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Recent progress on the GN model for a two-layer flow

B.B. Zhao[1,*], W.Y. Duan[1], R.C. Ertekin[2] and W.C. Webster[3]
[1] College of Shipbuilding Engineering, Harbin Engineering University, 150001 Harbin, China
Email: zhaobinbinheu@hotmail.com
[2] Department of Ocean & Resources Engineering, University of Hawai'i, Honolulu, HI 96822, USA
[3]Civil & Environmental Engineering, The University of California, Berkeley, California 94720-1710, USA

Highlights:

- The GN models for a two-layer flow with free surface and with a solid lid are developed.
- Generation of solitary waves by a moving body (with free surface) is simulated by the GN model and compared with Grue et al. (1997)'s results (with a solid lid); some differences are found.

1 Introduction

Grue et al. (1997) used a time-stepping method for the unsteady, fully nonlinear two-dimensional motion of a two-layer fluid. They performed a detailed investigation of upstream generation of solitary waves by a moving body, and observed that wave trains with amplitudes comparable to the thickness of the thinner layer are generated. The results indicate that weakly nonlinear theories in many cases have quite limited applications in modeling unsteady transcritical two-layer flows, and that a fully nonlinear method in general is required. In the case simulated in their paper, $\rho_2/\rho_1 = 0.7873$, and they used a fully nonlinear model with a solid lid.

Zhao and Duan (2012); Zhao et al. (2014) applied the high level Green-Naghdi (GN) equations to some wave transformation problems. Zhao and Duan (2013) developed the high-level GN model with the free surface for a two-layer flow for the first time. They used the gravity collapse method to generate an internal solitary wave of large amplitude.

To analyze the differences between the GN models with free surface and solid lid, we develop here the GN models with a solid lid and use Newton Iterative method to obtain the traveling solution of the GN models with free surface and solid lid. With the traveling solution, we can easily model an efficient wave-maker to generate an internal solitary wave in the GN model. Finally, we reproduce the case simulated in Grue et al. (1997). In their calculations, they use a fully nonlinear model with solid lid. Whereas, we calculate this case with the fully nonlinear GN model with free surface. Some differences are observed.

2 GN Theory

The GN equations for a two-layer flow with free surface is given by Zhao and Duan (2013). Here, we only give the theory on the GN two-layer model with a solid lid. For two-layer problems, we set the bottom as $z = \alpha(x,t)$, which is known, the interface as $z = \beta(x,t)$, which is unknown, and the rigid lid as $z = \gamma = h_2$ which is also known. The GN equations for a two-layer flow with solid lid are given as

$$\frac{\partial}{\partial x}\left(G_n^U + gS_n^U\right) + nE_{n-1}^U - \beta^n \frac{\partial}{\partial x}\left(G_0^U + gS_0^U\right) + \left(\gamma^n - \beta^n\right)\frac{\partial}{\partial x}\left(\frac{\hat{p}^U}{\rho^U}\right) = 0 \quad (1)$$

for $n = 1, 2, \cdots, K^U$.

$$\frac{\partial}{\partial x}\left(G_n^L + gS_n^L\right) + nE_{n-1}^L - \alpha^n \frac{\partial}{\partial x}\left(G_0^L + gS_0^L\right) + \left(\beta^n - \alpha^n\right)\frac{\partial}{\partial x}\left(\frac{\hat{p}^L}{\rho^L}\right) = 0 \quad (2)$$

for $n = 1, 2, \cdots, K^L$.

$$\frac{\partial \beta}{\partial t} = \sum_{n=0}^{K^U} \beta^n \left(w_n^U - \frac{\partial \beta}{\partial x} u_n^U\right) \quad (3)$$

$$\frac{\partial \beta}{\partial t} = \sum_{n=0}^{K^L} \beta^n \left(w_n^L - \frac{\partial \beta}{\partial x} u_n^L \right) \tag{4}$$

K^U and K^L are the GN levels in the upper and lower layers, respectively. When we use e.g., K^U=3 and K^L=3, we will name it the GN-3-3 (GN-K^U-K^L) model. The Eqs. (1)-(4) completes the GN models for a two-layer flow. However, we need to mention that the dynamic boundary conditions are given by $\hat{p}^L = p|_{z=\beta} = \bar{p}^U = \rho^U G_0^U + \rho^U g S_0^U + \hat{p}^U$.

