Interaction distance for scattered and radiated waves in large wave energy parks

Malin Göteman, Jens Engström, Mikael Eriksson, and Jan Isberg

Department of Engineering Sciences, Uppsala University, Uppsala, Sweden
Email address of presenting author: malin.goteman@angstrom.uu.se

Highlights

- A multiple scattering method with an interaction distance is presented, which includes hydrodynamical interactions between devices that are within an arbitrarily specified distance from each other.
- The method allows for faster modelling and optimization of wave energy parks to a chosen degree of accuracy, as compared to well-established software. Irregular waves measured at the west coast of Sweden are used as input in a time-domain model where the motion and power of the devices are computed.
- The paper is an extension to the approximate method presented at the previous workshop [1].

1 Introduction

Wave energy technology is currently at the stage where several developers are taking the step to full-scale, commercial energy production. For most of the concepts, large-scale electricity production requires that single wave energy converters (WECs) are combined into larger arrays, or parks. In particular, this is true for the wave energy concept developed at Uppsala University, where the individual WECs are relatively small and consist of surface buoys connected to linear direct-driven generators at the sea bed, see figure 1. The devices in a wave energy park are not independent but will interact both hydrodynamically and electrically, and the interactions may cause large increase or decrease in the produced electricity, depending on the park geometry, wave direction, distance between devices, etc. With the present advancement of several wave energy concepts into the commercial and full-scale phase, the need for reliable simulation tools that can model full wave energy parks is immense, and the area is receiving increasing attention [2, 3, 4, 5].

This paper focuses on the hydrodynamical interactions between large arrays of floating buoys by scattered and radiated waves. In general, when the number of interacting structures in an array grows, the complexity of the model increases, and the numerical simulations are a challenge that call for new methods and theories. Here, a nearest-neighbour approach is taken to enable simulations of large WEC arrays. A model based on the multiple scattering theory [6, 7] is presented, which includes full hydrodynamical interaction between all bodies within an arbitrarily defined *interaction distance*, but excludes the interaction between devices that are further apart than this distance. The interaction distance is so defined that either the interaction by scattered waves is excluded for devices sufficiently far apart (point-absorber approximation [8, 9]), or that the interaction by radiated waves is excluded, or both. As such, the model resembles somewhat the hierarchical multiple scattering theory [10] or similar models of semi-infinite WEC arrays using asymptotic approaches [11, 12], or modules of multiple scales [13]. This method is a cruder, yet perhaps more flexible way to include only the most relevant hydrodynamical interaction. No dependency of different length scales is needed, and any park geometry can be studied; the distance between the WECs may vary, and the positions may be completely arbitrary. The method presented here is a work-in-progress to address multiple parameter optimization of large wave energy parks.

The hydrodynamical coefficients computed with this semi-analytical method are used as input in a time-domain method where the motion of the buoys are computed as a convolution with incident irregular waves, measured at the Lysekil test side on the west coast of Sweden. The power of each device can then be computed and serves as data for comparisons and optimization methods for wave energy parks. The results of the method are compared with results where the hydrodynamics is computed using the state-of-the art software WAMIT.

2 Theory

Coupled equations of motion Arrays of N point-absorber wave energy converters (WECs) are studied, each consisting of a cylinder buoy with radius R and draft d, connected by a stiff line to a direct-driven generator.

