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# The dynamics and statistics of wave crest elevations and 2nd order near trapping for a semi-submersible platform

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**Highlights**

- Significant near trapping of wave energy is demonstrated below the deck of a realistic 4-leg semi-submersible.
- In random seas whether the structure is restrained vertically or freely floating, so it is free to ride the waves, has a dramatic effect on the probability that water reaches deck level.

## 1. Introduction

This contribution describes a numerical investigation into the free-surface motion below the deck of a large semi-submersible platform, building on the work of Walker et al [1]. We consider the free-surface motion to 2nd order in the wave amplitude, both for the structure held fixed and the structure freely floating. The linear components of the incoming wave field are assumed to be random in both amplitude and phase, chosen to be consistent with a severe storm with $H_s$=12*m* and spectral peak period $T_p$=15.2*s*. Such ($H_s$,$T_p$) combinations are typical of severe storms in many parts of the world, including both the North Sea and the Gulf of Mexico. For convenience the sea-state is assumed to be uni-directional, limiting the number of quadratic transfer function (QTF) combinations to be calculated using our 2nd order code DIFFRACT, with the approach direction taken as diagonal across the structure. The wave period is chosen such that 2nd order near-trapped modes will be excited by the incoming waves (e.g. Evans & Porter [2], Malenica et al [3]); for this structure there are several such near-trapped modes with periods around 7.5*s*. Then 2nd order nonlinear frequency doubling for the incoming wave components will lead to excitation of the near-trapped modes. We seek to determine the statistics of the free-surface response below the centre of the platform, where the 2 planes of symmetry of the structure intersect, at the origin as shown in Fig.1.

We choose a simplified geometry somewhat comparable to those of the largest production semi-submersible platforms currently in service. The semi-submersible geometry is assumed to consist of 4 vertical square columns with rounded corners, arranged at the corners of a square with a centre-to-centre spacing of 81.8*m*. Each column is of width 23*m*, the pontoons have rectangular cross-section 23*m* wide and 11.5*m* depth. The entire structure has a draft of 30*m*. The structure is assumed to be floating on water 350*m* deep. A quarter section of the structure, showing the surface mesh used for the diffraction calculations, is shown in Fig 1.

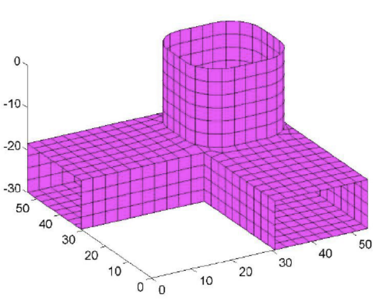
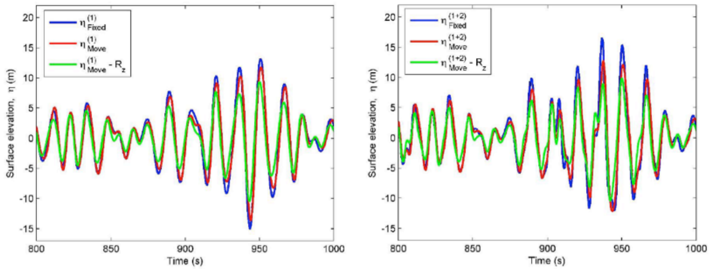


Fig.1 Quarter-plane model of semi structure




(a) First-order (b) Second-order

Fig.2 Example of the water surface elevation at the centre of the structure (a) linear, (b) linear + 2nd order

Examples of the motion of the water surface below the centre of the semi-sub in the same random waves are shown in Fig.2: on the left (a) only linear responses, on the right (b) the free surface motion including 2nd order. Each sub-plot shows 3 responses, the water surface motion for the structure held stationary in blue, the water motion with the structure freely floating in red, both water surface displacements measured relative to fixed mean sea level. For the freely floating structure, the 3rd green lines show the water surface in time measured relative to the vertical motion of the platform; clearly the key parameter in deciding whether water will reach deck level. We can see immediately that the 2nd order terms, while relatively small in the incoming undisturbed random sea, here are contributing significantly to the total response within the structure. Also allowing for the vertical motion of the structure significantly reduces both the absolute level that water reaches and more particularly, the relative level compared to the deck of the structure.

