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Transient flexural gravity wave motion in the presence of floating and submerged plate system

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Highlights

- The time dependent Green's functions for the floating and submerged plate system in three dimension in the presence of uniform current are derived using Laplace transform and method of stationary phase is used to derive their asymptotic forms.
- The derived Green's functions and the Green's identity are used to obtain the velocity potentials of the initial boundary value problem associated with the time dependent flexural gravity wave maker problem.
- As a special case of time dependent flexural gravity wavemaker problem, asymptotic solutions of floating and submerged plate deflections are obtained and analyzed.

1. Introduction

In the last decades, there has been a significant progress in the hydroelastic analysis of floating and submerged structures. Mohapatra and Sahoo [1] derived the Fourier-type expansion formulae for the velocity potentials associated with the floating and submerged plate system in the cases of water of both finite and infinite depths in two dimensions using eigenfunction expansion method. Recently, Mohanty et al. [2] studied the combined effect of current and compressive force on the gravity wave interaction with floating flexible structure in time-domain in both the cases of single-layer as well as two-layer fluid under the assumption of small amplitude water wave theory and structural responses using Laplace transform and Green's function technique in two-dimension. Montiel et al. [3] discussed the transient response of floating elastic plates in the presence of a vertical wavemaker using the Fourier transform technique in both time and frequency-domains in two dimensions.

2. Mathematical formulation

In the present study, the time dependent transient flexural gravity wave motion in the presence of floating and submerged plate system are analyzed in the three dimensional Cartesian coordinate system. It is assumed that the fluid domain is infinitely extended along the $x - z$ plane occupying the region $0 < x, z < \infty$, $0 < y < H$. In addition, it is assumed that there is a uniform current of speed U is flowing along the horizontal direction with components (U_1, U_2). A flexible plate is assumed to be floating on the surface of the water occupying the region $-\infty < x, z < \infty$ at the mean free surface $y = 0$ and another plate is submerged at at $y = h$, $-\infty < x, z < \infty$. The fluid region is divided into two regions namely, region 1, $-\infty < x, z < \infty$ $0 < y < h$ and region 2, $-\infty < x, z < \infty$, $h < y < H$. Assuming that the fluid is inviscid, incompressible and motion is irrotational, the velocity potentials $\oplus_j(x, y, z, t)$ are written as $\oplus_j(x, y, z, t) = U_1 x + U_2 z + \Phi_j(x, y, z, t)$ where subscripts $j = 1, 2$ correspond to regions 1 and 2 respectively. The deflections for the floating and submerged plates are denoted as $\eta_1(x, z, t)$ and $\eta_1(x, z, t)$ respectively. The velocity potentials $\Phi_j(x, y, z, t)$ satisfy the Laplace equation in the fluid domain along with the linearized kinematic conditions on the mean surface and interface of the floating and submerged structures given by

$$(\partial_t + U_1\partial_x + U_2\partial_z)\eta_1 = \partial_y\Phi_1, \quad \text{on} \quad y = 0, \tag{1}$$

$$(\partial_t + U_1\partial_x + U_2\partial_z)\eta_2 = \partial_y\Phi_1 = \partial_y\Phi_2, \quad \text{on} \quad y = h. \tag{2}$$

The combined kinematics and dynamic boundary conditions on the floating and submerged plates are given by (as in Mohanty et al. (2013))

$$\Big(D_1\partial_y^4 - Q_1\partial_y^2 + 1\Big)\Phi_{1y} = (1/g)\Big(\partial_t + U_1\partial_x + U_2\partial_z\Big)^2\Phi_1, \ \text{ on } \ y = 0, \tag{3}$$

$$\Big(D_2\partial_y^4 - Q_2\partial_y^2\Big)\Phi_{2y} = (1/g)\Big(\partial_t + U_1\partial_x + U_2\partial_z\Big)^2(\Phi_2 - \Phi_1), \ \text{ on } \ y = h, \tag{4}$$