The total force determining the dynamics of the float is given as a sum of the exciting force $F_{\text{exc}}(t)$ from the incident waves, the radiation force $F_{\text{rad}}(t)$ from the oscillations of the floats, the hydrostatic restoring force $F_{\text{stat}}(t) = -\rho g \pi R^2 z(t)$ and the damping power take-off force of the generator $F_{\text{exc}}(t) = -\gamma \dot{z}(t) - k_s z(t)$, where k_s is the spring constant of a retracting spring. In this paper, the buoys are constrained to move in heave only. To compute the dynamical forces, we assume non-steep waves and that the fluid is non-viscous, irrotational and incompressible. Then, the fluid velocity potential satisfies the Laplace equation, the boundary constraints can be linearized, and the total fluid velocity potential will be the superposition of incoming, scattered and radiated waves, $\phi = \phi_0 + \phi_S + \phi_R$. In the frequency domain, the time-dependence is factored out and the dynamical forces are given as integrals of the fluid velocity potentials over the wetted surface of the buoys. The vertical coordinate in the frequency domain can be solved for in terms of a transfer function $H(\omega)$, see [1]. In irregular waves, the position is then obtained by a convolution between the inverse Fourier transform of the transfer function with the amplitude of the incident waves, $z(t) = (h * \eta_{\text{in}})(t)$. With the position in time determined, the absorbed power of the WEC can be calculated as $P(t) = \gamma \dot{z}(t)^2$. The irregular waves used to actuate the model are measured outside the Swedish west coast, where the wave climate in general is relatively moderate and the linear approximation can be used with good agreement with full-scale experiments [14].

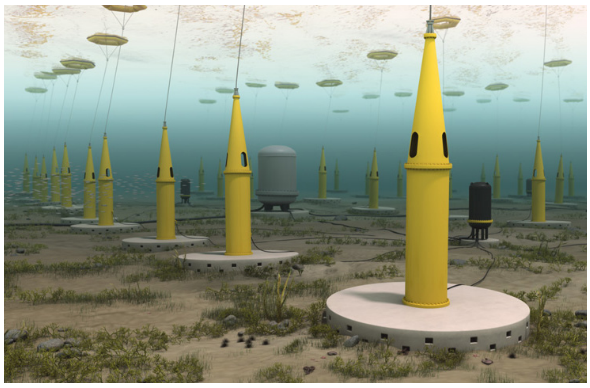

Figure 1: A wave energy park based on the concept of Uppsala University wave energy converters.

Multiple scattering theory Divide the fluid domain with depth h into interior and exterior domains underneath and outside each buoy. A general solution to the Laplace equation and the boundary conditions can be found by separation of variables. In local cylindrical coordinates (r_j, θ_j, z) with origin in the center of the cylinder, the solution in the exterior domain takes the form

$$\phi^{j,\text{ext}} = \sum_{n=-\infty}^{\infty} \left[\sum_{m=0}^{\infty} \psi_m(z) \left(\alpha^j_{mn} \frac{K_n(k_m r_j)}{K_n(k_m R)} + \beta^j_{mn} \frac{I_n(k_m r_j)}{I_n(k_m R)} \right) \right] e^{in\theta_j}, \tag{1}$$

where $\psi_m(z)$ are normalized vertical eigenfunctions. The wave number $k_0 = -ik$ is a root to the dispersion relation $\omega^2 = gk \tanh(kh)$ and $K_n(k_0 r) \propto H_n^{(1)}(kr)$ and $I_n(k_0 r) \propto J_n(kr)$ correspond to propagating modes. The wave numbers k_m, $m > 0$ are roots to the dispersion relation $\omega^2 = -gk_m \tan(k_m h)$ and correspond to evanescent modes. A general potential in the interior domain underneath the cylinder can be written in the form

$$\phi^{j,\text{int}} = \sum_{n=-\infty}^{\infty} \left[\frac{V^j}{2(h-d)^2} \left((z+h)^2 - \frac{r^2}{2} \right) + \gamma^j_{0n} \left(\frac{r_j}{R} \right)^{|n|} + 2 \sum_{m=1}^{\infty} \gamma^j_{mn} \cos(\lambda_m (z+h)) \frac{I_n(\lambda_m r_j)}{I_n(\lambda_m R)} \right] e^{in\theta_j}, \tag{2}$$

where $\lambda_m = m\pi/(h-d)$. Consider an incoming wave ϕ_0 propagating along the x-axis. In the diffraction problem, all buoys are considered fixed ($V^j = 0$ for all $j \in N$). The diffracted wave in the exterior domain of a buoy $i \in N$ will be a superposition of the incident wave and the scattered waves from the remaining cylinders incident on a buoy, which can be added by applying addition theorems for Bessel functions, $\phi_D^{i,\text{ext}} = \phi_0^i + \phi_S^j\big|_i + \phi_S^i$. By requiring continuity between the exterior and interior domains and truncating the infinite sums, the unknown coefficients α_{mn}, β_{mn} and γ_{mn} can be solved for according to the multiple scattering theory [7].

