A novel approach of QTFs for floating body

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1. INTRODUCTION

With the advent of ocean exploration toward the deep water, marine environment is becoming worse. The nonlinear effects on a multitude of offshore structures are significant. Due to these nonlinear effects the structure no longer follows a harmonic oscillation pattern and a time domain method is a more appropriate choice for predicting the body motion. Among the time domain methods, the one proposed by Cummins (1962) is effective and easily implemented. Following this method, the second-order sum or difference frequency exciting forces are calculated based on the QTF in the frequency domain. For a floating body, the body motion at the first-order contributes to the exciting force at the second-order. However, to the best of the author's knowledge, an accurate way to take account these components associated with first-order body motion into the second-order exciting forces has not been available if the Cummins's method is used. This is a basic motivation of the present study.

2. THE QUADRATIC TRANSFER FUNCTION (QTF)

In the presence of bichromatic waves with frequencies ω_p and ω_q and amplitudes A_p and A_q, the total first-order velocity potential can be written in the form:

$$\Phi^{(1)}(\mathbf{x},t) = \mathrm{Re}\left[\sum_{p=1}^{2} \phi^{p(1)}(\mathbf{x}) e^{-i\omega_p t}\right]. \tag{1}$$

For wave interaction with a floating body, the first-order velocity potential at the frequency ω_p is further decomposed into the incident potential, the diffraction potential, and the radiation potential as:

$$\phi^{p(1)}(\mathbf{x}) = \phi_I^{p(1)}(\mathbf{x}) + \phi_D^{p(1)}(\mathbf{x}) - i\omega_p \sum_{j=1}^{6} \xi_j^{(1)} \phi_j^{p(0)}(\mathbf{x}), \quad (p = 1, 2). \tag{2}$$

where $\xi_j^{p(1)}$ is the first-order translational or rotational complex displacements in the jth direction.

Under the excitation of bichromatic waves, the second-order exciting force has the components at the sum and the difference frequencies of the first-order waves. In the present study the components of second-order exciting force in the kth direction at the sum and the difference frequencies of ω_p and ω_q are decomposed into several parts:

$$\begin{aligned}
\tilde{f}_k^{pq(2)+} &= A^p A^q h_k^{pq(2)+} + \sum_{j=1}^{6}\left(\xi_j^{p(1)} A^q h_{k,j}^{pq(1)+} + A^p \xi_j^{q(1)} h_{k,j}^{qp(1)+}\right) + \sum_{j=1}^{6}\sum_{l=1}^{6} \xi_j^{p(1)} \xi_l^{q(1)} h_{k,jl}^{pq(0)+}, \\
\tilde{f}_k^{pq(2)-} &= A^p A^{q*} h_k^{pq(2)-} + \sum_{j=1}^{6}\left(\xi_j^{p(1)} A^{q*} h_{k,j}^{pq(1)-} + A^p \xi_j^{q(1)*} h_{k,j}^{qp(1)-*}\right) + \sum_{j=1}^{6}\sum_{l=1}^{6} \xi_j^{p(1)} \xi_l^{q(1)*} h_{k,jl}^{pq(0)-}.
\end{aligned} \tag{3}$$

In the above equation $h_k^{pq(2)\pm}$ is consistence with the definition of QTFs of a fixed body. The complete solution of $h_k^{pq(2)\pm}$ has been derived by some researchers (e.g., Kim and Yue (1990) and Eatock Taylor and Kernot (1999)) and will not be further elaborated in the present study; $h_{k,j}^{pq(1)\pm}$ represents the second-order exciting force due to linear unit incident waves with frequency ω_q and first-order unit body motion in the jth direction with frequency ω_p; $h_{k,jl}^{pq(0)\pm}$ represents the second-order exciting force due to first-order unit body motion in the jth direction with frequency ω_p and first-order unit body motion in the lth direction with frequency ω_q. The present study focuses on how to compute $h_{k,j}^{pq(1)\pm}$ and $h_{k,jl}^{pq(0)\pm}$.