**2. Free-surface statistics for a semi-sub held fixed**

We use DIFFRACT for the computation of all the hydrodynamic coefficients. This is a panel method, using quadratic elements. For linear computations only the surface of the submerged body needs to be discretised. For the 2nd order calculations, both the body surface and the adjacent water free surface need to be discretised.

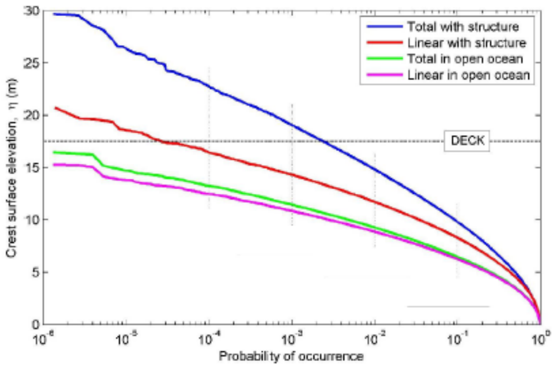


Fig.3 Free-surface crest responses for structure fixed.

We now perform random simulations of the water surface response within the structure, consistent with the random sea-state given above. The diffraction calculation provides both linear and quadratic transfer functions between the incoming wave components and the surface responses in the frequency domain. Calculating the QTFs is a slow process as individual pairs can require several hours computing using DIFFRACT. In contrast, the assembly of the entire realisation containing up to $10^6$ waves can be performed in a couple of hours on a standard pc, once the QTFs are available. In a typical sea-state, there will be perhaps $\sim 10^3$ waves, so we can go far out in the extreme tails of the probability distribution by such simple Monte Carlo simulations. The statistics of such a large number of waves is best investigated with short simulations combined together – this is satisfactory so long as both the amplitude and phase of the incoming wave components are treated as random variables. Figure 3 shows 4 lines for surface elevation statistics. The lowest one (purple) is for the undisturbed linear free surface crest without the structure present, next (green) is the undisturbed crest to combined linear and 2nd order. The third line up (red) accounts for the presence of the assumed stationary semi-sub with linear diffraction only. Clearly there is considerable linear diffraction required to take the undisturbed purple line up to the red one. Finally the top line (blue) accounts for both linear and 2nd order diffraction. There is a massive enhancement of the levels reached by the free-surface crests at long return periods. This is due to the excitation of the 2nd order near-trapped modes. With a hypothetical deck of the semi-sub at $+17m$, it is clear that this structure would get rather 'wet' at deck level. The 1 in $10^6$ wave is predicted to reach $+30m$ above mean-sea-level! However, it should be pointed out that we are assuming that 2nd order theory is valid for such large and locally steep wave motions, and that the structure does not move vertically for these results.

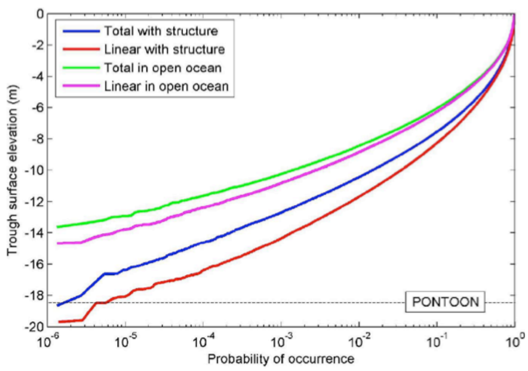


Fig.4 Trough statistics for fixed structure

Consistent with these 2nd order approximations we also show the distribution of water surface troughs in Fig. 4, using the same colours. Of course the linear responses are identical to the crests but inverted. But the surface response with 2nd order included (again in blue) is now numerically smaller (less negative) than the linear values, with the 1 in $10^6$ trough just down to the level of the top of the pontoons, though the statistics are given for the centre of the structure, away from the pontoons. The wiggles in the lines at low probability for both figures 3 and 4 arise from the use of only $10^6$ waves – there is significant sample variability at the left hand side of both figures.

**3. Free-surface statistics for a freely floating semi-sub**

We now present results for the same semi-sub geometry but with the structure freely floating, with representative values for all the necessary hydrodynamic parameters. The eigen-periods for all the modes other than heave are $> 60s$ . The model is assumed to be freely floating with no restraints, so, of course, the surge natural period is infinite. We do not make any attempt to model the soft moorings of a real floating platform or any of the risers and other equipment that would lead to significant fluid damping, as this is unlikely to affect the local free-surface motion around a floating structure at the main wave frequencies and higher. The only damping included in these simulations arises from wave radiation effects from the structure.