In the radiation problem, the buoys are free to move but there is no incident wave ϕ_0. The exterior potential will in general be a superposition of the outgoing radiated and scattered waves, and the incoming radiated and scattered waves from the remaining buoys, $\phi_D^{i,\text{ext}} = \phi_R^i + \phi_S^i + \phi_S^j\big|_i + \phi_R^j\big|_i$. Again, continuity requirements across the domain borders will imply a system of linear equations where the unknown coefficients can be solved for.

Interaction distance The multiple scattering approach includes a diffraction matrix with all the interaction terms. We introduce an interaction distance $D_{\text{int},S}$ for the diffraction problem, and only include the interaction terms corresponding to devices that are within this distance from each other. The resulting diffraction matrix will be sparse and the computational costs are lowered. Further, a separate interaction distance $D_{\text{int},R}$ for the radiation problem is introduced, which specifies which devices should interact by radiated waves, i.e. which non-diagonal terms in the radiation force are to be computed. This further speeds up the simulations.

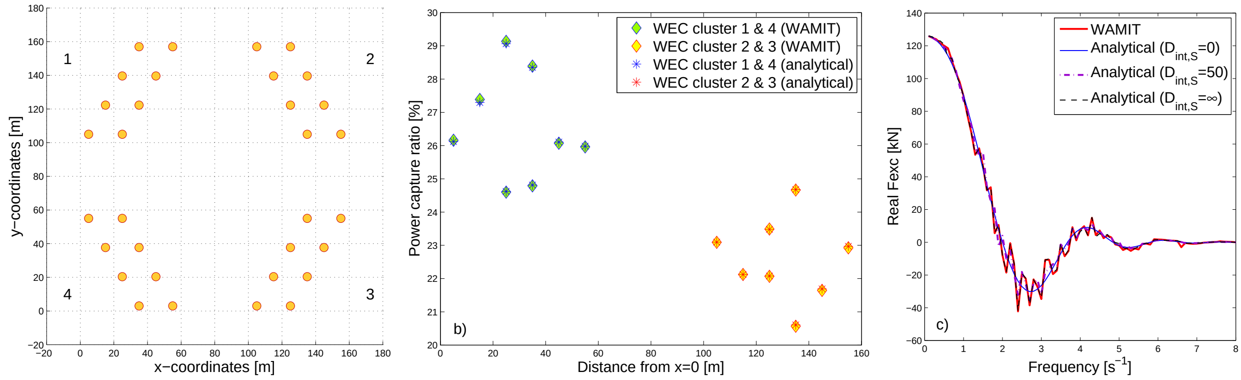

Figure 2: Array with 32 WECs in four clusters, numbered clockwise as in the left figure. b) Power capture ratio for all 32 devices, computed by both the analytical method with infinite interaction distance and with WAMIT. c) The real part of the excitation force in the array: without, with partial and with full multiple scattering.

3 Results

Accuracy and computational cost The hydrodynamical model has been implemented in a Matlab code and connected to a time-domain model with time-series of measured waves as input. The inversion of the diffraction matrix is computed using the object-oriented factorization algorithm of [15].

In figure 2, the power capture ratio $PCR = \bar{P}/(2JR)$, where J is the energy transport of the incident wave travelling along the x-direction, has been computed for each of the devices in an array with 32 WECs, using the analytical method with full multiple scattering (infinite interaction distance). As can be seen from the figure, due to symmetry, the WECs in the two clusters 1 and 4 (and 2 and 3) have identical PCR, and the agreement with WAMIT is good, but with only 28% of the computational cost of WAMIT. All simulations are performed on a standard desktop PC with Intel(R) Xeon(R) 3.07 GHz processor and 6 MB RAM.