where $D_j = E_j I_j/\rho_j g$, $Q_j = N_j/\rho_j g$, $\gamma_j = \rho_{ij} d_j/\rho_j$, $I_j = d_j^3/(12(1-\nu^2))$, $j = 1, 2$, E_j are the Young's modulus, N_j are the compressive force, ν is the Poisson's ratio, ρ_{ij} are the density of the floating ice sheet, ρ is the density of water and g is the acceleration due to gravity. The bottom condition on the flat and rigid bed is given by $\partial_y \Phi_2 = 0$ on $y = H$. The initial conditions at the floating and submerged plate covered surface are given by

$$\Phi_j, \partial_t \Phi_j = 0, \quad \text{on} \quad y = 0, h \quad \text{at} \quad t = 0. \tag{5}$$

Assuming that the the waves are propagating with an oblique angle with respect to $x-$axis, for sake of symmetry, it is assumed that velocity potentials and plate deflections are assumed to be of the forms $\Phi_j(x, y, z, t) = \mathrm{Re}\{\Psi_j(x, y, t)\mathrm{e}^{\mathrm{i}\nu z}\}$ and $\eta_j(x, z, t) = \mathrm{Re}\{\zeta_j(x, t)\mathrm{e}^{\mathrm{i}\nu z}\}$ respectively with $\nu = k \sin\theta$ and Re being the real part. Thus, the time dependent velocity potentials $\Psi_j(x, y, t)$ satisfy the reduced wave equation

$$\nabla_{xy}^2 \Psi_j - \nu^2 \Psi_j = 0, \tag{6}$$

the bottom boundary condition as in the case of $\Phi(x, y, z, t)$, the initial conditions as in Eq. (5). The boundary conditions on the plate covered surface are reduced to

$$(\partial_t + U_1 \partial_x + \mathrm{i}\nu U_2)\zeta_1 = \partial_y \Psi_1, \quad \text{on} \quad y = 0, \tag{7}$$

$$(\partial_t + U_1 \partial_x + \mathrm{i}\nu U_2)\zeta_2 = \partial_y \Psi_1 = \partial_y \Psi_2, \quad \text{on} \quad y = h, \tag{8}$$

$$\Big(D_1 \partial_y^4 - Q_1 \partial_y^2 + 1\Big)\Psi_{1y} = (1/g)\Big(\partial_t + U_1 \partial_x + \mathrm{i}\nu U_2\Big)^2 \Psi_1, \quad \text{on} \quad y = 0, \tag{9}$$

$$\Big(D_2 \partial_y^4 - Q_2 \partial_y^2\Big)\Psi_{2y} = (1/g)\Big(\partial_t + U_1 \partial_x + \mathrm{i}\nu U_2\Big)^2 (\Psi_2 - \Psi_1), \quad \text{on} \quad y = h. \tag{10}$$

2.1. Green's function for flexural gravity waves

In this subsection, the Green's function for flexural gravity wave problems are derived under the assumption that the motion is simple-harmonic along z-axis. For a point source of strength $m(t)$ located at (x_0, y_0) in the fluid domain, the Green's functions is rewritten as $G_0(x, y, z; x_0, y_0, t) = \mathrm{Re}\{G(x, y; x_0, y_0, t)\mathrm{e}^{\mathrm{i}\nu z}\}$. The Green's function $G(x, y; x_0, y_0, t)$ satisfies Eq. (6) in the fluid domain except at (x_0, y_0), along with the bottom boundary conditions and the initial conditions as described above and the boundary conditions in Eqs. (7)–(10) with $G = G_1$ for $0 < y < h$ and $G = G_2$ for $h < y < H$. In addition, the Green's function $G(x, y; x_0, y_0, t)$ satisfies the condition

$$G \sim -m(t)\mathrm{K}_0(\nu r_1), \quad \text{as } r_1 = \sqrt{(x - x_0)^2 + (y - y_0)^2} \to 0, \tag{11}$$

