# Water-exit problem with prescribed motion of a symmetric body

**Alexander Korobkin**

School of Mathematics, University of East Anglia, Norwich, UK, e-mail: a.korobkin@uea.ac.uk

**Tatyana I. Khabakhpasheva**

Department of Applied Hydrodynamics, Lavrentyev Institute of Hydrodynamics
Novosibirsk, Russia, e-mail: tkhab@ngs.ru

**Kevin J. Maki**

Department of Naval Architecture and Marine Engineering University of Michigan
Ann Arbor, MI, 48109 USA, e-mail: kjmaki@umich.edu

A two-dimensional problem of a symmetric rigid body with small deadrise angle, which is lifted from the liquid surface with a prescribed acceleration, is considered. The liquid is of infinite depth, inviscid and incompressible. Initially the liquid is at rest. The free surface of the liquid is flat and horizontal. Initial draft of the body is given. The body starts to move suddenly upwards from the liquid with a prescribed acceleration which varies in time. Gravity and surface tension effects are not included in the model. Boundary conditions on the liquid surface are linearized and imposed on the equilibrium position of the liquid surface. The hydrodynamic pressure is assumed continuous at the periphery of the wetted area which shrinks monotonically with time. The unknown size of the wetted area is determined by the condition that the speed of the contact points is proportional to the local velocity of the flow. Hydrodynamic forces acting on a lifting body are determined within the proposed linearized model and compared with the numerical results obtained by solving the Navier-Stokes equations. It is shown that the linearized water-exit model accurately predicts the hydrodynamic loads.

## 1. Introduction

The present study is motivated by naval hydrodynamics, where a section of the ship enters the water and exits from water thereafter, and aircraft ditching on the water surface within the '$2D + t$' approach. The first stage, which is referred to as the entry or impact stage, is well studied. However, the second stage, when the section exits from the water, has not yet been fully studied. It was shown (see Piro & Maki (2011, 2012, 2013) and Tassin et al. (2013)) that the negative hydrodynamic loads during the second stage can be as large in magnitude as the impact loads acting on the section during the impact stage. The physical processes that are important during the exit stage are different from those which dominant the impact stage (Greenhow, 1988). For many cases it is expected that viscous effects and even surface tension can play a significant role. A linearized model of exit was suggested by Korobkin (2013) and applied successfully to the problems of exit with constant acceleration. In this model, the flow is assumed potential and linear. The shape of the wetted part of the body is simplified by using the so-called "flat-plate approximation". Only inertia forces are included in the exit model of this paper.

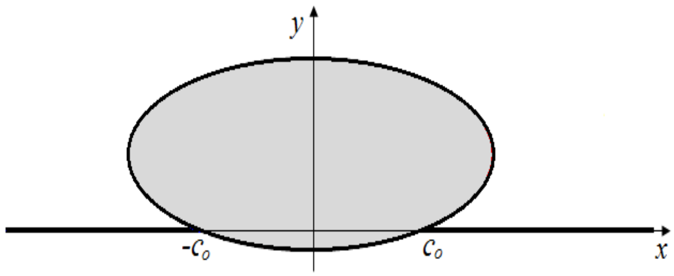


Figure 1. Initial configuration of the exit problem.

The present model is concerned with the negative hydrodynamic force acting on a body lifted from the liquid surface during the early stage (Figure 1). Viscous effects are taken into account through the equation for the velocity of the contact points, which is taken to be proportional to the local speed of the flow at these points.

## 2. Mathematical formulation of the exit model

Within the present exit model, the pressure $p(x,y,t)$ is given by the linearized Bernoulli equation $p(x,y,t) = -\rho\varphi_t(x,y,t)$, where the velocity potential $\varphi(x,y,t)$ satisfies the Laplace equation in the lower half plane $y<0$ and decays at infinity, where $x^2+y^2 \to \infty$. The boundary conditions are

$$\varphi_t(x,0,t) = 0 \quad (y=0, |x|>c(t)), \qquad \varphi_y(x,0,t) = h'(t) \quad (y=0, |x|<c(t)),$$

where the function $c(t)$ is calculated by using the condition

$$\frac{\mathrm{d}c}{\mathrm{d}t} = \gamma\varphi_x(c(t),0,t). \tag{1}$$

The coefficient $\gamma$ is undetermined in the present model and is chosen by using the numerical results by Piro & Maki (2011). It was found that $\gamma = 2$ corresponds to all available numerical results. We assume that the relation between the speed of the contact point $c'(t)$ and the local tangential velocity of the flow is linear with the coefficient $\gamma$ being dependent, in general, on the physical characteristics of both the liquid and the body surface, such as wettability of the body surface and viscosity of the liquid.
The solution of the mixed boundary problem with respect to the pressure provides

$$p(x,0,t) = -\rho h''(t)\sqrt{c^2(t)-x^2} \quad (|x|<c(t)), \qquad F(t) = \int\limits_{-c(t)}^{c(t)} p(x,0,t)\mathrm{d}x = -m_a h''(t), \tag{2}$$

where $m_a = 0.5\pi\rho c^2(t)$ is the added mass of the equivalent flat plate. It is seen that the hydrodynamic force can be calculated if the size of the wetted area, which is described by the function $c(t)$, is known.
The velocity potential and the velocity of the flow along the body surface are given by

$$\varphi(x,0,t) = \int\limits_0^t h''(\tau)\sqrt{c^2(\tau)-x^2}\mathrm{d}\tau \qquad \varphi_x(x,0,t) = -x\int\limits_0^t \frac{h''(\tau)\mathrm{d}\tau}{\sqrt{c^2(\tau)-x^2}}. \tag{3}$$

