Experimental and analytical study of the piston mode resonance inside WIRs (Water Intake Risers)

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So-called WIRs are elongated vertical tubes, hanging down from FLNGs, which serve the purpose of pumping deep sea water, typically more than 150 m below the free surface. Partly due to the pumping system inside, they are of variable internal cross section. The question then arises of possible resonant vertical motion of the water column inside, under the vertical motion of the WIR, induced by the heave, pitch and roll wave response of the FLNG.

We present here a simple analytical model to predict the vertical water motion inside the WIR, and we compare the model predictions with simple experiments.

Analytical model

We take $z = 0$ as the mean free surface, and the z axis vertically upwards. The internal cross section of the tube, $S(z)$, is assumed to be slowly-varying so that the flow is quasi-vertical.

We assume small vertical motion $Z(t)$ of the tube and we take $\eta(t)$ as the vertical motion of the internal free surface **relative to the tube**. So $Z(t) + \eta(t)$ is the absolute vertical motion of the free surface. Likewise $w(z,t)$ is the vertical velocity of the water, averaged over the section, relative to the tube, and $\dot{Z}(t) + w(z,t)$ is the absolute velocity.

The Euler equation, in the vertical direction, writes

$$w_t + w\, w_z = -\frac{1}{\rho}\, p_z - g - \ddot{Z} \tag{1}$$

while, from mass conservation

$$w(z,t)\; S(z) = \dot{\eta}\; S(0) \tag{2}$$

Neglecting the $w\, w_z$ term in equation (1), we get

$$\ddot{\eta}\, \frac{S(0)}{S(z)} = -\frac{1}{\rho}\, p_z - g - \ddot{Z} \tag{3}$$

Integrating in z, from the bottom of the pipe (in $z = -h$) to the free surface, we get

$$\ddot{\eta} \int_{-h}^{\eta} \frac{S(0)}{S(z)}\, \mathrm{d}z = -\frac{1}{\rho}\, (0 - \rho\, g\, h + \rho\, g\, Z) - (g + \ddot{Z})\, (h + \eta) \tag{4}$$

or, linearizing again:

$$\ddot{\eta} \int_{-h}^{0} \frac{S(0)}{S(z)}\, \mathrm{d}z + g\, \eta = -g\, Z - \ddot{Z}\, h \tag{5}$$

Here we have not accounted for added mass effects associated with the water exit and entrance at the bottom of the pipe. Taking an added mass of $2\, \rho\, b^3$, where b is the radius at the bottom, we see that it is equivalent to increase the draft h of the tube by $2\, b/\pi$.

As a result the natural period of the piston mode is given by

$$T_0 = 2\, \pi\, \sqrt{\left[\int_{-h'}^{0} S(0)/S(z)\; \mathrm{d}z\right]/g} \qquad \text{where } h' = h + \frac{2\, b}{\pi} \tag{6}$$

(e.g. see DNV-RP-205, 7.3.9.4[1])

From equation (5) we check that if the tube is of constant internal cross section $S(z) \equiv S(0)$ then $\eta(t) = -Z(t)$ meaning that the free surface inside the tube does not move, as expected from an inviscid flow model.

Experimental set-up

The experimental model is made of two concentric tubes, the interior one being shorter than the outer one. The outer tube has an internal diameter $2b$ of 100 mm, with an immersed length h of 540 mm. The inner tube has an outer diameter $2a$ of 70 mm and an immersed length d of 240 mm. It is sealed at its bottom, which is made either flat, or rounded or conical. In the first series of experiments reported here it is flat, meaning flow separation and viscous dissipation are taking place.

