Numerical and experimental study of the wave response of a floating support with partially filled tank

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Abstract: This paper presents a study of the sloshing in a rectangular tank put on the deck of a rectangular barge submitted to beam waves. The velocity potential formulations of Boussinesq-type models are applied to simulate sloshing in the tank with nonlinear free surface boundary conditions. A modal approach method based on linear theory is introduced to compare with the Boussinesq model. Experimental results performed at the BGO-First wave tank in la Seyne sur mer are presented to validate the numerical models.

Boussinesq-type models: A 2D rectangular tank partially filled is considered here (length: l; depth: h) and a tank-fixed coordinate system Oyz lies on the free surface at rest in the middle of the tank. Small-amplitude motions in sway and roll of the tank are considered, with the axis of rotation in the middle of the bottom. The velocity on the boundary of the tank is given by $[v_y, v_z] = [v - w_1(z+h), w_1 y]$, where v is the translational velocity and w_1 is the angular velocity.

Following Zakharov (1968), the free surface conditions are written in terms of the velocity potential $\tilde{\Phi} = \Phi(y, \eta, t)$ and the vertical velocity $\tilde{w} = (\Phi_z)_{z=\eta}$ defined on the free surface. The governing equations are given by:

$$\Phi_{yy} + \Phi_{zz} = 0; \qquad \Phi_y = v_y \quad y = \pm\frac{l}{2}; \qquad \Phi_z = v_z \quad z = -h; \tag{1}$$

$$\eta_t + (\tilde{\Phi}_y - v_y)\eta_y - (1+\eta_y^2)\tilde{w} = 0; \qquad \tilde{\Phi}_t - v_y\tilde{\Phi}_y - v_z\tilde{w} + g\eta + \frac{1}{2}\tilde{\Phi}_y^2 - \frac{1}{2}\tilde{w}^2(1+\eta_y^2) = 0 \quad z = \eta \tag{2}$$

In the numerical computations, the potential $\tilde{\Phi}$ and the elevation η can be advanced in time from the derivatives $\tilde{\Phi}_t$ and η_t if the vertical velocity $\tilde{w}$ is known. The potential Φ is separated into two parts: $\Phi = \phi + \varphi$, with φ a particular solution satisfying the Laplace equation and the no-flow condition on the walls given by:

$$\varphi = vy - w_1 y(z+h) + w_1 \sum_{n=1}^{\infty} \alpha_n \cos(\lambda_n(y + \frac{l}{2}))\frac{2\sinh(\lambda_n z)}{\lambda_n \cosh(\lambda_n h)}; \quad \lambda_n = \frac{n\pi}{l} \quad \alpha_n = -\frac{4l}{n^2\pi^2} \tag{3}$$

Following Bingham et al. (2009), the Taylor series expansion of the solution $\phi(y, z, t)$ about an arbitrary vertical position $z = \hat{z}(y)$ is given by

$$\phi(y, z, t) = \hat{\phi} + (z - \hat{z})\hat{w} + \frac{1}{2}(z-\hat{z})^2\hat{\phi}^{(2)} + \frac{1}{6}(z-\hat{z})^3\hat{\phi}^{(3)} + \dots \tag{4}$$

$$\hat{\phi} = \hat{\phi}^{(0)} = \phi(y, \hat{z}, t), \quad \hat{w} = \hat{\phi}^{(1)} = \frac{\partial\phi(y,z,t)}{\partial z}\Big|_{z=\hat{z}}, \quad \hat{\phi}^{(n)} = \frac{\partial^n\phi(y,z,t)}{\partial z^n}\Big|_{z=\hat{z}}, \quad for\ n = 2, 3, \dots, \infty. \tag{5}$$

A recurrence relation about $\hat{\phi}^{(n)}$ can be obtained with substituting them into the Laplace equation. Finally the velocity potential based on $\hat{\phi}$ and $\hat{w}$ is given. Setting $\psi = z - \hat{z}$ with $\hat{z} = -h$ and taking some enhancement techniques

$$\hat{\phi} = (1 + \frac{(\hat{z}\nabla)^2}{10} + \frac{(\hat{z}\nabla)^4}{120})\hat{\phi}^*, \quad \hat{w} = (1 + \frac{(\hat{z}\nabla)^2}{10} + \frac{(\hat{z}\nabla)^4}{120})\hat{w}^* \tag{6}$$

the velocity potential and the vertical velocity can be expressed with non-dimensional variables $\hat{\phi}^*$, $\hat{w}^*$ ($\nabla = \partial/\partial y$):

$$\phi(y,z,t) = (1 + (-\frac{\psi^2}{2} + \frac{\hat{z}^2}{10})\nabla^2)\hat{\phi}^* + (\psi + (-\frac{\psi^3}{6} + \frac{\hat{z}^2\psi}{10})\nabla^2)\hat{w}^* \tag{7}$$

$$w(y,z,t) = (-\psi\nabla^2 - (-\frac{\psi^3}{6} + \frac{\hat{z}^2\psi}{10})\nabla^4)\hat{\phi}^* + (1 + (-\frac{\psi^2}{2} + \frac{\hat{z}^2}{10})\nabla^2)\hat{w}^* \tag{8}$$