The nonlinear wave force on a body can be obtained by direct integration of hydrodynamic pressure over the body surface after velocity potential has been obtained. To obtain $h_{k,j}^{pq(1)\pm}$ and $h_{k,jl}^{pq(0)\pm}$, corresponding boundary value problems should be solved firstly.

3. THE BOUNDARY VALUE PROBLEM

The total velocity potential Φ can be expanded into a perturbation series in terms of the wave slope parameter ε. The governing equation for the velocity potential is the Laplace equation. A Cartesian coordinate system with the (x, y) plane in the quiescent free surface and the z axis pointing upward is used.

Consider the following two cases for the velocity potential. In the first case a body is placed in linear incident waves with frequency ω_p and amplitude A_p, and the body moves in the jth direction with frequency ω_q and amplitude $\xi_j^{q(1)}$. The first-order potential $\Phi^{(1)}(\mathbf{x},t)$ can be expressed as

$$\Phi^{(1)}(\mathbf{x},t) = \mathrm{Re}\left[-i\omega_p \xi_j^{(1)} \phi_j^{p(0)}(\mathbf{x}) e^{-i\omega_p t} + \phi_I^{q(1)}(\mathbf{x}) e^{-i\omega_q t} + \phi_D^{q(1)}(\mathbf{x}) e^{-i\omega_q t}\right], \tag{4}$$

In the other case incident waves do not exist and the body moves in both the jth and the lth directions with frequencies ω_p and ω_q and amplitude $\xi_j^{p(1)}$ and $\xi_l^{q(1)}$ respectively. The first-order potential $\Phi^{(1)}(\mathbf{x},t)$ can be expressed as

$$\Phi^{(1)}(\mathbf{x},t) = \mathrm{Re}\left[-i\omega_p \xi_j^{(1)} \phi_j^{p(0)}(\mathbf{x}) e^{-i\omega_p t} - i\omega_q \xi_j^{(1)} \phi_j^{q(0)}(\mathbf{x}) e^{-i\omega_q t}\right]. \tag{5}$$

Similar to the first-order velocity potential, the second-order velocity potential can be decomposed into the incident, the diffraction, and the radiation potentials. The second-order radiation potential includes the outgoing waves due to second-order body motion only and identical to that of the first-order radiation problem but at the sum and the difference frequencies respectively. The second-order radiation problem does not contribute to the second-order exciting wave force and hence will not be further elaborated. A convenient and consistent definition of the second-order diffraction potential $\phi_D^{pq(2)\pm}$ is to let it represent the combined diffracted potential due to the second-order incident waves as well as the forcing terms of all the quadratic contributions of the first-order quantities on the free surface and on the body surface. In the two cases under discussion the second-order incident waves are both not considered. Under the above decomposition, all difficulties in the computation of second order effects are confined to the diffraction problem, and correspondingly the boundary conditions for $\phi_D^{pq(2)\pm}$ are

$$-\left(\omega_p \pm \omega_q\right)^2 \phi_D^{pq(2)\pm}(\mathbf{x}) + g\frac{\partial \phi_D^{pq(2)\pm}(\mathbf{x})}{\partial z} = Q^{pq(2)\pm}(\mathbf{x}) \qquad on\ S_f; \tag{6a}$$

$$\frac{\partial \phi_D^{pq(2)\pm}}{\partial n} = B^{pq(2)\pm}(\mathbf{x}) \qquad on\ S_b; \tag{6b}$$

$$\frac{\partial \phi_D^{pq(2)\pm}}{\partial z} = 0 \qquad z = -d. \tag{6c}$$

$Q^{pq(2)\pm}(\mathbf{x})$ and $B^{pq(2)\pm}(\mathbf{x})$ are forcing terms of the quadratic products of the first-order bichromatic quantities on the free surface and the body surface. The expression of $Q^{pq(2)\pm}(\mathbf{x})$ in the first and the second cases are shown in Eq.7 and Eq.8 respectively.