where $\mathrm{K}_0(\nu r_1)$ is the zeroth order modified Bessel's function. Using Laplace transform in time variable t, the initial boundary value problem in G is reduced to a boundary value problem in the transformed variable $\bar{G}(x, y; x_0, y_0, p)$. The transformed Green's function $\bar{G}(x, y; x_0, y_0, p)$ associated with the boundary value problem satisfy Eq. (6) in the fluid domain except at (x_0, y_0) along with the bottom boundary conditions and the initial conditions as described above. $\bar{G}$ satisfies $\bar{G} \sim -\bar{m}(p)\mathrm{K}_0(\nu r_1)$ as $r \to 0$ with $\bar{m}(p)$ being the Laplace transform of $m(t)$. It may be noted that $\mathrm{K}_0(x) \sim -\ln x + \gamma$ as $x \to 0$ where γ is the Euler number and the modified Bessel's functions satisfy the integral identities

$$\mathrm{K}_0(\nu r_1) = \int_0^\infty \frac{\mathrm{e}^{-k|y - y_0|}}{k} \cos\mu(x - x_0)\mathrm{d}\mu, \; \mathrm{K}_0(\nu r_2) = \int_0^\infty \frac{\mathrm{e}^{-k|y + y_0|}}{k} \cos\mu(x - x_0)\mathrm{d}\mu, \tag{12}$$

where $r_2 = \sqrt{(x - x_0)^2 + (y + y_0)^2} \to 0$ and $k = \sqrt{\mu^2 + \nu^2}$. Further, the source point (x_0, y_0) is either in the lower layer fluid or in the upper layer fluid. Using Laplace transform and the boundary conditions, the transformed Green's function $\bar{G}(x, y; x_0, y_0, p)$ is obtained, whose details are discussed below. Assuming the source point (x_0, y_0) is in the upper layer, the transformed Green's function $\bar{G}(x, y; x_0, y_0, p)$ is derived as

$$\bar{G} = \begin{cases} \bar{m}(p)\Big\{\mathrm{K}_0(\nu r_2) - \mathrm{K}_0(\nu r_1) + \displaystyle\sum_{m=\mathrm{I}}^{\mathrm{II}} \int_0^\infty F^{\gamma_m} \mathrm{e}^{\mathrm{i}\gamma_m \mu(x - x_0)}\mathrm{d}\mu\Big\}, \; 0 < y < h, \\ \bar{m}(p) \displaystyle\int_0^\infty \cosh k(H - y)\Big\{C^+(k)\mathrm{e}^{\mathrm{i}\mu(x - x_0)} + C^-(k)\mathrm{e}^{-\mathrm{i}\mu(x - x_0)}\Big\}\mathrm{d}\mu, \quad h < y < H. \end{cases} \tag{13}$$

where $F^{\gamma_m} = \{A^{\gamma_m}(k)\cosh ky + B^{\gamma_m}(k)\sinh ky\}$, $\gamma_m = +,-$ for $m = \mathrm{I}, \mathrm{II}$ respectively. The unknowns $A^{\pm}$, $B^{\pm}$, $C^{\pm}$ are obtained using the boundary conditions as in Eqs. (7)–(10) and are given by

$$A^{\pm}(k) = \frac{\{P_{\pm u}^2 A_1(k) + B_1(k)\}\{1 + D_1k^4 - Q_1k^2\}}{G^{\pm}(k)\sinh kh \sinh k(H-h)}, \; B^{\pm}(k) = \frac{A^{\pm}(k)P_{\pm u}^2}{(1 + D_1k^4 - Q_1k^2)k} + \frac{2\mathrm{e}^{-ky_0}}{k}, \tag{14}$$

$$C^{\pm}(k) = -\frac{P_{\pm U}^2\{P_{\pm U}^2 \sinh ky_0 + (1 + D_1k^4 - Q_1k^2)k\cosh ky_0\}}{G^{\pm}(k)k\sinh kh \sinh k(H-h)}, \quad P_{\pm u} = (1/\sqrt{g})\{p \pm \mathrm{i}(\mu U_1 \pm \nu U_2)\}, \tag{15}$$

