LAGRANGIAN NUMERICAL WAVE-CURRENT FLUME

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SUMMARY

Lagrangian formulation for surface waves with vorticity is used to create a numerical wave-current flume. The numerical flume is then used to reproduce a physical experiment on focused wave groups in sheared currents. The numerical results include evolution of the free surface of focused wave groups in still water and over in-line and opposing currents and flow kinematics under such waves. Numerical results are compared with experiment and demonstrate good agreement.

1 INTRODUCTION

The natural way of modelling strong deformations of a fluid domain is using equations of fluid motion in the Lagrangian form which is solved in a fixed domain of Lagrangian labels. Lagrangian models are capable of efficient modelling of very steep and overturning waves. Advantages of the Lagrangian approach for numerical modelling of steep waves were demonstrated in Buldakov (2013*b*), Buldakov (2013*a*) were a simple fully Lagrangian numerical model was developed and applied for various wave problems.

Another advantage of the Lagrangian formulation is very simple representation of vortical flows. The vorticity in Lagrangian coordinates is constant in time and is specified by initial velocity conditions. This paper exploits this advantage of the Lagrangian formulation. We generalise the previously developed numerical method for free-surface flows with arbitrary sheared currents. Certain practical problems of numerical implementation of the method, such as excessive deformation of physical computational domain, are addressed and successfully solved. The method is then used to create a numerical wave-current flume. The numerical flume is used to reproduce physical experiments on evolution of wave groups over currents. An iterative methodology of generating focused wave groups on currents (Stagonas *et al.*, 2014) is used for both physical and numerical experiments. The results for surface elevation and wave kinematics are obtained and good comparison is achieved between numerical and experimental results.

2 LAGRANGIAN 2D WATER-WAVE FORMULATION WITH VORTICITY

A general Lagrangian formulation for two-dimensional flow of inviscid fluid with a free surface can be found in Buldakov *et al.* (2006). We consider time evolution of Cartesian coordinates of fluid particles $x(a,c,t)$ and $z(a,c,t)$ as functions of Lagrangian labels (a,c). The formulation includes the Lagrangian continuity equation and the Lagrangian form of vorticity conservation

$$\frac{\partial(x,z)}{\partial(a,c)} = J(a,c)\,; \quad \frac{\partial(x_t,x)}{\partial(a,c)} + \frac{\partial(z_t,z)}{\partial(a,c)} = \Omega(a,c)\,, \tag{1}$$

and the dynamic free-surface condition

$$x_{tt}x_a + z_{tt}z_a + g\,z_a\Big|_{c=0} = 0\,. \tag{2}$$

Functions $J(a,c)$ and $\Omega(a,c)$ are given functions of Lagrangian coordinates. $J(a,c)$ is defined by initial positions of fluid particles associated with labels (a,c). We select $(a,c) = (x_0,z_0)$, which gives $J = 1$. $\Omega(a,c)$ is the vorticity distribution and is defined by the velocity field at $t = 0$. A sheared current can be defined by specifying vorticity depending only on the vertical Lagrangian coordinate c. For our choice of Lagrangian labels the parallel current can be specified as $x = a + V(c)t;\ z = c$, where $V(c) = V(z_0)$ is the current profile. Substitution to the second equation of (1) gives

$$\Omega(a,c) = \Omega(c) = V'(c)\,. \tag{3}$$

Therefore, waves on a sheared current with an undisturbed profile $V(z_0)$ are described by equations (1) with the free surface boundary condition (2) and the vorticity distribution given by (3). A particular problem within a general formulation is defined by initial conditions and boundary conditions on the bottom and side boundaries.

