# Hydroelastic behavior of a vertical plate subjected to third-order wave interactions

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## 1. Introduction

The hydrodynamics research group of Ecolé Centrale Marseille comprehensively investigated in a series of recently published studies the effect of tertiary interactions on the wave run-up on a vertical plate subjected to (initially) regular wave excitations (see e.g. [1-2]). The method they developed accounted for the continuous interactions between the incoming and the reflected (by the plate) waves that eventually change completely the wave field. To achieve that, the authors exploited the formulae proposed by Longuet-Higgins and Phillips [3] that describe the modification of the wave number and the change of the direction of the reflected waves. The studies presented in [1-2] accounted only for a confined wave field adapting the environment of a wave basin. Accordingly, Chatjigeorgiou and Molin [4] extended the existing theory to assimilate the conditions of an infinite wave field. That was achieved describing the plate as an elliptical cylinder with nearly zero semi-minor axis and the associated effort avoided the reflections due to the lateral walls of the confined wave field which for long distance travelling waves eventually modify the wave run-up on the plate. The investigation of the third-order interactions is still under way as there are many more interesting features that require proper answers such as for example the disability to achieve steady-state conditions for large wave steepness [5]. In the present study we advance one step further investigating the dynamic behavior of the plate subjected to third-order interactions. In the present initiatory effort we employ linear theory for the structural dynamics of the plate assuming that it has hinged edges and we emphasize on the differences between the linear component on the wave run-up and the actual (final) run-up on the plate after a steady-state condition has been established and the wave field has obtained a steady pattern. The hydrodynamic phenomenon is treated assuming the infinite wave field option [4].

## 2. The diffraction problem in the infinite wave field

The basics of the followed methodology have been extensively analyzed in reference [4] where the interested reader is referred. Succinctly, the plate is simulated as a degenerated elliptical cylinder with nearly zero semi-minor axis. The "elliptical" plate is considered stationary and subjected to regular waves with amplitude $A_I$ and frequency $\omega$ in an infinite water depth. Under these conditions the incident and diffracted components of the velocity potential will be given by

$$\varphi_I = -2i\frac{g}{\omega}A_I e^{kz}\left\{\sum_{m=0}^{\infty} i^m \mathrm{Mc}_m^{(1)}(u;q)\mathrm{ce}_m(v;q)\mathrm{ce}_m(\alpha;q) + \sum_{m=1}^{\infty} i^m \mathrm{Ms}_m^{(1)}(u;q)\mathrm{se}_m(v;q)\mathrm{se}_m(\alpha;q)\right\} \quad (1)$$

$$\varphi_D = -i\frac{g}{\omega}A_I e^{kz}\left\{\sum_{m=0}^{\infty} i^m B_m \mathrm{Mc}_m^{(3)}(u;q)\mathrm{ce}_m(v;q) + \sum_{m=1}^{\infty} i^m C_m \mathrm{Ms}_m^{(3)}(u;q)\mathrm{se}_m(v;q)\right\} \quad (2)$$

where $g$ is the gravitational acceleration, $k$ is the wavenumber, $u$ and $v$ are the elliptic coordinates, $\alpha$ is the angle of incidence, $q$ is the Mathieu parameter $q=(ka\epsilon/2)^2$, where $a$ denotes herein the semi-major axis, $\epsilon$ is the elliptic eccentricity, $\mathrm{ce}_m$ and $\mathrm{se}_m$ are the even and odd periodic Mathieu functions and $\mathrm{Mc}_m$, $\mathrm{Ms}_m$ are the even and odd modified Mathieu functions. The indices (1) and (3) denote the kind of the Mathieu functions. The unknown expansion coefficients of the diffraction component $B_m$ and $C_m$ are obtained by employing the zero velocity condition on the plate making use of the orthogonality relations of the even and odd periodic Mathieu functions. In fact, this is the first step of the global procedure that provides the linear solution and accordingly the linear wave run-up. Subsequently the reflected wave-filed interacts with the (continuously coming) regular wave train and the wave field is modified. At each stage the modified wave field is obtained by applying the parabolic system theory developed in Molin at al. [1-2]. The global procedure is put into an iterations process whilst, each iteration literally represented an interaction process. If the method is convergent, meaning that the wave run-up eventually stabilizes, the entire wave filed attains a steady state condition. As shown in Molin et al. [5] analyzing experimental results this is not always a fact as for large wave steepness the wave field changes continuously between specific patterns which accordingly are reflected on the wave run-up. Here, we investigate only convergent cases where convergent is achieved after a number of iterations. For long travelling waves, more iterations are required. A specific example of the wave run-up on a 10m plate after convergence is shown in the following Fig. 1.