$$\left.\begin{aligned}
&A_1(k) = -\{\sinh k(h-y_0)\sinh k(H-h) + \cosh k(h-y_0)\cosh k(H-h)\},\\
&G^{\pm}(k) = P_{\pm U}^4 s_1 + P_{\pm U}^2 s_2 + s_3, \quad B_1(k) = k(D_2k^4 - Q_2k^2)\cosh k(h-y_0)\sinh k(H-h),\\
&s_1 = \coth kh \coth k(H-h) + 1, \quad s_3 = k^2(D_2k^4 - Q_2k^2)(1 + D_1k^4 - Q_1k^2),\\
&s_2 = (1 + D_1k^4 - Q_1k^2)k\{\coth k(H-h) + \coth kh\} + k(D_2k^4 - Q_2k^2)\coth kh.
\end{aligned}\right\}$$

Further, assuming the source point (x_0, y_0) is in the lower layer, the transformed Green's function $\bar{G}(x, y; x_0, y_0, p)$ can be derived in a similar manner. Next, Using inverse Laplace transform and the convolution theorem, the time dependent Green's function $G(x, y, x_0, y_0, t)$ having source in the upper layer is derived as

$$\begin{aligned}
G = m(t)\mathcal{D}_1(\nu) \quad &+ \quad \int_0^{\infty}\int_0^t m(t-\tau)\Big[\mathrm{e}^{-\mathrm{i}\nu\tau U_2}\mathcal{D}_2\cos\mu(x - x_0 - U_1\tau)\\
&\times \quad \Big\{\mathcal{A}(k,y)\sin\tau C_1 + \mathcal{B}(k,y)\sin\tau C_2\Big\} + \frac{\delta(\tau)}{s_1}\mathcal{M}(k,y)\cos\mu(x-x_0)\Big]\mathrm{d}\tau\mathrm{d}\mu,
\end{aligned}\tag{16}$$

$$\begin{aligned}
\text{where} \quad \mathcal{A}(k,y) \quad &= \quad \begin{cases} s_1C_1^2\mathcal{M}(k,y) - (s_2C_1^2 - s_3)\mathcal{N}(k,y) - s_{11}\mathcal{T}(k,y), & 0<y<h,\\ s_1C_1^2\mathcal{R}(k,y) - (s_2C_1^2 - s_3)\mathcal{S}(k,y), & h<y<H,\end{cases}\\
\mathcal{B}(k,y) \quad &= \quad \begin{cases} (s_2C_2^2 - s_3)\mathcal{N}(k,y) - s_1C_2^2\mathcal{M}(k,y) + s_{11}\mathcal{T}(k,y), & 0<y<h,\\ (s_2C_2^2 - s_3)\mathcal{S}(k,y) - s_1C_2^2\mathcal{R}(k,y), & h<y<H,\end{cases}\\
\mathcal{M}(k,y) \quad &= \quad -\frac{g}{\sinh kh}\Omega_1(D_2k^2 - Q_2)k^4\cosh k(H-y_0)\cosh ky, \quad \mathcal{T}(k,y) = \frac{\mathcal{N}(k,y)\tanh ky}{\Omega_1 k},\\
\mathcal{N}(k,y) \quad &= \quad -\Omega_2\Omega_1\cosh k(H-y_0)\sinh ky, \quad \mathcal{R}(k,y) = -\Omega_2\Omega_1\cosh ky_0\cosh k(H-y)\\
\mathcal{S}(k,y) \quad &= \quad -\Omega_2\sinh ky_0\cosh k(H-y), \quad \mathcal{D}_1(\nu) = \begin{cases} \mathrm{K}_0(\nu r_2) - \mathrm{K}_0(\nu r_1), & 0<y<h,\\ 0, & h<y<H,\end{cases}
\end{aligned}$$