$$Q^{pq(2)-} = \xi_j^{p(1)} A^{q*}\left[\frac{1}{2}\omega_p\left(\omega_p - \omega_q\right)\nabla\phi_j^{p(0)}\cdot\nabla\phi_B^{q(1)*} - \frac{1}{4}\frac{\omega_p^3\omega_q}{g}\frac{\partial\phi_j^{p(0)}}{\partial z}\phi_B^{q(1)*} + \frac{1}{4}\frac{\omega_p^2\omega_q^2}{g}\phi_j^{p(0)}\frac{\partial\phi_B^{q(1)*}}{\partial z} - \frac{\omega_p^2}{4}\phi_j^{p(0)}\frac{\partial^2\phi_B^{q(1)*}}{\partial z^2} + \frac{\omega_p\omega_q}{4}\frac{\partial^2\phi_j^{p(0)}}{\partial z^2}\phi_B^{q(1)*}\right], \tag{7a}$$

$$Q^{pq(2)+} = \xi_j^{p(1)} A^{q}\left[\frac{1}{2}\omega_p\left(\omega_p + \omega_q\right)\nabla\phi_j^{p(0)}\cdot\nabla\phi_B^{q(1)} + \frac{1}{4}\frac{\omega_p^3\omega_q}{g}\frac{\partial\phi_j^{p(0)}}{\partial z}\phi_B^{q(1)} + \frac{1}{4}\frac{\omega_p^2\omega_q^2}{g}\phi_j^{p(0)}\frac{\partial\phi_B^{q(1)}}{\partial z} - \frac{\omega_p^2}{4}\phi_j^{p(0)}\frac{\partial^2\phi_B^{q(1)}}{\partial z^2} - \frac{\omega_p\omega_q}{4}\frac{\partial^2\phi_j^{p(0)}}{\partial z^2}\phi_B^{q(1)}\right], \tag{7b}$$

$$Q^{pq(2)-} = \xi_j^{p(1)}\xi_l^{q(1)*}\left[\begin{array}{l}\frac{1}{2}i\omega_p\omega_q\left(\omega_p - \omega_q\right)\nabla\phi_j^{p(0)}\cdot\nabla\phi_l^{q(0)*} - \frac{1}{4}\frac{i\omega_p^3\omega_q^2}{g}\frac{\partial\phi_j^{p(0)}}{\partial z}\phi_l^{q(0)*} + \frac{1}{4}\frac{i\omega_p^2\omega_q^3}{g}\phi_j^{p(0)}\frac{\partial\phi_l^{q(0)*}}{\partial z}\\ + \frac{i\omega_p\omega_q^2}{4}\frac{\partial^2\phi_j^{p(0)}}{\partial z^2}\phi_l^{q(0)*} - \frac{i\omega_p^2\omega_q}{4}\phi_j^{p(0)}\frac{\partial^2\phi_l^{q(0)*}}{\partial z^2}\end{array}\right], \tag{8a}$$

$$Q^{pq(2)+} = \xi_j^{p(1)}\xi_l^{q(1)}\left[\begin{array}{l}-\frac{1}{2}i\omega_p\omega_q\left(\omega_p - \omega_q\right)\nabla\phi_j^{p(0)}\cdot\nabla\phi_l^{q(0)} - \frac{1}{4}\frac{i\omega_p^3\omega_q^2}{g}\frac{\partial\phi_j^{p(0)}}{\partial z}\phi_l^{q(0)} - \frac{1}{4}\frac{i\omega_p^2\omega_q^3}{g}\phi_l^{p(0)}\frac{\partial\phi_j^{q(0)}}{\partial z}\\ + \frac{i\omega_p\omega_q^2}{4}\frac{\partial^2\phi_j^{p(0)}}{\partial z^2}\phi_l^{q(0)} + \frac{i\omega_p^2\omega_q}{4}\phi_j^{p(0)}\frac{\partial^2\phi_l^{q(0)}}{\partial z^2}\end{array}\right]. \tag{8b}$$