With the boundary conditions on the bottom and the initial values η and ϕ on the free surface, we can solve the unknowns $\hat{\phi}^*$ and $\hat{w}^*$ which will be used for the computation of the vertical velocity on the free surface. The hydrodynamic loads are given by the integrations of the pressure on the wetting surface as follows:

$$\vec{F} = \int p\ \vec{n}\ dl; \quad \vec{M} = \int p\ \vec{r} \times \vec{n}\ dl; \quad p = -\rho(\Phi_t - v_y\Phi_y - v_z\Phi_z + \frac{1}{2}(\Phi_y^2 + \Phi_z^2) + gz) \tag{9}$$

Linear theories: A modal approach is used to represent the motion of the liquid in the 2D tank. Following Molin et al. (2002), the eigen-modes of the two-dimensional rectangular tank are given by

$$\Phi_n = -\frac{A_{n0}\ g}{w_n}\frac{\cosh\ \lambda_n(z+h)}{\cosh\ \lambda_n h}\ \cos\ \lambda_n(y+\frac{l}{2})\ \sin(w_n t+\theta_n) \qquad w_n^2 = g\lambda_n\ \tanh\ \lambda_n h \quad with \quad \lambda_n = \frac{n\pi}{l} \tag{10}$$

and the free surface elevation are superposition of $\eta_n(y,t) = A_{n0}\ \cos\lambda_n(y + l/2)\ \cos(w_n t + \theta_n)$.

The tank undergoing forced motion is considered. The $A_n(t)$ of the sloshing motion n and the forced motion $v = \dot{X}_2(t)$ and $w_1 = \dot{X}_4(t)$ are related with a pendulum equation. The force and moment are given by taking the integration of pressure over the wetting surface.

$$\ddot{A}_n + B_{1n}\dot{A}_n + w_n^2 A_n = \sum_j D_{nj}\ddot{X}_j \qquad F_j = \sum_n A_n(t) f_{nj} - \mathbf{M}_a(\infty)\ddot{X}_j \tag{11}$$

where B_{1n} are the damping terms, $\mathbf{M}_a(\infty)$ is the infinite frequency added mass matrix and D_{nj}, f_{nj} are the coefficients which can be derived analytically (Molin et al. 2002).

The motions of sway and roll are considered in our computations. For even n, f_{nj} and D_{nj} are equal to zero. That means the elevation on the free surface is only the superposition of odd modes.

Coupled motion computations: The diffraction-radiation problem of the barge is solved by a semi-analytical method, applicable to two-dimensional bodies of rectangular shapes (Cointe et al. 1990). This method defines a succession of three rectangular sub-domains where eigen-function expansions are used to express the velocity potential. Matching of the potentials and horizontal velocities at the successive boundaries gives the solution to the problem.

Based on the equations of motion in the frequency domain, the linear modal approach method used for describing the flow inside the tank is coupled with the eigen-function expansions model for the fluid outside of the barge. For the nonlinear Boussinesq-type model, forces and moments produced by the internal liquid are introduced into the equations of motion in the time domain.

$$\sum_j \{(M_{ij} + m_{ij})\ddot{x}_j + \int_0^t K_{ij}(t-\tau)\dot{x}_j(\tau)d\tau + C_{ij}x_j\} = F_i(t) \tag{12}$$

the frequency-independent added masses m_{ij} and the retardation functions K_{ij} are calculated by the hydrodynamic coefficients produced by the eigen-function expansions method. Since our study only deals with beam seas, the hydrodynamic coefficients given by the 2D model are just multiplied by the equivalent lengths of the barge and of the tank for comparing with the actual experiments.

Experimental campaign: The experiments were carried out in the BGO-First wave tank, at la Seyne sur mer (fig. 1). This facility is 16 m wide and 3 m deep, for a total length around 40 m. On the deck of a barge model of 3 m long and 1 m wide, with a draft of 10.8 cm, two rectangular glass tanks 80 cm long and 25 cm wide were installed. The free surface motion inside the tanks was measured by five capacitive probes. Three were installed in the tank 2 at 25, 180 and 350 mm from the wall closest to the wavemaker (numbered probe 3,2,1)and two were installed in tank 1, at 25 and 180 mm from the same wall (numbered probe 4,5). The motion of the barge was measured through an optical tracking system. More details can be found in Molin et al. (2002).