$C_{1,2} = \sqrt{s_2 \pm \sqrt{s_2^2 - 4s_1s_3}/2s_1}$, $\mathcal{D}_2 = \dfrac{1}{C_2 s_1^2(C_1^2 - C_2^2)}$, $\Omega_1 = (D_1k^4 - Q_1k^2 + 1)$, $\Omega_2 = \dfrac{2g}{\sinh kh\sinh k(H-h)}$, $s_{11} = s_1$, δ being the Dirac delta function, in $C_{1,2}$, $1, 2$ denote the positive and negative sign respectively. For simplicity it is assumed that the waves and the current are moving in the same direction and making an angle zero degree with positive direction of x-axis which yield $U_2 = 0$ and $U_1 = U$ and $m(t) = 1$. It may be noted that various terms in the Green's function derived above are highly oscillatory in nature. To understand the characteristics of Green's function for large values of space and time, the oscillatory integrals in Eq. (16) are computed using the method of stationary phase. In the subsequent discussion various non-dimensional parameters $\tilde{x} = x/H$, $\tilde{y} = y/H$, $\tilde{y}_0 = y_0/H$, $\tilde{x}_0 = x_0/H$, $\tilde{t} = (g/H)^{1/2}t$, $\tilde{G}(\tilde{x},\tilde{y},\tilde{t}) = G(x,t)/H$, $\tilde{U} = U/(gH)^{1/2}$, $\tilde{D}_1 = D_1/gH^4$, $\tilde{Q}_1 = Q_1/gH^2$, $\tilde{D}_2 = D_2/gH^4$, $\tilde{Q}_2 = Q_2/gH^2$ are used. Using the non-dimensional parameters and applying method of stationary phase, the non-dimensional Green's function $\tilde{G}(\tilde{x},\tilde{y};\tilde{x}_0,\tilde{y}_0,\tilde{t})$ when the source (x_0, y_0) is in the upper layer is obtained as

$$\begin{aligned}
\tilde{G}(\tilde{x},\tilde{y};\tilde{x}_0,\tilde{y}_0,\tilde{t}) = \mathcal{D}_1(\nu) \quad &+ \quad \int_0^{\infty}\tilde{\mathcal{H}}_9(k,y)\cos kH(\tilde{x}-\tilde{x}_0)\mathrm{d}k + \sqrt{\frac{\pi}{2}}\sum_{m=1}^{2}\Bigg[\sum_{n=0}^{n_1}\frac{\tilde{\mathcal{H}}_{2m}(\alpha_n,y)}{\tilde{t}\tilde{f}_m''(\alpha_n)}\cos\{\tilde{t}\tilde{f}_m(\alpha_n) + \pi/4\}\\
&- \quad \frac{\tilde{\mathcal{H}}_{3m+2}(\alpha_n,y)}{\tilde{t}\tilde{f}_{m+2}''(\alpha_n)}\cos\{\tilde{t}\tilde{f}_{m+2}(\alpha_n) + \pi/4\}\Bigg],
\end{aligned}\tag{17}$$

$$\begin{aligned}
\text{where} \quad \mathcal{H}_{1,2}(k,y) \quad &= \quad \frac{\mathcal{A}(k,y)}{U - C_1}, \quad \mathcal{H}_{3,4}(k,y) = -\frac{\mathcal{A}(k,y)}{U + C_1}, \mathcal{H}_{5,6}(k,y) = \frac{\mathcal{B}(k,y)}{U - C_1},\\
\mathcal{H}_{7,8}(k,y) \quad &= \quad -\frac{\mathcal{B}(k,y)}{U + C_1}, \quad \mathcal{H}_9(k,y) = \frac{\mathcal{M}(k,y)}{s_1} - \frac{C_1}{U^2 - C_1^2} - \frac{C_2}{U^2 - C_2^2},
\end{aligned}$$

$$f_{1,3}(k) = (x-x_0)\frac{k}{t} + k(C_1 \pm U) = -f_{2,4}(k), \quad f_{5,7}(k) = (x-x_0)\frac{k}{t} - k(C_2 \pm U) = f_{6,8}(k),$$

and α_n's are real roots of $\tilde{f}_n'(kH) = 0$. The values of n for which there is no real root of $\tilde{f}_n'(kH) = 0$, do not contribute to the asymptotic results in (17). Further, the value of n_1 in Eq. (17) computed for specific physical problems. Similarly, the asymptotic form of the time dependent Green's function when the source is in the lower layer is derived. It may be