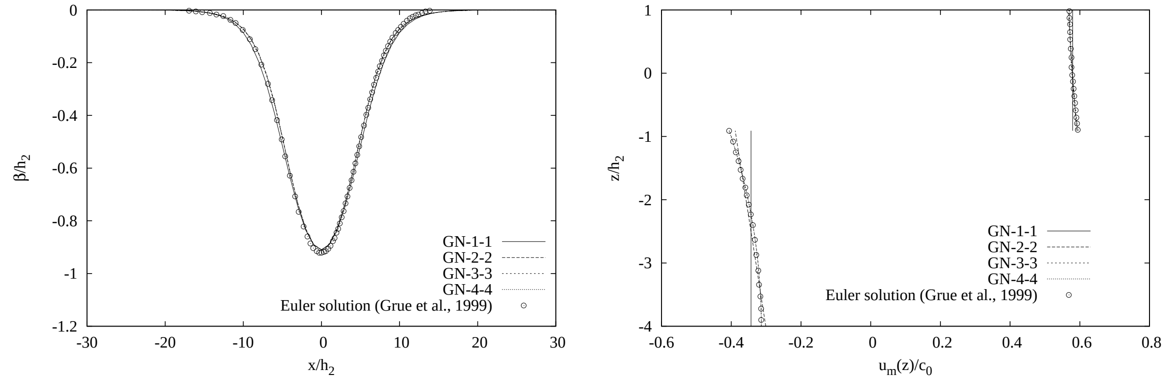
The upper surface in the upper layer is assumed to be $z = \gamma = h_2$, where h_2 is the depth of the upper layer before it is disturbed. We mention that $\hat{p}^U = p(x, h_2, t)$ is not zero. When we consider the traveling solution for the GN models, we assume that the bottom of the lower layer is $z = \alpha = -h_1$, where h_1 is the depth of the lower layer before it is disturbed. When we simulate the case with a time-varying bottom, the dynamic bottom boundary condition is similar to the one just given, except that we change β to α. As to the numerical algorithm used here, see Zhao and Duan (2013).

3 Test cases

3.1 $\rho_2/\rho_1 = 0.977$, comparison between with free surface and rigid lid.

Grue et al. (1999) conducted physical experiments which are calibrated with a layer of fresh water above a layer of brine. The depth of the brine was h_1 = 62cm and the depth of the fresh water was h_2 = 15cm. The density of brine was 1.022g/cm^3 and the density of fresh water was 0.999g/cm^3.

We present here traveling wave solutions for GN-1-1, GN-2-2, GN-3-3 and GN-4-4 with free surface and with solid lid. For $a/h_2 = 0.91$, the results are shown in Figure 1. Figure 1 shows the results with solid lid.