In above expressions $\phi_B^{p(1)}(\mathbf{x}) = \phi_I^{p(1)}(\mathbf{x}) + \phi_D^{p(1)}(\mathbf{x}) \quad \left(p = 1,2\right)$. The superscript * indicates the complex conjugate. The expression of $B^{pq(2)\pm}(\mathbf{x})$ will be shown at the workshop. A boundary-integral equation method is then used to determine the second-order diffraction potential and the method will be elaborated in the next section.

4. A BOUNDARY-INTEGRAL EQUATION METHOD FOR THE SECOND-ORDER DIFFRACTION POTENTIAL

The first-order and the second-order boundary value problems can be solved by the boundary integral equations formulated by applying Green's theorem to the fluid domain. The second-order diffraction problem is much more difficult. The oscillating source is used as Green's function, which satisfies the linear free surface boundary condition, the radiation condition at infinity and the impermeable condition on the horizontal seabed. Then by introducing Green function $G^{\pm}$ corresponding to an oscillating source at the frequencies $\omega_p \pm \omega_q$, and applying Green's theorem to $\phi_D^{pq(2)\pm}$ and $G^{\pm}$, a Fedholm integral equation of the second kind for the second-order diffraction potentials can be obtained.

$$C(\mathbf{x_0})\phi_D^{pq(2)\pm}(\mathbf{x_0}) - \iint_{S_B} \phi_D^{pq(2)\pm}(\mathbf{x}) \frac{\partial G^{\pm}(\mathbf{x},\mathbf{x_0})}{\partial n} ds = -\iint_{S_B} G^{\pm}(\mathbf{x},\mathbf{x_0}) B^{pq(2)\pm}(\mathbf{x}) ds - \frac{1}{g}\iint_{S_F} G^{\pm}(\mathbf{x},\mathbf{x_0}) Q^{pq(2)\pm}(\mathbf{x}) ds. \tag{9}$$

Here $\mathbf{x}$ and $\mathbf{x_0}$ are the field and the source points, respectively. Evaluation of the infinite free surface integral is the key point in processing the second-order solution. In the present study the approach proposed by Chau and Eatock Taylor (1992) is used to treat the infinite integral.

After solving the first-order and the second-order potentials, the second-order pressure in the fluid domain can be obtained according to Bernoulli equation. Then the second-order exciting force $f_{k,j}^{pq(2)\pm}$ and $f_{k,jl}^{pq(2)\pm}$ can be calculated directly and $h_{k,j}^{pq(1)\pm}$ and $h_{k,jl}^{pq(0)\pm}$ can be determined from Eq. 11 and Eq. 12.

$$h_{k,j}^{pq(1)+} = f_{k,j}^{pq(2)+} \Big/ \left(\xi_j^{p(1)} A^q\right), \quad h_{k,j}^{pq(1)-} = f_{k,j}^{pq(2)-} \Big/ \left(\xi_j^{p(1)} A^{q*}\right). \tag{10}$$

$$h_{k,jl}^{pq(0)+} = f_{k,jl}^{pq(2)+} \Big/ \left(\xi_j^{p(1)} \xi_l^{q(1)}\right), \quad h_{k,jl}^{pq(0)-} = f_{k,jl}^{pq(2)-} \Big/ \left(\xi_j^{p(1)} \xi_l^{q(1)*}\right). \tag{11}$$

As an example, calculation is carried out for a truncated cylinder with a radius of 1*m* and a draft of 3*m* in a water depth of 10*m*. Fig .1 shows some results of $h_{k,j}^{pq(1)-}$ and $h_{k,jl}^{pq(0)-}$ for the cylinder.