Some comparisons:

1. Fig.2 and fig.3 give the spectral density of the sway and roll motion. With the same filling levels 19 cm and 19 cm in the two tanks, the angular frequency of the first sloshing mode is 4.95 rd/s, and the roll natural frequency is around 4.8 rd/s (when the water in the tanks is frozen). The wave spectra considered in the study were JONSWAP, with $\gamma = 2$. The peak periods and significant wave-heights are 1.6 second and 3.4 cm respectively. In all calculations, the C_d coefficient arising in the roll damping moment was taken equal to 0.2. The results *Linear* obtained from the linear modal approach in the frequency domain computations coincide well with the Boussinesq method calculated in the time domain. Due to the symmetry of the tank and the barge, the second sloshing mode (8.2 rad/s) is not present in the roll and sway motion of the barge.

2. In fig.5, fig.6 and fig.7, the spectral densities of the relative free surface elevations are given. A fair agreement between the linear theory and the Boussinesq approach can be noticed similarly in the first sloshing mode. But a significant difference can be found in the second sloshing mode in the gauge 1 and gauge 3. Due to the fact that gauge 2 corresponds to a node, the nonlinear Boussinesq model gives no contribution to the free surface elevations. This is different for the gauge 1 and gauge 3, close to the center and to the wall.

3. Fig.4 gives the time series of roll and fig.8 gives the time series of the relative free surface elevations at the gauge 3. The Fourier transform has been used to obtain the values of amplitudes and phases of the incident waves in the tank. Based on the values, the numerical motions of roll and relative free surface elevations varying with time are compared with the experimental time series.

References

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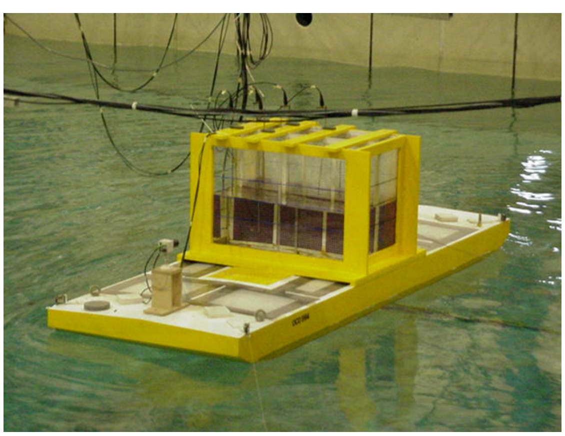