3 LAGRANGIAN NUMERICAL WAVE-CURRENT FLUME

The problem formulated in the previous section is solved numerically using a finite-difference technique. Detailed description of the numerical method can be found in Buldakov (2013*a*) and Buldakov (2013*b*). The numerical method for the formulation with vorticity is mostly identical to the irrotational formulation. Here we only mention differences relevant to the formulation with a sheared current used for construction of a numerical wave-current flume. In the numerical formulation we do not use vorticity distribution directly. We differentiate

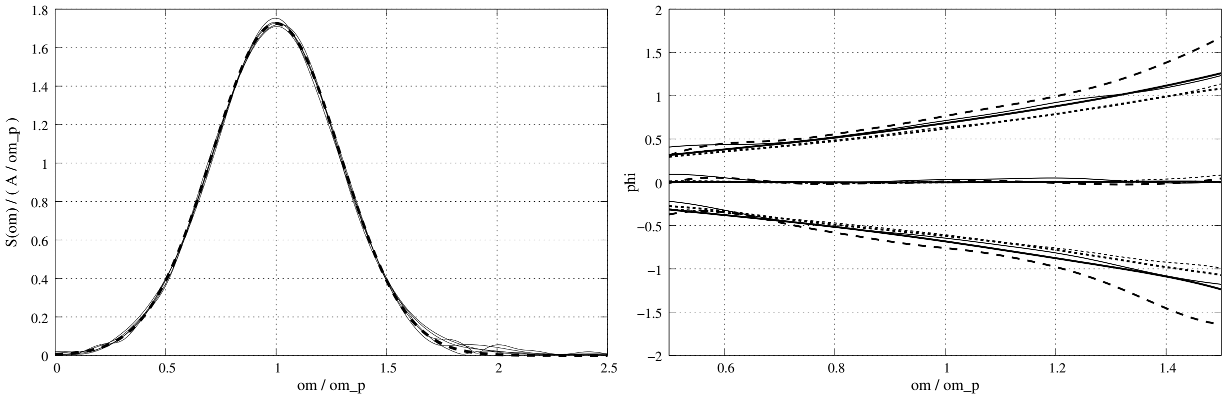

Figure 1: Left: thick dashed – target amplitude spectrum; thin solid – linearised amplitude spectra for all computational and experimental cases at the focus point ($x = 0$). Right: phase spectra at the focus point ($x = 0$) and at positions $x = \pm 0.7h = \pm 0.108\lambda$. Thin – experimental, thick – computational. Solid – no current, dotted – in-line current, dashed – opposing current (computational only). Frequency is scaled by the peak frequency ω_p and amplitude spectral density by the ratio of the linear focus amplitude A to the peak frequency A/ω_p.

the vorticity conservation equation with respect to time to exclude Ω. Vorticity is implicitly defined by the initial condition, when we specify x and z for three initial time steps, which are required to start time marching. The numerical wave-current flume is created by specifying boundary conditions allowing safe inlet and outlet of the current flow into the computational domain, wave generation and absorption of waves reflected from domain boundaries. The two latter requirements are satisfied using re-formulated free-surface boundary condition (2) which includes time-varying pressure gradient and artificial dissipation

$$x_{tt}x_a + z_{tt}z_a + g\, z_a + k(a)\left((x_t - V(c))\, x_a + z_t z_a + g\, z_a\right) = P_x(a,t)\,\Big|_{c=0}.$$

The last term in the right-hand side represents the artificial surface dissipation with the space-varying dissipation coefficient k, and the term on the left-hand is the surface pressure gradient. The dissipation coefficient is selected to be zero in the working section of the flume and gradually grows to a large value near the inlet and outlet boundaries. As the result, the free surface at the boundaries remains relatively steady and does not move from its original position. This serves a double purpose. First, reflections from the boundaries are significantly reduced. Second, the boundary conditions at the inlet and outlet boundaries can be specified simply as the undisturbed velocity profile at the inlet ($x_t(a_{in}, c, t) = V(c)$) and as a parallel flow at the outlet ($z_a(a_{out}, c, t) = 0$). The wave in the flume is generated by creating an area in front of one of the wave absorbers where pressure distribution of a prescribed shape is defined. Time-varying amplitude of this pressure disturbance is used as a control input for wave generation. An additional difficulty with numerical realisation of the Lagrangian formulation with a sheared current is sheared deformation of the original domain in physical coordinates which indefinitely increases with time. The shape of the domain eventually becomes impractical as it moves out of an area of interest. Besides, accuracy of computations for strongly deformed computational cells considerably reduces. To avoid these difficulties we perform sheared deformation of the Lagrangian domain to compensate the deformation of the physical domain. Such deformation takes place after several time steps and moves boundaries of the physical domain back to the original vertical lines. After this Lagrangian labels are re-assigned to new values to preserve the rectangular shape of the Lagrangian computational domain with vertical and horizontal lines of the computational grid.