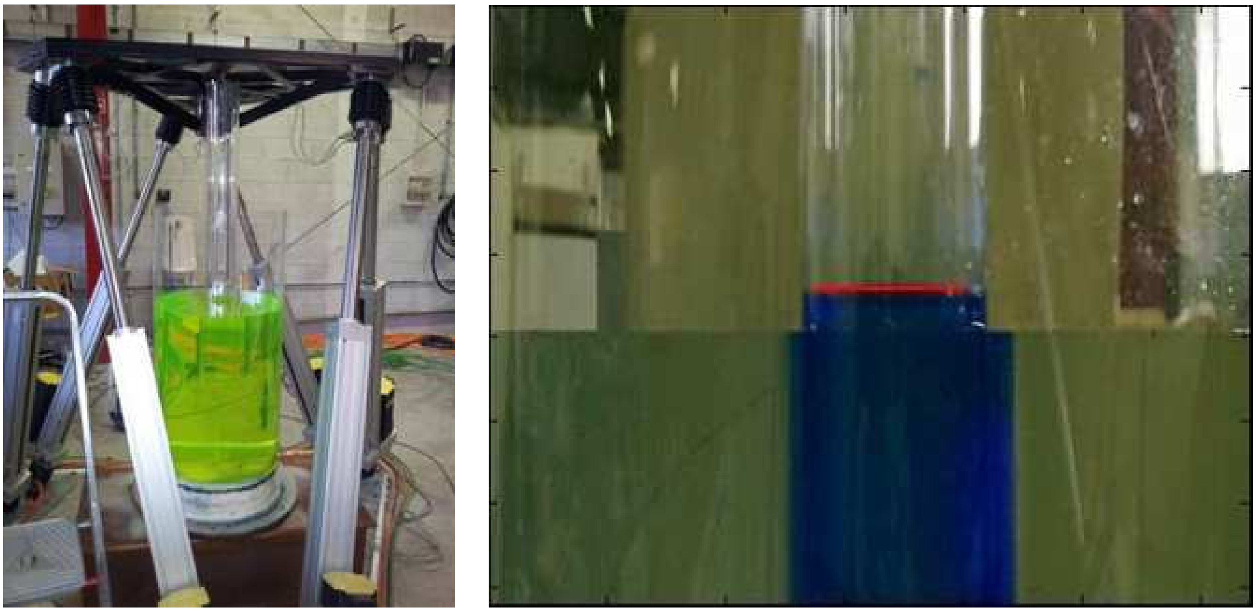

Figure 1: Experimental set-up: the dual tube system dipping in a cylindrical container below the Hexapode (lefft) and optical detection of the free surface (right).

The two tubes are solidly linked together, and attached to the forced motion bench Hexapode of Ecole Centrale Marseille. They are dipping in a cylindrical container, with a diameter of 440 mm (see figure 1). In the experiments reported here the waterheight in the cylindrical container was set at 640 mm, meaning a 100 mm clearance from the base of the model to the bottom of the container.

Tests were performed under two types of forced excitation: harmonic and irregular. The vertical motion of the free surface inside the tube is recorded with a video camera. Subsequent image processing, under Matlab, yields the time series of the free surface elevation $\eta(t) + Z(t)$. To improve the detection of the free surface, the water inside the tube is dyed in green or blue (see figure 1).

Introducing damping effects

The fluid motion inside the tube is damped through two types of viscous effects: friction along the vertical walls of the inner and outer tubes, and flow separation at the lower ends of the inner and outer tubes.

Considering friction first, the thickness of the oscillatory boundary layer is expected to be of the order of $\sqrt{\nu T_0}$, that is with $\nu = 10^{-6}$ m^2/s and $T_0 = 1.3$ s, roughly 1 mm, hence much smaller than

[1] Note that in this RP the vertical added mass for the circular cross section is different from what we have taken here ($8/3\,\rho\,b^3$ vs $2\,\rho\,b^3$) because in RP-205 the moonpool bottom is assumed to be surrounded by a solid horizontal plane going to infinity.

the cylinder radii. As for assuming laminar boundary layer, with a maximum amplitude A of 0.1 m, the oscillatory Reynolds number $A^2\,\omega_0/\nu$ is around 5 10^4, so the boundary layer should still be laminar (Faltinsen, 1990). This means that the Stokes model of the oscillatory flat plate can be used to predict the viscous dissipation.

According to the Stokes model, the friction stress that opposes the relative velocity $w(z,t)$ of the fluid with respect to the wall is

$$\sigma(z,t) = -\rho\,\sqrt{\frac{\omega\,\nu}{2}}\,w(z,t) \tag{7}$$

with ω the angular frequency.

In the upper part of the assembly the relative velocity is $w = \dot\eta$ and the friction area is $2\,\pi\,(a+b)\,d$. In the lower part the velocity is reduced to $w = \dot\eta\,(b^2-a^2)/b^2$ and the friction area is $2\,\pi\,b\,(h-d)$.

Accounting for this damping effect, equation (5) transforms into

$$\left[d' + (h'-d')\,\frac{b^2-a^2}{b^2}\right]\,\ddot\eta + \left[\frac{d}{b-a} + \frac{(h-d)\,(b^2-a^2)}{b^3}\right]\,\sqrt{2\,\nu\,\omega}\,\dot\eta + g\,\eta = -g\,Z - h'\,\ddot Z \tag{8}$$

still valid for an oscillation frequency ω.

The effects of flow separation at the lower ends of the tubes are collapsed into a single drag force expressed as

$$F_d = -\frac{1}{2}\,\rho\,C_D\,\pi\,a^2\,\dot\eta\,|\dot\eta| \tag{9}$$

meaning a term $1/2\,C_D\,a^2/(b^2-a^2)\,\dot\eta\,|\dot\eta|$ is added up to equation (8).