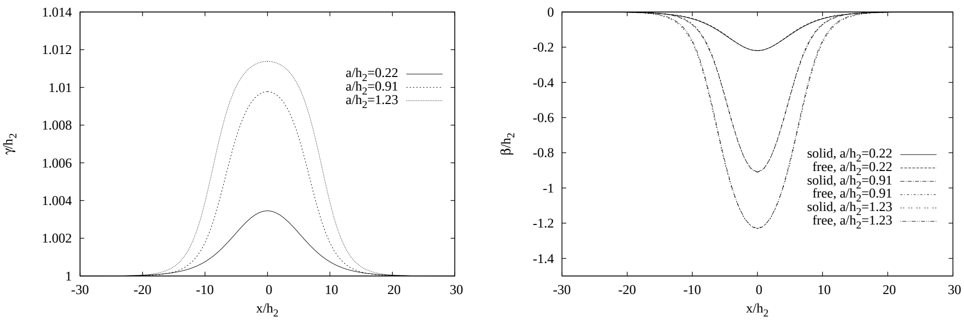


(a) the internal wave profile

(b) the velocity distribution along the water depth at the maximum displacement

Figure 1: GN results with solid lid, $\rho_2/\rho_1 = 0.977$

If we use the GN model with free surface, the free surface is disturbed, and this is shown in Figure 2(a).



(a) the wave profile on the free surface

(b) the wave profile on the interface

Figure 2: GN results with free surface, $\rho_2/\rho_1 = 0.977$

From Figure 2(a), we find that as the amplitude of internal wave goes up, the wave amplitude on the free surface goes up too. When the internal wave amplitude reaches $1.23h_2$, the amplitude on the free surface reaches $1.14\%h_2$. This is a small amount compared with the depth of upper layer. Therefore, it is reasonable to assume that the free surface can be assumed to be a solid lid, see Figure 2(b) (it shows the comparison of the internal wave profile calculated by the GN-3-3 model with free surface and with solid lid). We can see that the differences between them are small. More results will be presented during the workshop.

3.2 $\rho_2/\rho_1 = 0.8$, comparison between with free surface and rigid lid.

Next, we change the density ratio to $\rho_2/\rho_1 = 0.8$. The other parameters are the same as given in Section 3.1. We present the wave profiles on the free surface first, see Figure 3. Figure 3 shows that the amplitude

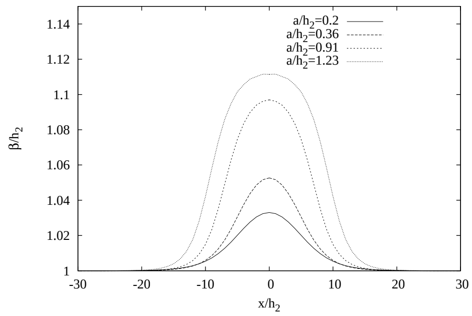
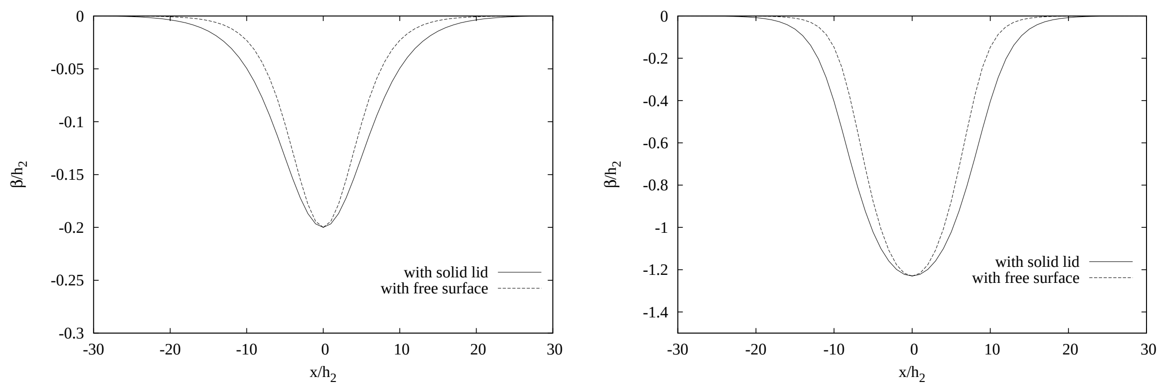


Figure 3: wave profiles on the free surface, $\rho_2/\rho_1 = 0.8$

of the wave on the free surface may reach $11.145\%h_2$ when the internal wave amplitude reaches $1.23h_2$. This disturbance is almost ten times compared with the value $1.14\%h_2$ in the case of $\rho_2/\rho_1 = 0.977$.