fig. 1: The barge with tanks

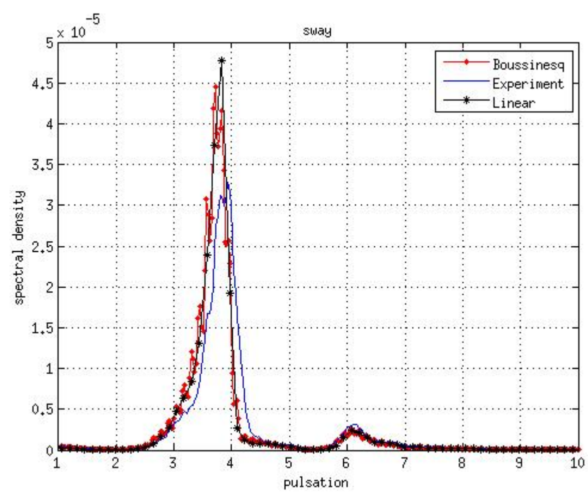

fig. 2: The spectrum of sway

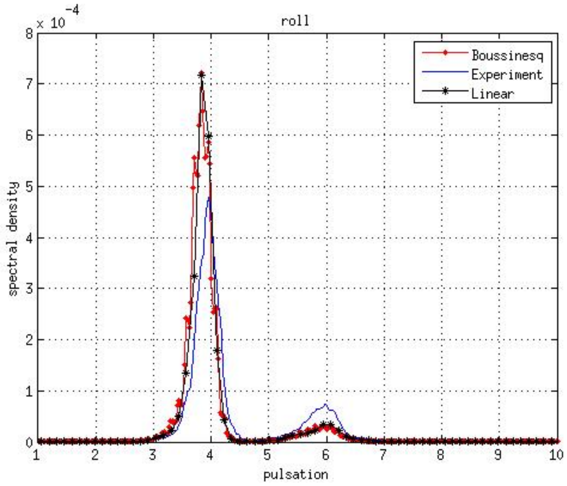

fig. 3: The spectrum of roll

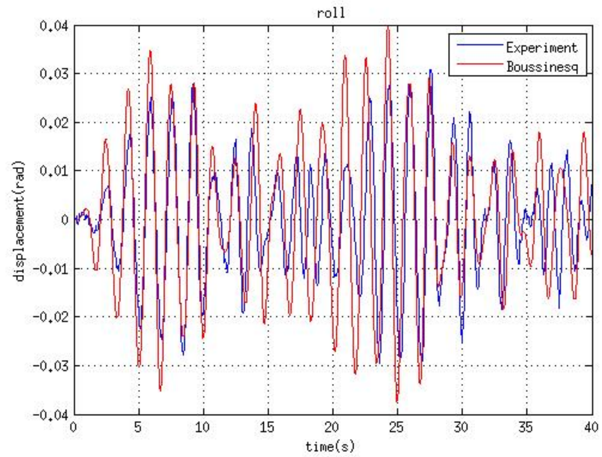

fig. 4: The displacements of roll

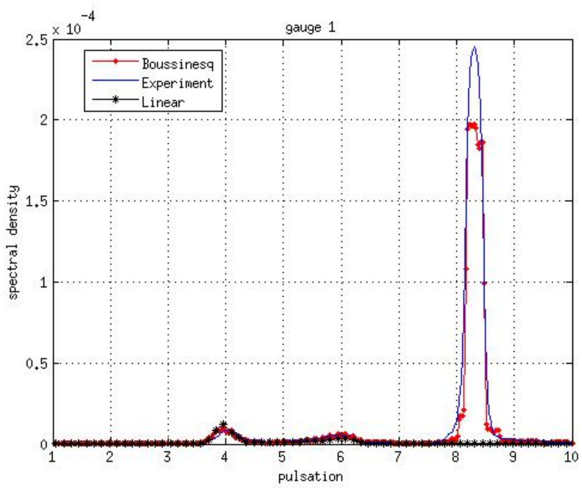

fig. 5: The spectrum at gauge 1

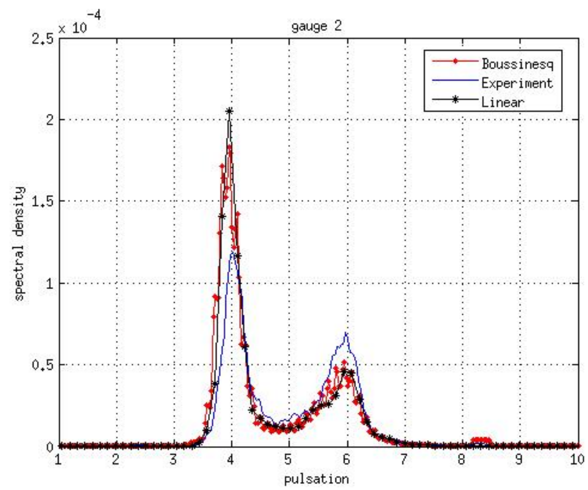

fig. 6: The spectrum at gauge 2

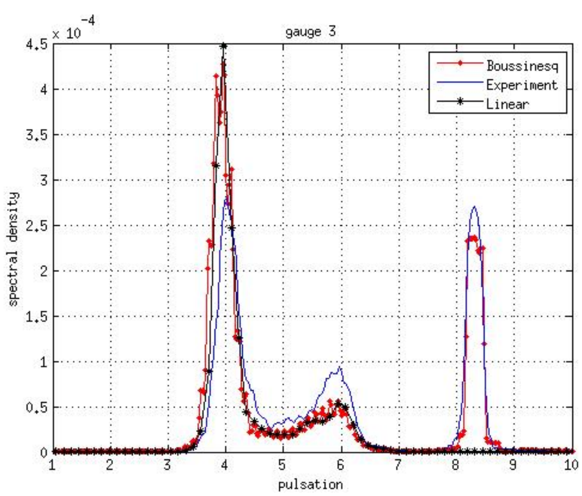

fig. 7: The spectrum at gauge 3

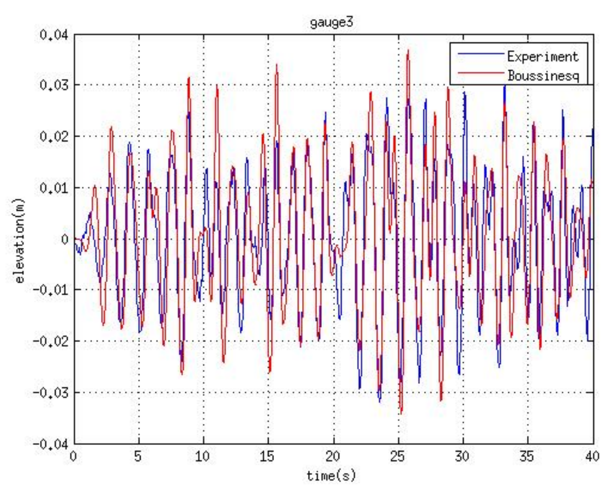

fig. 8: The relative elevations at gauge 3