4 NUMERICAL AND EXPERIMENTAL SETUPS

We use the numerical wave-current flume to reproduce results obtained during experimental study of focused wave groups over sheared currents performed in the coastal recirculating flume in the fluids laboratory of the Department of Mechanical Engineering at UCL. The flume has the width of $1.2\,m$ and the distance between two piston wavemakers is about $16\,m$. The depth for all tests was set to $h = 0.5\,m$. A recirculating system with three parallel pumps and vertical inlets $13\,m$ apart is used to create a current. A paddle on the right end of the flume is used as a wave generator and the opposite paddle as an absorber. Blocks of wire mesh of trapesiodal shape are installed on top of the inlet and outlet to condition the flow and create a desired current profile. Surface elevation at selected points along the flume is measured by resistance wave probes and a PIV system is used to measure flow kinematics. Wave groups are generated with 4 constant phase shifts within the

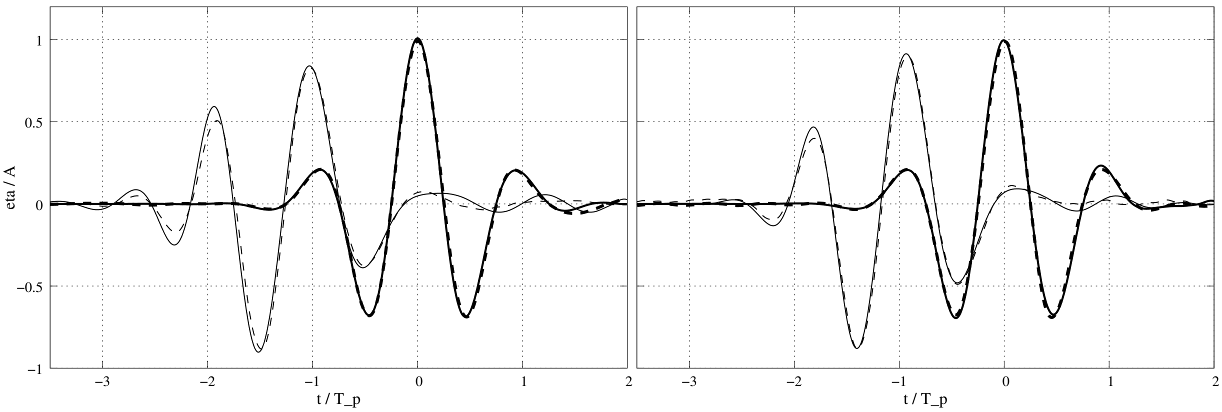

Figure 2: Comparison of computational (solid) and experimental (dashed) time histories of linearised surface at the focus position $x = 0$ (thick) and at position $x = -6.34h = -0.98\lambda$ (thin). Left: no current. Right: in-line current. Time is scaled by the peak period T_p and surface elevation is scaled by the linear focus amplitude A.

