Coupled MPS-FEM Model for Violent Flows-Structures Interaction

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Highlights:
- Coupled MPS-FEM model is developed for strongly nonlinear fluid-structure interaction problems.
- In this method, semi-implicit MPS and explicit FEM are time-marching alternately to consider the coupling between fluids and structures.
- The developed model is validated through comparisons with a model experiment of 2-D dam-breaking test.

1. Introduction
Nonlinear fluid-structure interaction problems are important issue for ships and offshore structures, e.g. green water impacts. Nonlinear fluid-structure interaction is so complex phenomenon that advanced numerical approaches are desired for representing the nonlinear dynamic fluid-structure coupled response. There are several pioneer works to tackle this problem by adopting the Smoothed Particle Hydrodynamics (SPH) and finite element approach, e.g. [1]. The SPH has good advantage for representing strongly nonlinear flows while mesh-based CFD has problems for modelling nonlinear flows. The FE model has been established and well validated for the structures behavior under extreme pressure loads. The SPH method easily handles fragmentations and reconnections of free surface and no complex tracking scheme is needed for free-surface capturing. From these points, the SPH-FEM coupling model would be a promising method for nonlinear fluid-structure interactions.

The Moving Particle Semi-implicit method (MPS) is also a particle method developed by Koshizuka and Oka for incompressible fluids [2]. The pressure Poisson equation is solved, and hence larger time-step is allowed for time evolution. Since the particle number density sharply increases when particles are being closed, which results in a large repulsion force, the undesirable penetration and collision of particles are well prevented. Specific boundary condition (e.g. wave-inlet and transparent boundary [3]) can be easily introduced without increasing CPU-cost. In this paper, we have developed a MPS-FEM coupling method to be applicable for violent flows/structures interaction problems. Then the developed method is validated though comparisons with a model experiment of 2-D dam-breaking test.

2. Numerical Method
2.1 Semi-implicit MPS
The MPS method is one of the meshfree particle methods, which can deal with violent free surface flows, e.g. sloshing, slamming and breaking waves. The governing equations of the MPS method, dealing with incompressible fluids, are expressed as Eq.1-2. The first and second order differential operators in Eq.2 are calculated with discrete models called particle interaction models using a weight function. Since the MPS method is fully laplacian scheme, considering particles as discrete points, the advection term does not appear in the momentum equation. The gravity and the viscous terms are solved explicitly and the Poisson equation for the pressure is solved implicitly. In this study, COMPS (Computational code for Moving Particle Simulation) is used as the MPS solver. The weight function shown in Eq.3 is used in this research. In COMPS, ghost particles are not necessary to deal with solid boundary condition by adopting a symmetry boundary (mirror symmetry) [4]. Therefore, thin plate structures can be treated with larger distance of adjacent particles.

\[
\frac{D \rho}{Dt} = 0
\]

\[
\frac{Du}{Dt} = \frac{1}{\rho} \nabla p + \nu \nabla^2 u + g
\]

\[
w(r) = \begin{cases} 
0 & r \leq r_c \\
\frac{r_c}{r_c + r \times 0.05} & 0 < r < r_c \\
\frac{r}{r_c} & r \geq r_c 
\end{cases}
\]

(1) (2) (3)
2.2 Explicit FEM
In the explicit FEM, the solution at \( t=t+\Delta t \) is obtained from the equation of motion at \( t \) using the central difference scheme. The equation of motion for spatially discretized FE model is expressed as Eq.4.

\[
[M] \{\ddot{u}\}^{(n)} + [C] \{\dot{u}\}^{(n)} + [K] \{u\}^{(n)} = \{f\}^{(n)}
\]  
(4)

With Taylor expansion for displacements at \( t=t+\Delta t \) and \( t=t-\Delta t \), the following central differences are obtained.

\[
\{u\}^{(n)} = \frac{1}{2\Delta t} \left( \{u\}^{(n-1)} - \{u\}^{(n+1)} \right)
\]

\[
\{u\}^{(n)} = \frac{1}{\Delta t^2} \left( 2\{u\}^{(n)} - \{u\}^{(n-1)} + \{u\}^{(n+1)} \right)
\]  
(5)

From (4) and (5), the displacement at \( t=t+\Delta t \) can be expressed as follows.

\[
\{u\}^{(n)} = \left[ [M] + \frac{\Delta t}{2} [C] \right] \{u\}^{(n)} + \left[ \Delta t \left( [f]^{(n)} - [K] \{u\}^{(n)} \right) + 2[M] \{u\}^{(n)} - \left[ [M] - \frac{\Delta t}{2} [C] \right] \{u\}^{(n-1)} \right]
\]  
(6)

If the mass matrix \([M]\) and damping matrix \([C]\) are diagonal matrices, we can avoid solving simultaneous equations. In this study, the lumped mass is adopted and the damping matrix is assumed to be linear to the lumped mass matrix. \( ([C]=a[M]) \) The explicit FEM is efficient for parallel computing, and hence is applicable for large-scale structural analysis.