In figure 2c), the real part of the excitation force in the same park with 32 WECs is computed without multiple scattering ($D_{\text{int},S} = 0$), with partial multiple scattering ($D_{\text{int},S} = 50$) and with full multiple scattering ($D_{\text{int},S} = \infty$). The partial multiple scattering includes diffraction between devices within 50 m distance from each other; the resulting excitation force will be a combination of single-body and full multiple scattering.

No hydrodynamical interaction for distant structures In figure 3, the power per device for an array with 9 WECs in a square configuration (shown in the figure) has been computed for different values of the interaction distance. The horizontal and vertical distance D between adjacent WECs in the array is increased from 4 m to 80 m. For interaction distances $D_{\text{int},S,R} = 0$ (blue crosses), no hydrodynamical interaction is computed, which implies that the average power per device is constant, independent of the separation distance between devices. For $D_{\text{int},S,R} = 50$ (purple plus signs), only hydrodynamical interaction between devices within 50 m from each other is accounted for. When full hydrodynamical interaction is included (black dots), the agreement with WAMIT is very good. From the figure it is clear that the approximation with $D_{\text{int},S,R} = 50$ agrees with the full multiple scattering simulation when the separating distance between adjacent devices is $D < D_{\text{int},S,R}/2\sqrt{2} = 17.7$, since all devices are within 50 m distance from each other. When D exceeds $D_{\text{int},S,R}/2\sqrt{2}$, the corner WECs lose hydrodynamical contact, and for D exceeding $D_{\text{int},S,R}/\sqrt{5}$, $D_{\text{int},S,R}/2$ and $D_{\text{int},S,R}/\sqrt{2}$, more of the hydrodynamical interactions within the park are lost. At $D > D_{\text{int},S,R}$, no hydrodynamical interactions are computed. These distances are indicated by vertical lines in figure 3. The computational time (lower plot) decreases with reduced number of hydrodynamical interaction terms to be computed.

No multiple scattering for distant structures It is also possible in the model to choose a different interaction distance for the radiation $D_{\text{int},R}$ and diffraction $D_{\text{int},S}$ problems. In figure 4, the average power per device has been computed as function of number of devices in square array with WECs at 20 m distance from each other. The agreement between the analytical method with full multiple scattering and WAMIT is very good, the deviation between the methods is never larger than 0.2%. Interaction distance $D_{\text{int},R} = D_{\text{int},S} = 50$ gives an approximation whose error increases with the number of WECs: 3% for an array with 25 WECs and 7% for 49 WECs. A better approximation, which gives the same time savings, is to include interaction by radiation for all WECs ($D_{\text{int},R} = \infty$), but neglect multiple scattering ($D_{\text{int},S} = 0$). Also here, the error increases with the number of devices, but is still less than 3% for an array with 49 WECs.

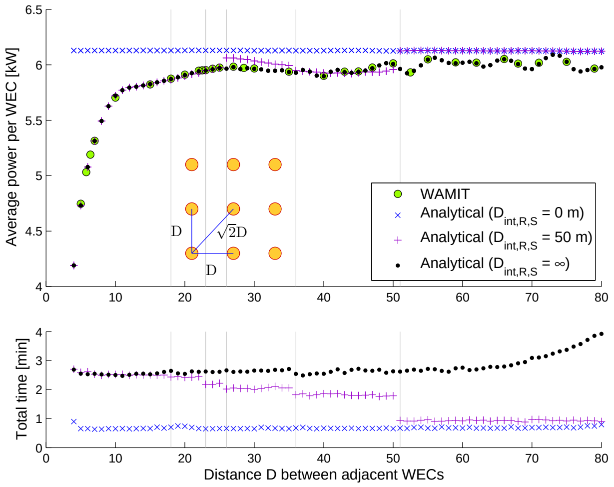

Figure 3: Power per device computed with the analytical model with different interaction distances in the array with 9 WECs, compared to the value computed using WAMIT. The lower plot shows the total computational time for each of the simulations.