Equations (1) and (3) yield the following equation for the function $c(t)$

$$\frac{\mathrm{d}c}{\mathrm{d}t} = -\gamma c(t)\int\limits_0^t \frac{h''(\tau)\mathrm{d}\tau}{\sqrt{c^2(\tau)-c^2(t)}}. \tag{4}$$

To solve equation (4), we introduce new non-dimensional variables $\alpha$ and $\sigma$ such that $c^2(t) = c_0^2(1-\sigma)$, $c^2(\tau) = c_0^2(1-\alpha)$, where $\alpha$ and $\sigma$ are equal to zero when $t=0$ and $\tau=0$, correspondingly, and $\alpha=\sigma$ at $\tau=t$. A new unknown function $f(\sigma)$ is introduced by the equation

$$h''(t) = f(\sigma)\frac{\mathrm{d}c^2}{\mathrm{d}t}.$$

The equation (4) leads to two equations with respect to $f(\sigma)$ and $t(\sigma)$

$$h'(t) = -c_0^2 \int_0^\sigma f(\alpha)\mathrm{d}\alpha, \qquad h''(t) = 2\gamma c_0^3(1-\sigma)f(\sigma)\int_0^\sigma \frac{f(\alpha)\mathrm{d}\alpha}{\sqrt{\sigma-\alpha}}. \tag{5}$$

Initial asymptotic behaviour of the solution to (5) determines the integration scheme. If $h(t) \sim At^m$ as $t \to 0$, then $f(\sigma) = -q_0\sigma^{-k}G(\sigma)$, where $G(0)=1$ and $k=(3-m)/(2m)$. The formula for $q_0$ is not shown here. We introduce two non-dimensional functions $V(t) = h'(t)/(c_0^2 q_0)$ and $W(t) = h''(t)/(2\gamma c_0^3 q_0^2)$. These functions are given.

## 3. Numerical solution and its comparison with the CFD results

The interval $0<\sigma<1$ is divided into $N$ subintervals with $\sigma_n = n/(N+1)$ and new unknowns $G_n = G(\sigma_n)$ and $t_n = t(\sigma_n)$. The function $G(\sigma)$ is interpolated linearly in each subinterval. The system provides

$$G_n = (V(t_n) - V(t_{n-1}) + G_{n-1}\beta_n)/\alpha_n, \qquad W(t_n) = 2(1-\sigma_n)\sigma_n^{1/2-2k}G_n(G_nA_{nn}+P_n), \tag{6}$$

where $\beta_n$, $\alpha_n$ and $A_{nn}$ are given coefficients, and $P_n$ depends on $G_j$, $1 \le j \le n-1$. The system (6) is solved by the bisection method with respect to the time instants $t_n$.

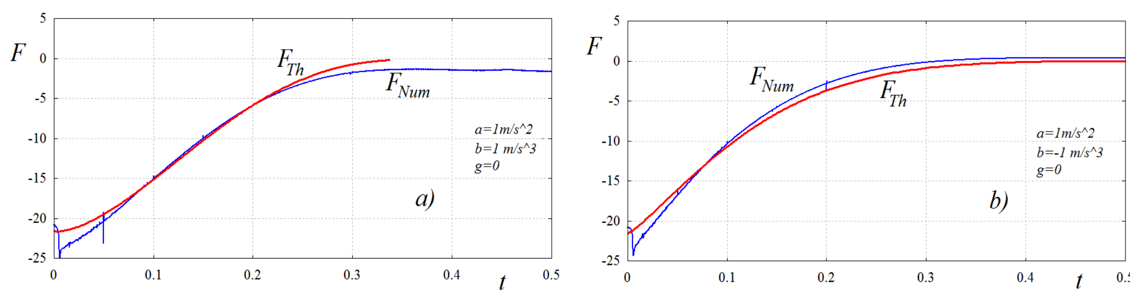


Figure 2. The dimensional force in $N/m$ acting on a half of the body as a function of time in seconds for (a) $b=+1$ m/s$^{-3}$ and (b) $b=-1$ m/s$^{-3}$.