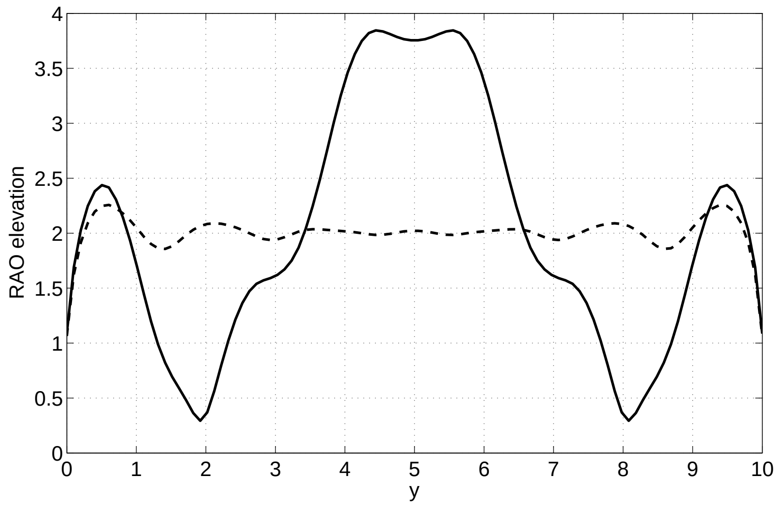


**Fig. 1.** Wave run-up on a 10m plate.

The assumed conditions that resulted in the calculations shown above correspond to wave period $T$=1.01s and wave steepness $\varepsilon=kA_I$=2.5%. The origin of the wave train was assumed to be 200m ahead of the plate. For references, Fig. 1 contains also the linear run-up (first iteration; dashed line). It can be easily seen that the final wave run-up is significantly larger that the (nearly uniform) linear one. The depicted data have been normalized by $A_I$. Note that the third-order interactions method developed in [1-2, 4] takes into account the half plate assuming mirroring effects. The data shown in Fig. 1 have been artificially mirrored to the other half for solving the structural dynamics (hydroelastic) problem of the plate. The computation of the wave run-up allows the calculation of the potential on the plate and accordingly the hydrodynamic pressure.

## 3. The dynamics of the plate

The plate employed in [1-2, 4] was 10m long (including the mirror part), 2.33m height having draught 2m. Here the plate is considered hinged along all four edges. Linear theory is employed and the forced vibrations of the plate comply with the following governing equation

$$\partial^4 w / \partial y^4 + 2\partial^4 w / \partial y^2 \partial z^2 + 2\partial^4 w / \partial z^4 + M / D \partial^2 w / \partial t^2 = p(y,z;t) / D \quad (3)$$

where $w(y, z; t)$ denotes the normal (bending) deformations of the plate, $y$ and $z$ are the horizontal and vertical Cartesian coordinates (fixed on the origin of Fig. 1), $M$ is the mass per area of the plate and $D=Eh^3/12/(1-v^2)$ is the plate bending stiffness. Note that $E$ is the Young's modulus of elasticity, $h$ is the thickness and $v$ is Poisson's ratio. For a steel plate $E$=209GPa, the material density is 7850kg/m$^3$, and $v$=0.3. Here we assumed that $h$=3mm. In Eq. (3) $p$(y, z; $t$) denotes the hydrodynamic pressure obtained by wave run-up as $p(y,z;t) = i\omega\rho\varphi$ where $\rho$ is the water density and $\varphi = \varphi_I + \varphi_D$ is the total velocity potential [see Eqs. (1)-(2)]. The total velocity potential is written as $\varphi = -iA_I g / \omega e^{kz} \zeta(y)$ where $\zeta(y)$ is the wave run-up obtained according to the method developed in [1-2, 4]. Note that the origin of the vertical ($z$) coordinate in the previous relation is fixed on the free surface. Hence, in order to apply Eq. (3) $z$ must be transformed in order to take into account the draught ($f$=2m) of the plate.

A solution that satisfies the boundary conditions (zero bending moments and motions) of the perimeter of the plate is expanded as

$$w = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} A_{mn}(t) \sin\frac{m\pi y}{a} \sin\frac{n\pi z}{b} \quad (4)$$

where $a$ and $b$ now denote the length and the height of the plate. Using the regular assumptions of the linear theory we write $A_{mn}(t) = \mathrm{Re}\{a_{mn} e^{-i\omega t}\}$ where $a_{mn}$ are complex expansion coefficients depending on $\omega$ that need to be evaluated.

Introducing Eq. (4) into Eq. (3) and making use of the orthogonality of trigonometric functions the complex expansion coefficients yield from the simple formula $a_{mn} = F_{mn} / \left(\omega_{mn}^2 - \omega^2\right)$ where the eigen-frequencies $\omega_{mn}$ are given by

$$\omega_{mn} = \left[(m\pi / a)^2 + (n\pi / b)^2\right]\sqrt{D / M} \tag{5}$$

and the coefficients $F_{mn}$ are

$$F_{mn} = \frac{4\rho g A_I}{abM} \frac{b\left[n\pi e^{-kf} - n\pi \cos(n\pi f / b) + kb \sin(n\pi f / b)\right]}{(kb)^2 + (n\pi)^2} \int_0^a \zeta(y) \sin(m\pi y / a) dy \,. \tag{6}$$

Given the complicated form of the final run-up on the plate (see for instance Fig. 1), the integral in Eq. (6) is obtained numerically. Note that it was assumed that the hydrodynamic pressure above the draught dimension $f$ (above the free surface) is zero. The computation of the complex expansion coefficients $a_{mn}$ allows obtaining the values for the structural vibrations of the plate $w(y, z; t)$ through Eq. (4). Clearly, the solution of the structural vibrations problem dictates the solution of the third-order hydrodynamic problem, namely it requires performing the computations described in [4].