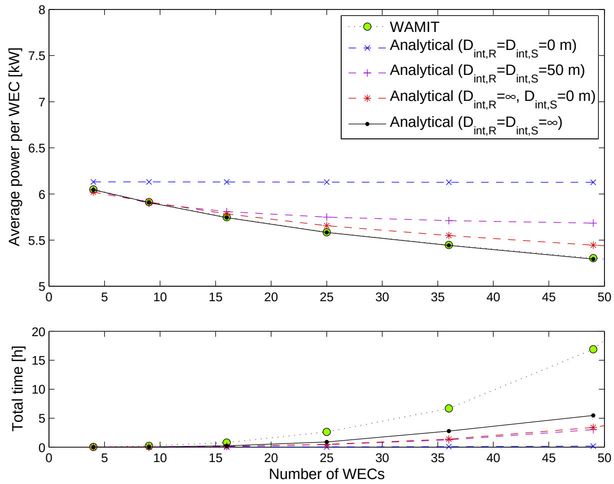

Figure 4: Average power per device computed with WAMIT, with the analytical method, and with the approximate analytical method with different interaction distances, as well as the corresponding computational time for each simulation.

4 Discussion

An advantage of having introduced the interaction distance in the model can be seen in figures 3: when WECs are far away from each other, the error of neglecting their hydrodynamical interactions is less than when they are close. Indeed, the point-absorber approximation has been found to give accurate results when the separating distance between devices is $D > 5R$ [16]. Hence, in a large park, a suitable interaction distance can be defined, such that both an acceptable accuracy and computational speed is achieved. Depending on the accuracy needed, this interaction distance can be increased or decreased as wanted. As figure 4 shows, for large parks it may be advantageous to choose different interaction distances for the radiation and diffraction problems.

Up to now, most optimization studies of wave energy parks have either compared different array configurations by trial-and-error, changed one-parameter at a time or studied only regular waves. Multiple parameter optimization of large-scale wave energy parks requires a hydrodynamical model which is both reasonably accurate and fast. This model is a work-in-progress to address optimization modelling of wave energy parks in a more systematic way and with realistic, irregular waves as input.

References

[1] M. Göteman, J. Engström, M. Eriksson, J. Isberg, and M. Leijon. In *29th IWWWFB*, Osaka, Japan, 2014.
[2] J. Cruz, R. Sykes, P. Siddorn, and R. Eatock Taylor. In *Proc. of the 8th EWTEC*, Uppsala, Sweden, 2009.
[3] B.F.M. Child and P. Laporte Weywada. In *Proc. of the 10th EWTEC*, Aalborg, Denmark, 2013.
[4] M. Vicente, M. Alves, and A. Sarmento. In *Proc. of the 10th EWTEC*, Aalborg, Denmark, 2013.
[5] J. Engström, M. Eriksson, M. Göteman, J. Isberg, and M. Leijon. *J. Appl. Physics*, 114:204502, 2013.
[6] M. Ohkusu. In *Intl. Symp. on Dynamics of Marine Vehicles and Structures in Waves*, volume 12, pages 107–112, London, UK, 1974.
[7] H. Kagemoto and D.K.P. Yue. *J. Fluid Mech.*, 166:189–209, 1986.
[8] D. Evans. pages 213–249. Power from Sea Waves. Academic Press, 1980.
[9] J. Falnes. *Appl. Ocean Res.*, 2:75, 1980.
[10] M. Kashiwagi. In *The 14th IWWWFB*, Port Huron, USA, April 1999.
[11] M. A. Peter and M. H. Meylan. *J. Fluid Mech.*, 575:473–494, 2007.
[12] B.G. Carter and P. McIver. *J. Eng. Math.*, 81:9–45, 2013.
[13] X. Garnaud and C.C. Mei. *J.Fluid Mech.*, 635, 2009.
[14] M. Eriksson, R. Waters, O. Svensson, J .Isberg, and M. Leijon. *J. Appl. Physics*, 102(084910), 2007.
[15] T.A. Davis. *ACM Transactions on Mathematical Software*, 39(4):28:1–28:18, 2013.
[16] S.A. Mavrakos and P. McIver. *Applied Ocean Research*, 19(5–6):283 – 291, 1997.