2.3 MPS-FEM coupling
The semi-implicit MPS/explicit FEM coupling procedure is as follows:
1) Structural behavior is solved by explicit FEM with the pressure load at \( t \) and node’s displacements at \( t \) and \( t-\Delta t \).
2) Velocities of wall particles, corresponding to structural deformation, are calculated from node’s displacement at \( t+\Delta t \), and are put in MPS as solid boundary displacement. Here it should be noted that COMPS does not use ghost particles to impose solid boundary condition, and hence ghost particle generation/rearrangement is not necessary.
3) Fluid behavior is solved by semi-implicit MPS considering the structural deformation at \( t+\Delta t \).
4) Pressure loads at \( t+\Delta t \) are obtained from calculated pressure field by MPS, and are put in FEM as external loads. Since pressure on each boundary particle is not solved in COMPS, weighted average pressure is calculated from the neighbor particles with use of the weight function, on desired locations.

The same time step cannot be used for MPS and FEM solvers because the accuracy of structure is generally much higher than that of fluids. Therefore the different time step are to be used for FE analysis, which is determined in advance with taking account of courant condition (\( \Delta t<\Delta u/c \)). This is one of disadvantages as compared to the SPH-FEM model where the same time step can be used in both explicit SPH and implicit FEM solvers.

3. Validation
3.1 Model Experiment
Model experiments for validation of nonlinear fluid-structure interaction were planned and conducted by Prof. Changhong Hu from Research Institute for Applied Mechanics (RIAM) of Kyushu University. The experimental result is kindly provided for our validation study. The schematic view of tank and its dimensions for 2-D dam-breaking tests are shown in Fig.1. A partition colored in black can move vertically along linear guides and pulled up by free fall weight. To observe fluid-structure coupled behaviors, elastic plates are attached to the bottom of tank at 600 mm from the left side. Physical quantity of the elastic plate is shown in Table 1. Poli-150 has sufficient rigidity and can be regarded as a rigid structure while Poli-30 is done as elastic structure.

![Fig. 1 Dimensions of tank.](image)

<table>
<thead>
<tr>
<th>Table 1 physical quantity of plates.</th>
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<tbody>
<tr>
<td>Elastic plate</td>
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<tr>
<td>Material</td>
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<tr>
<td>Height [m]</td>
</tr>
<tr>
<td>Width [m]</td>
</tr>
<tr>
<td>Thickness [m]</td>
</tr>
<tr>
<td>Young ratio [GPa]</td>
</tr>
<tr>
<td>Density [g/cm³]</td>
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</table>
3.2 Results and Discussion

Numerical parameters used in MPS and FEM solvers are shown in Table 2-3. The comparisons between the MPS-FEM calculation and the model experiment for the rigid plate (Poli-150) and the elastic plate (Poli-30) are shown in Fig. 2-5. For the rigid case, water behavior after the impact can be well represented by the MPS method. Here the fluid motion is calculated by MPS and the structural response is not solved. The free fall partition is neglected in the present calculation, and the prediction accuracy could be slightly improved if it is taken into account. For the elastic case, the calculated horizontal displacement of plate is underestimated as compared to the experiment. In the experiment, the elastic plate has an initial angle, which is comparably large to the structural displacement. Therefore numerical simulation with taking the initial inclination into account is executed. By considering the initial angle of the plate, the prediction accuracy of the water behavior after impact is improved and the dynamic behaviors of structural response to the dam breaking wave show reasonable agreement with the experiment. The calculated pressure load (converted from 3-D to 2-D), and horizontal displacement at several nodes are shown in Figs. 6-7. The maximum pressure load appears in n1 at $t=0.246s$ and the maximum displacement does in n5 at $t=0.282s$.

Table 2 Parameters for MPS.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tr>
<td>Particle distance</td>
<td>0.001 m</td>
</tr>
<tr>
<td>No. of water particles</td>
<td>80000</td>
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<tr>
<td>Time step</td>
<td>0.000115 s</td>
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<tr>
<td>Multiplication coeff.</td>
<td>1.05</td>
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<tr>
<td>Relaxation factor</td>
<td>1.0</td>
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</table>

Table 3 Parameters for FEM.

<table>
<thead>
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<th>Parameter</th>
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<tr>
<td>No. of divisions</td>
<td>1×100×1</td>
</tr>
<tr>
<td>No. of nodes</td>
<td>404</td>
</tr>
<tr>
<td>No. of elements</td>
<td>100</td>
</tr>
<tr>
<td>Time step</td>
<td>0.00000025 s</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>426.0</td>
</tr>
</tbody>
</table>

4. Conclusions

The coupled semi-implicit MPS/explicit FEM model is developed for violent flows and structure interaction problems. The numerical results represented the fluid and structural behaviors for dam breaking tests. Further/Careful validation studies are needed for the quantitative assessment of the MPS-FEM model. The developed code itself is 3-D, so that applications/validations for 3-D problems are expected as well.

Acknowledgement

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References

Fig. 3 Comparison of water and structure behaviors for elastic plate (Poli-30).

Fig. 4 Enlargement around the plate (Poli-150).

Fig. 5 Enlargement around the plate (Poli-30).

Fig. 6 Calculated time histories of pressure load and horizontal displacement at 5 node points (Poli-30).