## 4. Numerical results

Here we provide some numerical results for the conditions outlined in Sections 2 and 3. The wave-run up on the plate is shown in Fig. 1 for the linear (first-impact) problem and at the end of iterations after the steady-state condition has been established.

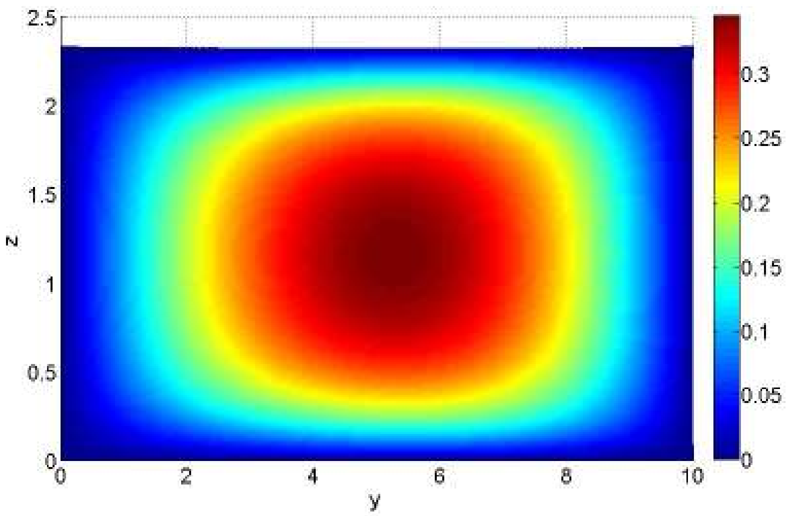


**Fig. 2.** Bending vibrations of the plate $w(y, z)/h$ at the first iteration (linear problem-first impact).

The magnitudes shown in Figs. 2 and 3 have been normalized by the plate thickness $h$. It is clearly seen that the final magnitudes (Fig. 3) are considerably larger than the original at the first impact. On the contrary at the first impact the deformations cover nearly the complete impacted area whereas when the phenomenon is stabilized the normal deformations are focalized at the center. The pattern evolves from nearly rectangular to elliptical. In addition the maximum deformations occur in the plate's center although the hydrodynamic pressure obtain its maximum at the free surface (here at $f$=2m) from the bottom of the figures. It is noted also that the patterns are slightly asymmetrical with respect to the center (here at $y$=5m).

The actual values of $w(y, z)$ are relatively large compared to the plate thickness. Nevertheless, it should be noted that no stiffeners were assumed on the back surface of the plate. Although the present results were obtained by employing linear theory for the structural dynamics of the plate, it should be mentioned that the linear theory contributes the most on the final conditions.

This work is ongoing. The next steps that have been identified by the authors will be: (i) use of more realistic connections, e.g. clamped ends and change of the expansion (4); (ii) use of nonlinear theory for the structural dynamics of the plate and (iii) employment of the finite elements approximation together with the third-order hydrodynamic interactions. During the workshop some new material will be presented that provide additional understanding of the investigated phenomenon, such as the loading exerted on the plate as well as details of the evolution of the wave run-up when it is impossible to achieve a steady state condition for large wave steepness.

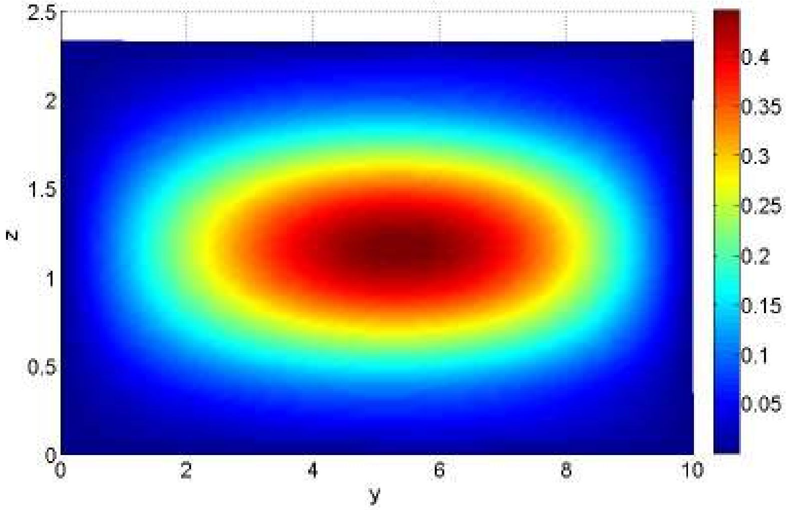


**Fig. 3.** Bending vibrations of the plate $w(y, z)/h$ at the end of iterations ($64^{th}$) that assure convergence

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