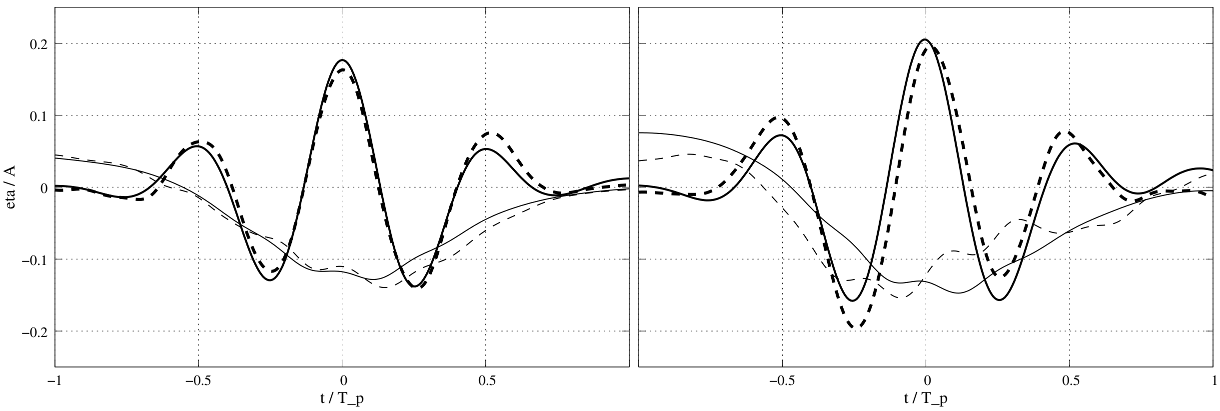

Figure 3: Comparison of computational (solid) and experimental (dashed) time histories of plus (thick) and minus (thin) second-order components at the focus position $x = 0$. Left: no current. Right: in-line current. Time is scaled by the peak period T_p and surface elevation is scaled by the linear focus amplitude A.

same amplitude spectrum. The resulting signals for surface elevations are used to extract linear part of the signal as well as non-linear components including second order subharmonics and second, third and fourth order super-harmonics. An iterative procedure is used to focus the wave group at a prescribed location and time. More details of the experimental setup and the methodology can be found in Stagonas *et al.* (2014). At this stage only tests for waves without current and over in-line currents are completed.

To validate the numerical results we select a moderately non-linear wave group with a Gaussian linear amplitude spectrum. The peak frequency of the spectrum is $f_p = 0.6\,Hz$ for water depth $h = 0.5\,m$. The corresponding linear wave length is $\lambda_p = 6.488\,h$. The linear focus amplitude of the wave is $A = 0.1\,h$. The normalised linear target spectrum is presented on figure 1. The numerical waves were generated in still water and over in-line and opposing currents of maximal velocity $V_0 = 0.09\,C$, where $C = \sqrt{gh}$ is linear shallow water celerity. The profile for computations was created using preliminary ADV measurements of current velocity. The working section of the numerical flume free from wave generator and absorbers is set to about $20\,h$. The origin of the coordinate system is set to the focusing position at the center of the flume with a horizontal axis pointing against the wave propagation direction. The waves in the numerical flume were generated using the same iterative focusing procedure as in the physical flume. Calculations are performed for waves without current as well as for in-line and opposing currents. Calculation results include time histories of surface elevation at the same positions as in physical experiment and flow kinematics at the focus point.

5 RESULTS

Results of numerical tests and their comparison with physical experiment are demonstrated on figures 1-5. Figure 1 demonstrates efficiency of the focusing procedure for all experimental and computational cases. The evolution of phases near the focus point is physically relevant for different current cases and compares well between computations and experiment. The behaviour of the linearised wave at the focus point is identical for

