Second-order resonance among an array of two rows of vertical circular cylinders
- Comparisons of theoretical calculations and reality –

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Highlights

First and second order free-surface displacements among an array of two rows of cylinders are dealt with. Characteristic features of the free-surface dynamics are examined experimentally and theoretically.

1. Introduction

In the last workshop, we presented experimental results on the free-surface displacements among an array of two rows of vertical circular cylinders and showed that, in waves of certain periods, large free-surface displacements could be induced [1]. Depending on the period of the incident wave, in some cases, the large displacement had the same oscillation period as that of the incident wave, but, in another cases, it oscillated with the period one-half of that of the incident wave. We also carried out theoretical calculations of harmonic and bi-harmonic components of the free-surface displacements among the cylinders. As the result of the comparisons, we showed that free-surface displacement could be quite large at certain places among an array of two rows of vertical cylinders not only in theory but also in reality.

In this workshop, on the other hand, based on the results of additional experiments conducted after the last workshop, we show that at some places among the array large free-surface displacement predicted in theory is attenuated significantly in reality.

We thought that the most probable reason for this discrepancy between the theory and the reality may be the viscous dissipation taking place in reality. In order to examine the rightfulness of this speculation, we carried out calculations based on the modified theory which approximately account for the viscous dissipation taking place over the wetted body surfaces [2].

2. Experiment

As already presented in the last workshop, water-tank experiments were conducted using an array of two rows of vertical truncated circular cylinders shown in Figure 1. The cylinders were fixed in regular head waves and the free-surface displacements at four points (Point A, Point B, Point C and Point D) were measured.

3. Theory

The theory used in the present calculations is the same one as that we used in the last workshop, which was proposed by Sanada et. al [3]. As described in the Introduction, we carried out calculations based on the modified theory which approximately account for the viscous dissipation taking place over the wetted surface of the cylinders [2]. In the theory, the usual non-penetrating body boundary condition on the cylinders is modified as:

$$\frac{\partial \phi}{\partial r} = -i\varepsilon \frac{\phi}{a} \qquad (1)$$

where ϕ represents the velocity potential and a stands for the cylinder radius. ε is a certain real positive number that represents the effect of viscous dissipations. In the reference [2], it is suggested that $\varepsilon = 0.03$ well reproduces the wave decay along a long array of 50 cylinders. In the present calculation, the body boundary condition (1) was applied for the first-order velocity potential only.

4. Results and Discussion

Figures 2(a), 2(b), 2(c) & 2(d) show experimental and theoretical calculation results of the first-order (harmonic) and second-order (bi-harmonic) water-surface displacement amplitudes at points A, B, C & D respectively.

The vertical axes of the figures represent the ratio of the harmonic and bi-harmonic components of the water-surface displacement amplitudes to incident-wave amplitude ζ_a. (Second-order free-surface displacement is usually normalized by $k\zeta_a{}^2$ instead of ζ_a, where k stands for wave number, because theoretically it should be proportional to $k\zeta_a{}^2$. In the Figure 2, however, it is shown while being normalized by ζ_a, because it is easier to compare to the first-order component, and also because it is justified as both the experiments and the theoretical calculations were conducted in waves of about the same wave height (H).)

From the figures, the following facts are observed.

(1) First-order harmonic free-surface displacement

(1-1) Large free-surface displacements are observed at T=0.60sec at Point A & C, and at T=0.73sec at Point B & D, which suggests that there are more than one resonant modes of the free-surface among the cylinders. At these wave periods, experimental results are as large as those predicted by the theoretical calculations.

(1-2) The theoretical calculations that account for the viscous dissipations excessively attenuate the free-surface displacement compared to the experimental results.

(2) Second-order bi-harmonic free-surface displacement

(2-1) Large second-order free-surface displacements are predicted by the theoretical calculations at Point A & C, while at Point B & D, no relevant second-order free-surface displacements are predicted in the theoretical calculations.

(2-2) Experimental results at Point A & B agree well with the theoretical predictions. It is notable that the peak value of the second-order free-surface displacement at Point A could really be as large as the first-order displacement.

(2-3) On the other hand, at Point C, the real second-order free-surface displacements are significantly small compared to the theoretical predictions, although the first-order free-surface displacements are really as large as the theoretical predictions.

(2-4) Considering the fact that Point C is close to the cylinder surface, the above mentioned discrepancies between the theoretical predictions and reality may be attributed to the viscous dissipations taking place over the cylinder surface, but, then, there remains the question why the real first-order displacement at the same point (Point C) rather agree well with the theoretical predictions that do not account for the viscous dissipation.

References

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[3] T. Sanada, N. Mizutani and K. Iwata: Second-order approximate solution of nonlinear wave diffraction due to vertical cylinder array, *Intl. Journal of Offshore and Polar Engineering*, Vol.7, No.3, 1997.