Here we focus on results from tests with imposed irregular motion. Stochastic linearization is applied to the quadratic damping term, that is $\dot\eta\,|\dot\eta|$ is replaced with $\sqrt{8/\pi}\,\sigma_{\dot\eta}\,\eta$ where $\sigma_{\dot\eta}$ is the standard deviation of $\dot\eta$. For a given spectrum $S_Z(\omega)$ of the forced motion, the RAO of η is obtained from

$$\Bigg\{-\omega^2\,\left[d' + (h'-d')\,\frac{b^2-a^2}{b^2}\right]$$

$$-\mathrm{i}\,\omega\,\left[\left(\frac{d}{b-a} + \frac{(h-d)\,(b^2-a^2)}{b^3}\right)\,\sqrt{2\,\nu\,\omega} + \sqrt{\frac{2}{\pi}}\,C_D\,\frac{a^2}{b^2-a^2}\,\sigma_{\dot\eta}\right] + g\Bigg\}\,RAO_\eta = -g + h'\,\omega^2$$

while

$$\sigma^2_{\dot\eta} = \int_0^\infty \omega^2\,RAO^2_\eta(\omega)\,S_Z(\omega)\,\mathrm{d}\omega$$

This is solved through iterations over the value of $\sigma_{\dot\eta}$.

Comparisons between experimental and numerical RAOs

Imposed vertical motions with the Hexapode were generated from spectral density constant over the frequency range from 0.5 Hz through 2 Hz.

Figure 2 shows the experimental and numerical RAOs obtained in the case of an irregular heave motion with a standard deviation equal to 1 cm (meaning maximum excursions, roughly, close to 4 cm). The drag coefficient C_d was adjusted to get the best fit, which gave $C_d = 0.85$. Experimental RAOs obtained under harmonic motion of 1 cm amplitude are also shown as symbols in the figures, in close agreement with the two curves.

Finally figure 3 shows results analogous to figure 2 in the 2 cm standard deviation case. The best fit is obtained with a drag coefficient of 0.65. These drag coefficients are consistent with literature (e.g. see Thiagarajan & Troesch, 1993).

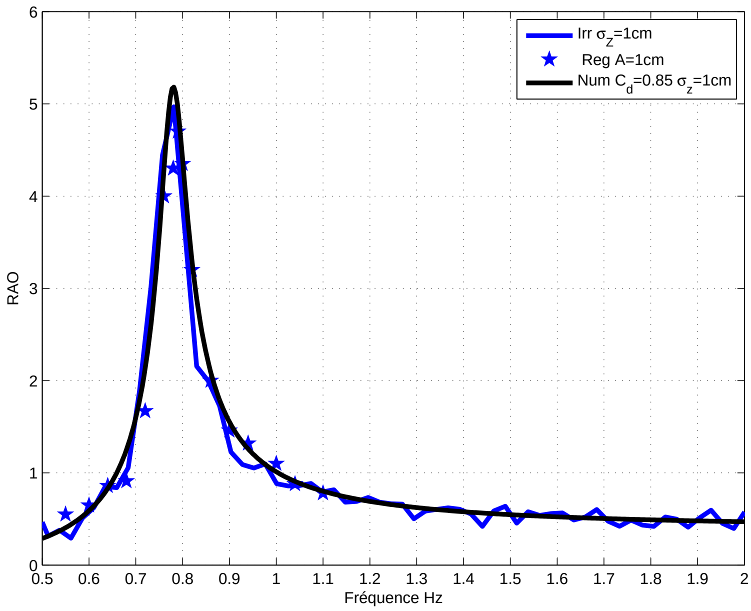

Figure 2: Experimental and numerical RAOs in irregular motion tests with 1 cm standard deviation.

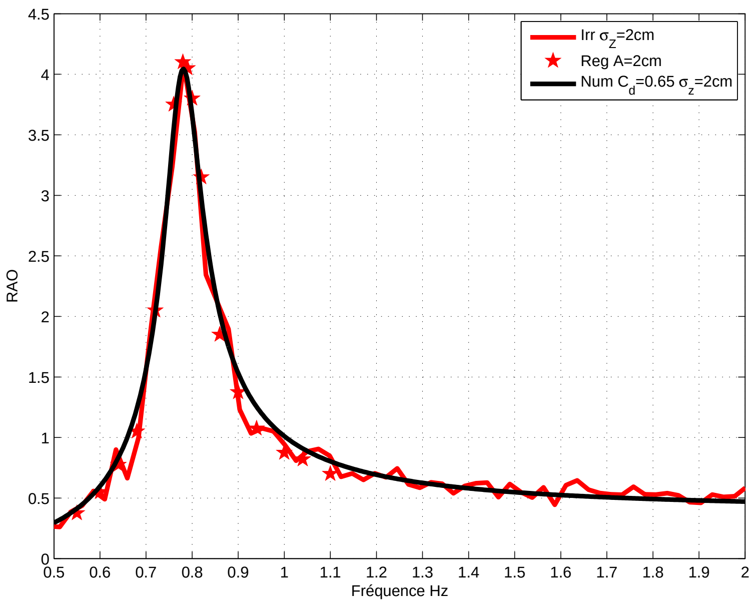

Figure 3: Experimental and numerical RAOs in irregular motion tests with 2 cm standard deviation.

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