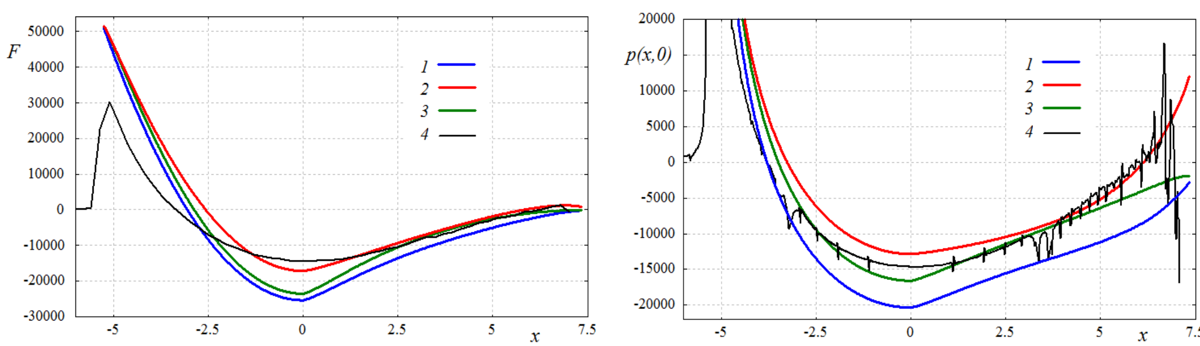

Fig. 2 The hydrodynamic sectional force (on the left) and the pressure at $y = 0$ (on the right) as functions of the longitudinal coordinate x. The correspondences of the lines are the same as in Fig. 1 (see the text).

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