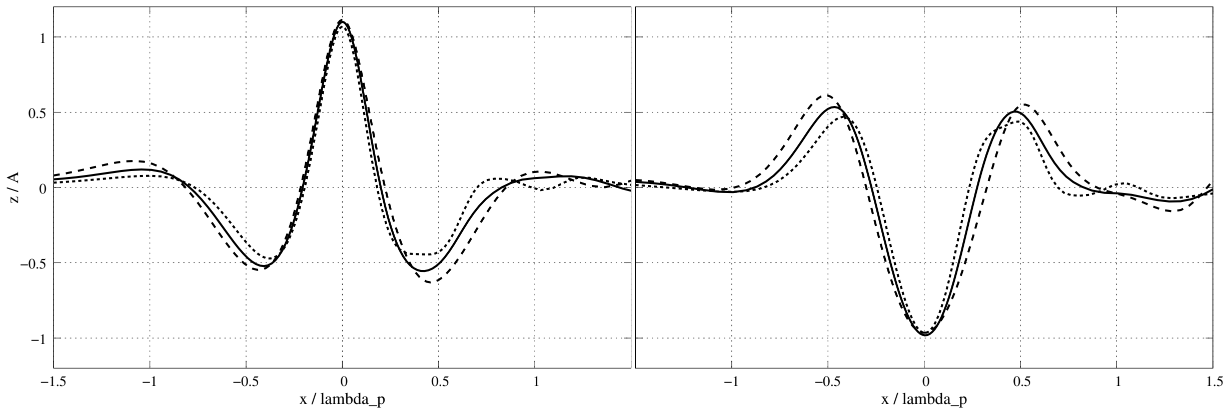

Figure 4: Computational surface profiles at the focus time $t = 0$. Solid – no current; dashed – opposing current; dotted – in-line current. Left: peak focused. Right: trough focused. Vertical coordinate is scaled by the linear focus amplitude A and horizontal coordinate by the peak wave length λ_p.

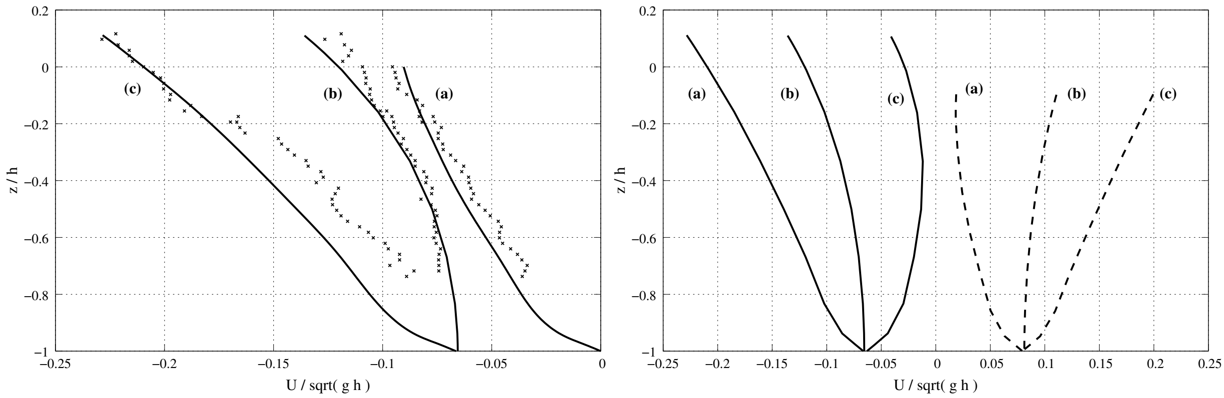

Figure 5: Left: Comparison of computational (lines) and experimental (dots) horizontal velocity profiles for the incoming current (a) and under the crest of the peak focused wave at focus position $x = 0$ and focus time $t = 0$ without current (b) and over in-line current (c). Right: Computational profiles of horizontal velocity under peak (solid) and trough (dashed) focused waves. (a) – in-line current; (b) – no current; (c) – opposing current. Velocity is scaled by the shallow-water celerity $C = \sqrt{gh}$ and vertical coordinate is scaled by depth h.

all cases (figure 2), which is not surprising as this was the aim of the iterative focusing procedure. However, good comparison at the position one wave length before the focusing point demonstrates that the dispersion relation is represented by the numerical model with good accuracy. Figure 3 shows that non-linear terms are also well captured by the numerical model. The calculated wave profiles presented for at focus time for different current direction for peak and trough focused waves are shown on figure 4. Finally, figure 5 demonstrate good agreement with measured wave kinematics. The discrepancy observed for the in-line current case is explained by the defect in the incoming current profile for this experimental run. It is hoped that further analysis of experimental results and generating more experimental cases will allow to remove this discrepancy.

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