5. SECOND-ORDER WAVE EXCITING FORCE IN TIME DOMAIN

The second-order exciting forces on a body due to stationary Gaussian random seas can in general be expressed as a two-term Voterra series in the time domain:

$$\begin{aligned}
\tilde{F}_k^{(2)pq\pm}(t) &= \int_{-\infty}^{+\infty}\int_{-\infty}^{\infty} H_k^{(2)pq\pm}\left(t-\tau_p, t-\tau_q\right)\eta^p\left(\tau_p\right)\eta^q\left(\tau_q\right)d\tau_p d\tau_q \\
&+ \sum_{j=1}^{6}\left\{\int_{-\infty}^{+\infty}\int_{-\infty}^{\infty} H_{k,j}^{pq(1)\pm}\left(t-\tau_p, t-\tau_q\right)\Xi_j^{p(1)}\left(\tau_p\right)\eta^q\left(\tau_q\right)d\tau_p d\tau_q + \int_{-\infty}^{+\infty}\int_{-\infty}^{\infty} H_{k,j}^{qp(1)\pm}\left(t-\tau_p, t-\tau_q\right)\Xi_j^{q(1)}\left(\tau_q\right)\eta^p\left(\tau_p\right)d\tau_p d\tau_q\right\} \\
&+ \sum_{j=1}^{6}\sum_{l=1}^{6}\left\{\int_{-\infty}^{+\infty}\int_{-\infty}^{\infty} H_{k,jl}^{pq(0)\pm}\left(t-\tau_p, t-\tau_q\right)\Xi_j^{p(1)}\left(\tau_p\right)\Xi_l^{q(1)}\left(\tau_q\right)d\tau_p d\tau_q\right\}.
\end{aligned} \tag{12}$$

where $H_k^{(2)pq\pm}, H_{k,j}^{pq(1)\pm}, H_{k,j}^{pq(1)\pm}$ and $H_{k,jl}^{pq(0)\pm}$ are quadratic impulse response functions. $H_k^{(2)pq\pm}, H_{k,j}^{pq(1)\pm}, H_{k,j}^{qp(1)\pm}$ and $H_{k,jl}^{pq(0)\pm}$ represent the second-order exciting force at time *t* due to two unit amplitude inputs at τ_p and τ_q, and can be obtained by Fourier transform of $h_k^{(2)pq\pm}$, $h_{k,j}^{pq(1)\pm}$, $h_{k,j}^{qp(1)\pm}$ and $h_{k,jl}^{pq(0)\pm}$ respectively. $\eta^p\left(\tau_p\right)$ $\left(p=1,2\right)$ is the ambient wave elevation at a reference point and $\Xi_j^{p(1)}\left(\tau_p\right)$ $\left(p=1,2\right)$ is the first-order body motion in the time domain.

6. NUMERICAL RESULTS FOR A TRUNCATED CYLINDER

According to Cummins (1962)'s method, the body motion equation in the time domain can be written as:

$$\sum_{j=1}^{6}\left\{\left(M_{kj}+m_{kj}\right)\ddot{\xi}_j(t) + \int_{-\infty}^{t}\dot{\xi}_j(t)K_{kj}(t-\tau)d\tau + C_{kj}\xi_j(t)\right\} = \tilde{F}_k^{(1)}(t) + \tilde{F}_k^{(2)\pm}(t). \tag{13}$$

For comparing with the frequency domain result, we divide the body motion into the first and the second order terms. The second-order body motion in the time domain can be simulated by solving the following equation:

$$\sum_{j=1}^{6}\left\{\left(M_{kj}+m_{kj}\right)\ddot{\xi}_j^{(2)}(t) + \int_{-\infty}^{t}\dot{\xi}_j^{(2)}(t)K_{kj}(t-\tau)d\tau + C_{kj}\xi_j^{(2)}(t)\right\} = \sum_{p=1}^{2}\sum_{q=1}^{2}\left[\tilde{F}_k^{pq(2)\pm}(t) + E_k^{pq(2)\pm}\right]. \tag{14}$$

$E_k^{pq(2)\pm}$ is associated with the first-order rotational displacements and the moments of inertia of the body.