The structural motion in heave is controlled by the heave resonance and the frequencies when the net vertical hydrodynamic force drops to zero. At very long periods the structure moves in heave in phase with the waves. The heave resonance is at 25.9*s* with the first zero of the vertical force at 24.3*s*. So, in the narrow wave period range 25.8-24.3*s* the structure moves out of phase with the incoming wave. For wave periods shorter than 24.3*s*, the structure moves in phase with the wave until between the next two force zeroes at 9*s* and 4*s* where the response is small but out of phase. This issue of whether the heave response is in phase or out of phase with the waves is obviously important for determining whether the platform air-gap will be increased or eroded by platform motion. Clearly with a storm with a wave spectral peak $T_p$=15.2s, the linear heave response of the semi-sub will be predominately in phase with the incoming waves – the structure moves upwards as the wave crest passes through it. Thus we would expect a floating semi-sub to be 'drier' than would occur if the same structure were rigidly restrained vertically.

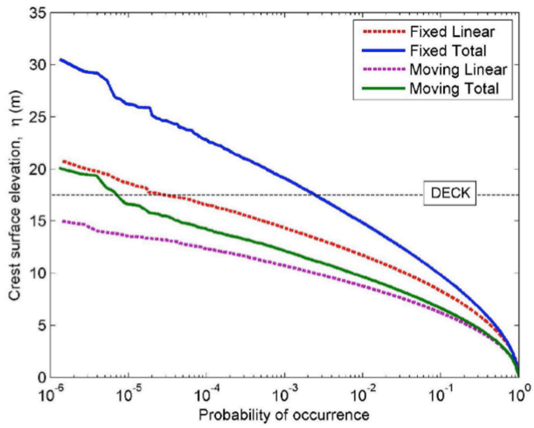



Fig. 5 Crest statistics with/without structural motion

Figure 5 shows the statistics of wave crests, again at the geometric centre of the semi-sub. The dashed lines correspond to linear wave interaction with and without linear structural motion. The solid lines correspond to linear and 2$^{nd}$ order wave elevation (labelled as 'total'). When the structural motion is included, this is still calculated linearly (2$^{nd}$ order structural motions will be insignificant on such a large structure). The solid blue line is the same as in figure 3, with the structure held fixed but the water free surface calculated to 2$^{nd}$ order. The solid green line is the water crest elevation relative to the position of the equilibrium waterline on the structure. So relative to the structure, the upward water projection is greatly reduced in magnitude. This reduction is sufficient that even at long return periods of 1 in $10^5$ waves, water only just reaches the deck level.

**4. Time histories of extreme response and the undisturbed wave fields that produced them**

We now investigate the local structure of the wave field occurring around an extreme crest event. Figure 6 (below) contains the average of the top 500 events in a sea state containing $10^6$ waves, thus the figure shows the average response time histories associated with a return rate of ~1 in 2000 waves.