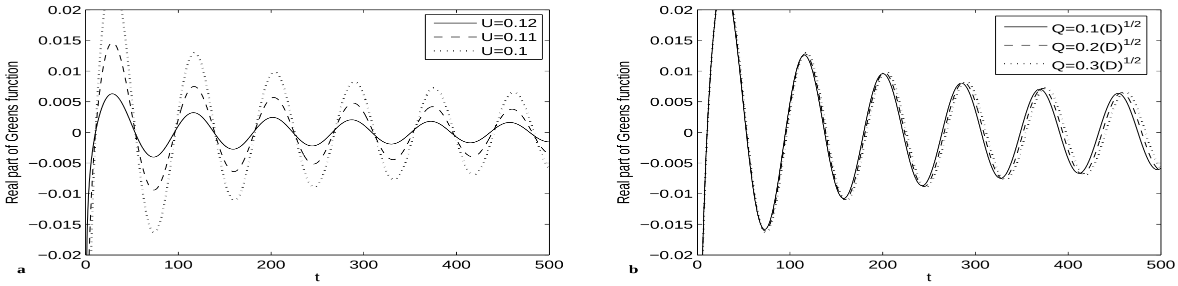

Fig. 1. Real part of the Green's function versus time for different values of (a) current speed (b) compressive force

noted that using $\bar{m}(p) = 1$ and $p^2 = -\omega^2$ in Eq. (13) and Cauchy residue theorem, the time harmonic Green's function can be obtained easily. From Fig. 1(a) it is observed that as the current speed increases the amplitude of the real part of the Green's function decreases also for large time the amplitude decreases. From Fig. 1(b) it is observed that compression has very small impact on the real part of the Green's function in the presence of current.

2.2. Transient flexural gravity wavemaker problem

On the vertical wavemaker located at $x = 0$, the boundary condition is given by $\partial_x \Psi = V_1(y,t) + U$ with $U_2 = 0$ as defined in the previous subsection. Proceeding in a similar manner as in Mohanty et al. [1] and using the Green's identity the expansion formulae for the velocity potentials associated with the transient flexural gravity wavemaker problems are derived. Using $V_1(y,t) = \sin\sqrt{(H/g)}t$ and Eqs. (9)-(10), the floating and submerged deflections are obtained whose details are deferred here. Some of the numerical computations are presented below. From Fig. 2(a) it is observed that as

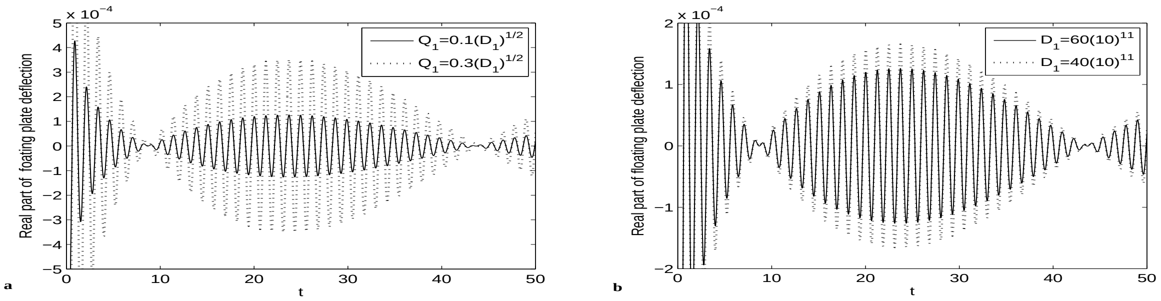

Fig. 2. Real part of the plate deflection versus time for different values of (a) compressive force (b) flexural rigidity

the compressive increases the amplitude of the real part of the plate deflection decreases. From Fig. 2(b) it is observed that flexural rigidity has significant impact on the plate deflection. Various other results will be presented in the workshop.

References

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