If we assume the free surface to be a solid lid, the internal wave profile will be much more different compared with the results for the free surface. The comparisons are shown in Figure 4. From Figure 4, we



(a) $a/h_2 = 0.2$

(b) $a/h_2 = 1.23$

Figure 4: Comparison on the internal wave profile between GN results with free surface and with solid lid, $\rho_2/\rho_1 = 0.8$

also see that even in the small amplitude case (Figure 4(a)), there are significant differences in the internal wave profile between the GN results with free surface and with solid lid. More results will be presented during the workshop.

3.3 Density ratio = 0.7873, upstream generation of internal solitary waves by a moving body

In this section, we reproduced the case simulated in Grue et al. (1997). In this case, $h_2/h_1 = 4$, $h_1 = 0.03m$, $h_2 = 0.12m$. The body on the bottom moves as $U = 1.1c_0$, where the non-dimensional linear shallow water speed c_0 is given by ${c_0}^2 = (gh_1h_2(\rho_1 - \rho_2)) / (\rho_1 h_2 + \rho_2 h_1)$. The shape of the moving body on the bottom can be described by a half-elliptical with horizontal half-axis $10h_1$ and vertical half-axis $0.1h_1$. And, we make the body's velocity increase from 0 to U in $5s$. Here, we use GN-1-1, GN-3-3 and GN-4-4 theories to simulate the same case with the same parameters. In Grue et al. (1997)'s calculations, a fully nonlinear model with a solid lid was used. We calculated this case with fully nonlinear GN model with free surface. And, some difference are observed.

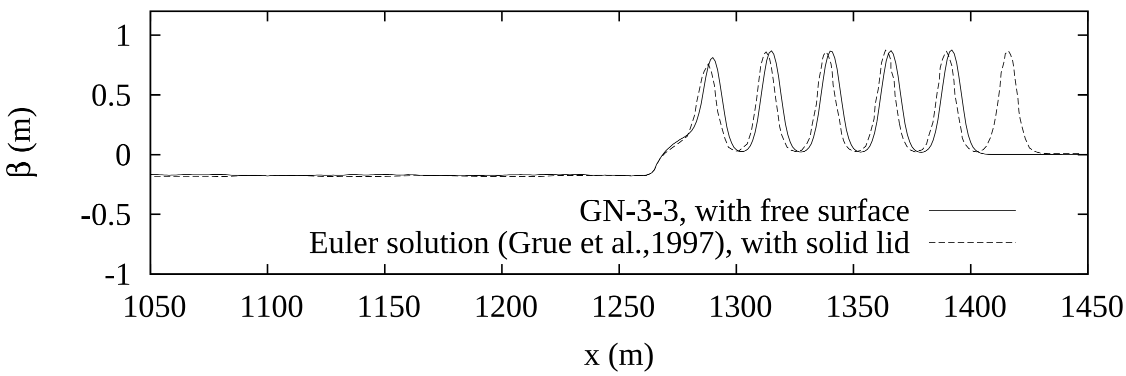


Figure 5: Time snapshot at $t(g/h_1)^{1/2} = 2760$, $\rho_2/\rho_1 = 0.7873$

4 Conclusions

The GN models with free surface and with solid lid are developed. And, we use Newton Iterative method to get the traveling solution for the GN models. Only a few results are presented here. When $\rho_2/\rho_1 = 0.977$, assuming that the free surface to be a solid lid is reasonable. When $\rho_2/\rho_1 = 0.8$, however, assuming that the free surface to be a solid lid is inaccurate. Finally, we reproduce the case simulated in Grue et al. (1997). In their calculations, $\rho_2/\rho_1 = 0.7873$, and they use a fully nonlinear model with solid lid. In our calculations, with fully nonlinear GN models with free surface, we observed some differences.

Acknowledgments

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