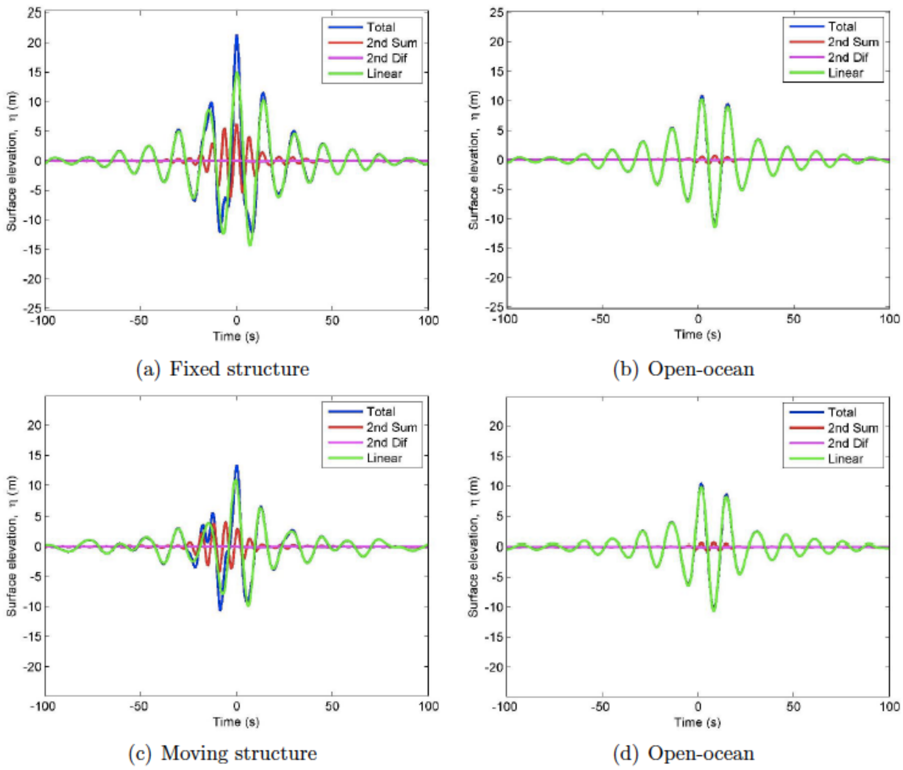
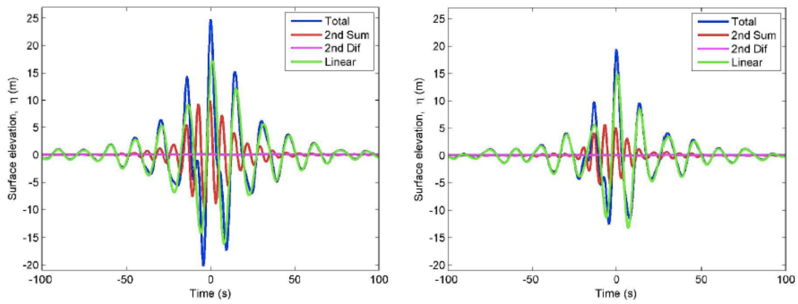



Fig. 6 Each of the sub-plots shows the free-surface response at the centre of the structure as the total time history, and also broken down into the individual components: linear, 2$^{nd}$ order sum and difference. Sub-plot (a) is for the structure fixed, (b) is the average of the incoming waves which produced the fixed structure response, (c) is for the structure

allowed to freely move – with the surface elevation taken relative to mean-sea-level, and (d) shows the average of the incoming waves which produced the moving structure surface response.

It is striking how close to symmetric the average extreme response is in case (a), and that the incoming wave group has a deep trough at the centre. The 2$^{nd}$ order sum term is large and again close to symmetric, and its maximum occurs exactly when the maximum of the linear response occurs. For the moving body, the results are more complex. The phasing is less good, with the envelope of the 2$^{nd}$ order sum term shifted relative to that of the linear component. Presumably, the motion of the structure is affecting the hydrodynamics. In both the fixed and floating cases, the 2$^{nd}$ order difference terms are negligible, both in the incoming deep water wave but also in the field with the model structure in place.



(a) Fixed (b) Moving

Fig.7 Averaged 1 in 2000 surface response in front of rear leg.

We conclude by showing similar averaged 1 in 2000 wave responses but at a different point beneath the structure, figure 7. The position is now moved to just upstream of the rear leg (recall that the wave approach direction is diagonally across the structure). The responses are now larger than in the centre of the structure, and for the fixed structure, water reaches an elevation of >2x the significant wave height.

## 5. Conclusion

The paper has dealt with the occurrence of water at very high levels around a multi-leg large volume structure. If near-trapped modes can be excited by frequency doubling, then high levels are to be expected in severe sea-states. These levels can be much greater than in the external undisturbed wave field, and the results for the averaged shape of extreme events suggest that these local events can be triggered by localised energetic wave groups.

We predict a dramatic difference in the likelihood of wave-in-deck problems for semi-submersible platforms as compared to tension leg platforms. Both consist of multiple columns, usually 4, joined by submerged pontoons. Both structural types will be susceptible to the excitation of near-trapped modes between the columns. For structures of sufficient size, these modes will be predominately excited by 2$^{nd}$ order frequency doubling interactions for severe storms. Given the ability of a semi-sub to ride the wave crests, this is one reason why the relative level that water reaches is lower. A second reason is associated with the nature of near trapping for moving structures, the phenomenon being somewhat different if the structure is free to move.

More examples will be shown at the workshop.

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