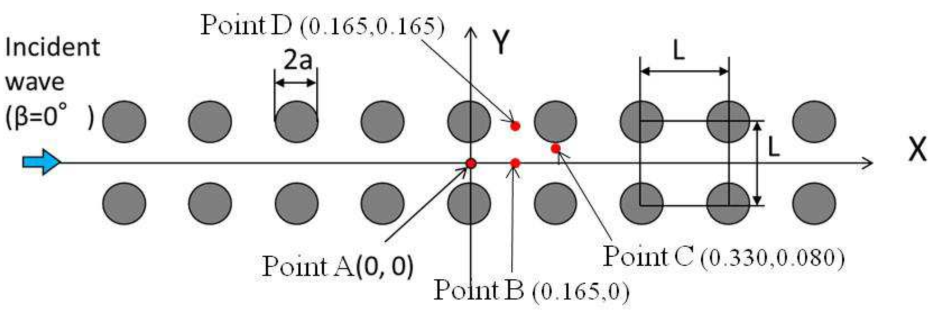
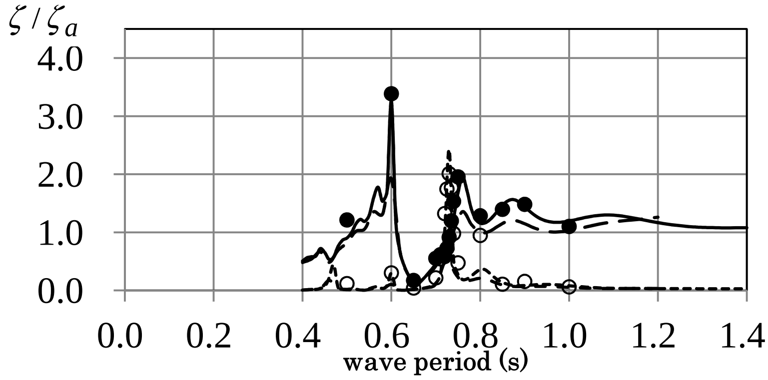
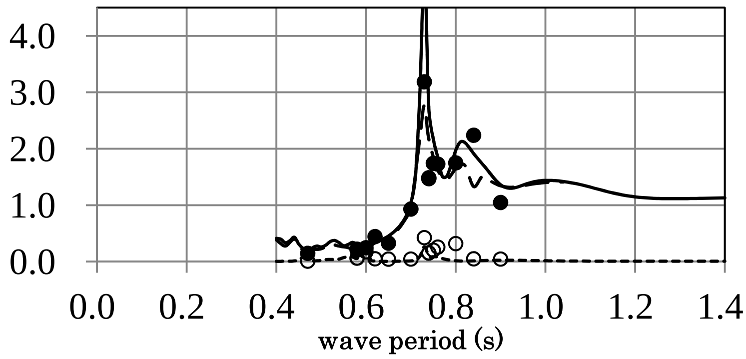


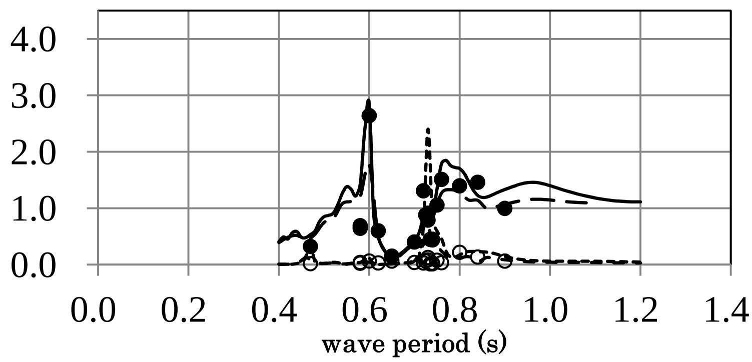
Fig.1 The set-up of the tank experiment



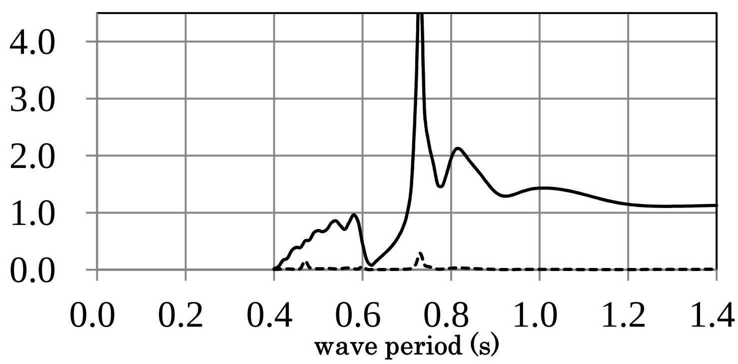
(a) Point A (H=20mm)



(b) Point B (H=10mm)



(c) Point C (H=10mm)



(d) Point D (H=10mm)

Fig.2 Experimental and theoretical results at Point A, B, C, & D (● Exp-1st, ○ Exp-2nd, ——— Cal-1st, ·········· Cal-2nd, — — — Cal-1st (eps=0.03), - - - - Cal-2nd (eps=0.03))