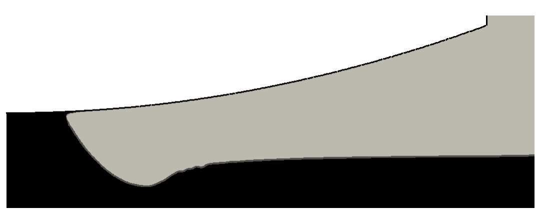

Figure 3. CFD prediction of free-surface location at time 0.3 s for case shown in Figure 2a.

The problem of water exit is also solved numerically with a VOF-based Navier-Stokes solver from the OpenFOAM library. Computations are performed for the parabolic contour $y = x^2/(2R) - h_0 + h(t)$, where $R \approx 1.4$ m, $h_0 = 1$ cm and $h''(t) = a + 2bt$, $a = 1$ m/s$^{-2}$, $b = \pm 1$ m/s$^{-3}$. Here $c_0 = \sqrt{2Rh_0}$. Gravity is not included in the computations. The results for the vertical force on the body are presented in Figure 2. It is seen that the theoretical model well describes the evolution of the hydrodynamic force. However, the model does not describe all details of the force just after the start of the body motion, and the interaction lasts longer in CFD than in the theory for $b = +1$ m/s$^{-3}$. The behavior for large time in the this case is due to the nonlinear free-surface which is still attached to the body (Figure 3).

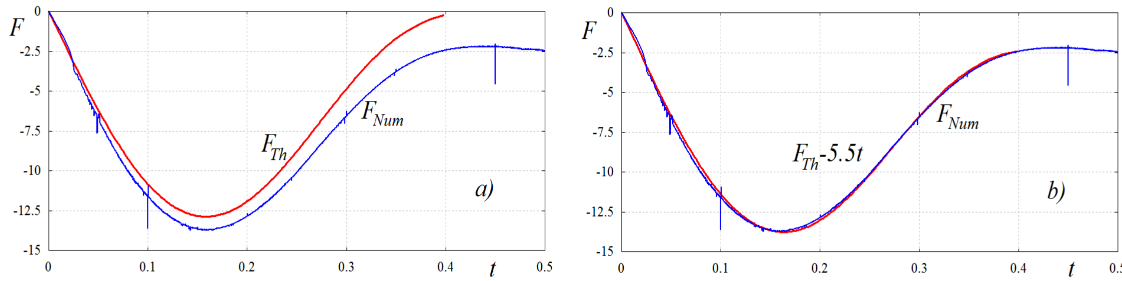


Figure 4. The dimensional force in $N/m$ acting on a half of the parabolic contour lifted from water with linear acceleration, $h(t) = at^3$, where $a = 1$ m/s$^{-3}$.

If the body acceleration linear function of time, $h(t) = at^3$, then the system (5) can be written in the form independent of any parameters

$$\tau^2 = 4\int_0^\sigma g(\alpha)\mathrm{d}\alpha, \qquad \tau = (1-\sigma)g(\sigma)\int_0^\sigma \frac{g(\alpha)\mathrm{d}\alpha}{\sqrt{\sigma-\alpha}}, \tag{7}$$

where $t = \tau(c_0/48\gamma a)^{\frac{1}{3}}$ and the force $F(t) = F_{sc}\tau(\sigma)(\sigma-1)$, $F_{sc} = 0.25\pi\rho(36a^2c_0^7/\gamma)^{\frac{1}{3}}$. The computations were performed for the same shape as in Figures 2 and 3. The system (7) is solved by the power series method. The results are shown in Figure 4a. It was found that the difference between the CFD and theoretical solutions can be approximated as $5.5t$ (see Figure 4b). This implies that the added mass in the CFD results differs from the theoretical one by a constant. One could expect that this constant describes the effect of the liquid left on the lifting surface. The latter effect is not taken into account in the theoretical model.

**Acknowledgement:** This work has been supported by the NICOP research grant "Fundamental Analysis of the Water Exit Problem" N62909-13-1-N274, through Dr. Woei-Min Lin. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the Office of Naval Research.

## References

Greenhow M. (1988) Water-entry and -exit of a horizontal circular cylinder. Appl. Ocean Res. Vol. 10(4), pp. 191-198.

Piro D. J. & Maki K. J. (2011) Hydroelastic wedge entry and exit. In 11th International Conference on Fast Sea Transport. Honolulu, Hawaii, USA.

Piro D. J. & Maki K. J. (2012) Water exit of a wedge-shaped body. In Proceedings 27th IWWWFB, Copenhagen, Denmark, pp. 141-144.

Piro D. J. & Maki K. J. (2013) Hydroelastic analysis of bodies that enter and exit water. Journal of Fluids and Structures. Vol. 37, pp. 134-150.

Tassin A., Piro D. J., Korobkin A. A., Maki, K. J., & Cooker M. J. (2013). Two-dimensional water entry and exit of a body whose shape varies in time. Journal of Fluids and Structures. Vol. 40, pp. 317-336.

Korobkin A. A. (2013) A linearized model of water exit. Journal of Fluid Mechanics. 737, pp. 368-386.