Calculation is carried out for a truncated cylinder. Proper structural damping and linear stiffness are considered during the calculation. Fig.2 shows the second-order difference frequency body motion results at ω_p=3.0rad/s and ω_q=2.8rad/s. The results based on the QTFs of the present method agree well with the frequency domain results while differ much from those based on QTFs of fixed bodies. Even though the first-order motion is small in magnitude, its contribution to the second-order motion is considerable. Above findings suggest that the study of QTFs of moving bodies is necessary and the influence of first-order motion to the higher order motion should be taken into account.

ACKNOWLEDGEMENTS

This work was supported by the National Natural Science Foundation of China (Grant Nos. 51379032, 51221961) and the National Basic Research Program of China (“973” Program) (Grant No. 2011CB013703).

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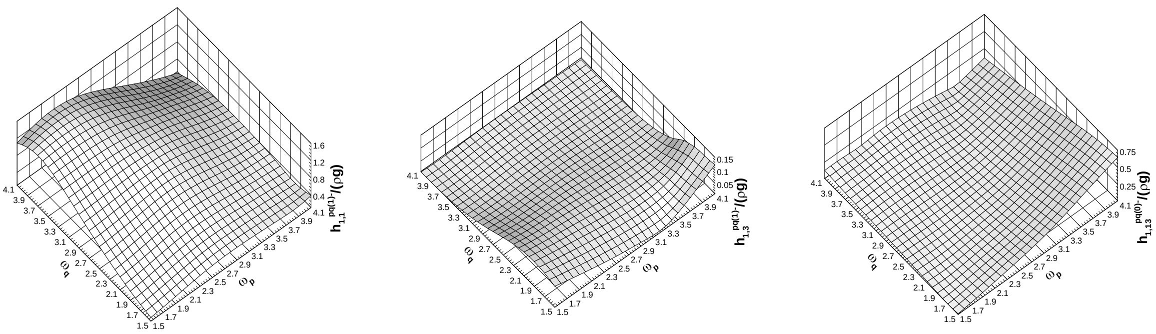
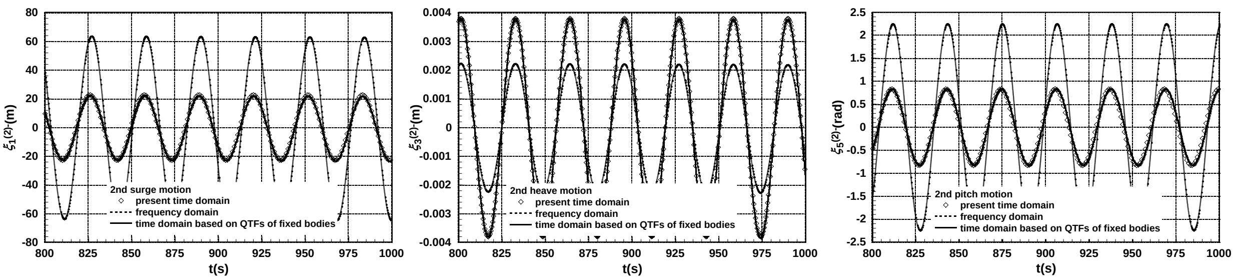


(a) $h_{1,1}^{pq(1)-}/(\rho g)$ (b) $h_{1,3}^{pq(1)-}/(\rho g)$ (c) $h_{1,13}^{pq(0)-}/(\rho g)$

Fig.1 Results of $h_{k,j}^{pq(1)-}$ and $h_{k,jl}^{pq(0)-}$ for a truncated cylinder.



(a) surge (b)heave (c) pitch

Fig.2 Second-order difference frequency body motion results.