Analytical and numerical approaches to optimizing fluid-structure interactions in wave energy parks

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Highlights

- A semi-analytical method is presented that enables fast parameter studies of wave energy parks. The results are compared with simulations from well-established software.
- The simulations focus on finding guidelines for optimized parks with high and steady power output.
- In particular, here we study how power fluctuations and average output power depends on the separating distance between units and on the number of interacting units.

1 Introduction

In this paper, the interactions between water waves and floating buoys are studied. The buoys are connected to linear generators on the seabed, modelling the wave energy converter (WEC) developed at Uppsala University [1]. One single wave energy converter does not provide sufficient power for wave energy to be an effective energy source. To produce a power of more than a few MW and enable an even power distribution, future designs will necessarily include arrays of many absorbing units. As the individual units in these wave power parks interact by scattered and radiated waves, the complexity of the model increases rapidly with the number of interacting structures, and the numerical simulations are a challenge that call for new methods and theories.

In certain situations, assumptions such as the point-absorber [2, 3] or plane-wave [4, 5] methods can simplify the calculations. However, future wave energy parks will most likely be forced to deploy devices in close proximity, and full hydrodynamical interactions between all units should be considered. The exact multiple-scattering theory originally established in [6, 7] has been further developed to enable simulations of a large number of units in, e.g., [8, 9, 10]. In this paper, a simplified semi-analytical method based on the multiple-scattering model is presented.

One of the largest costs associated with the installation of wave energy parks is the expensive and oversized electrical system needed to handle large power fluctuations. An overall aim of this paper is to optimize the design of wave energy parks to maximize the power output while minimizing the power fluctuations and used ocean area. Several parameters may affect the performance: the number of devices, the separating distance between adjacent units, the global and local geometry of the array, sea state and incoming wave direction, etc. In two earlier papers [11, 12], several of these parameters were studied by calculating hydrodynamical coefficients of full linear hydrodynamical interactions using the boundary element potential flow solver WAMIT. Whereas the method is rigid and well-established, each array configuration must be studied separately by trial-and-error, and the computational costs are high.

As an alternative, we develop a semi-analytical method to enable fast parameter studies. Instead of studying each park configuration separately, the parameters can be varied continuously and give hints of optimal configurations and parameter values. In the first stage presented here, scattering between the buoys is neglected, but single-body diffraction and interaction by radiated waves between all units are included. The method is very fast and gives remarkably good prediction on the studied wave energy park properties. The simulations are compared with simulations using the numerical approach of the earlier papers [11, 12].

2 Theory

Linear potential flow theory Consider a volume of fluid with finite depth h and define a global coordinate system (x, y, z) such that $z = -h$ at the seabed and $z = 0$ at the undisturbed free sea surface, and N floating cylinders with radius R and draft d, labelled by indices $j \in [1, N]$, and constrained to move in heave only. Divide the fluid domain into interior and exterior domains underneath and outside each buoy. Under the assumption of non-compressibility, homogeneous fluid density and negligible viscosity and vorticity, the governing equation reduces to the Laplace equation $\Delta\Phi = 0$. Under the assumption of non-steep waves, the non-linear boundary condition at the free sea surface can be linearised and the first order approximation taken. In addition, the fluid is not penetrating the seabed or the floating bodies, and the full linear boundary conditions are

$$\frac{\partial \Phi}{\partial t} + gz\Big|_{z\approx 0} = 0, \qquad \frac{\partial \Phi}{\partial z}\Big|_{z=-h} = 0, \qquad \frac{\partial \Phi}{\partial n}\Big|_{S_B} = 0, \tag{1}$$

where n is the normal direction of the body surface. The time-dependence is sinusoidal and can be factored out as $\Phi(x,y,z,t) = \mathrm{Re}(\phi(x,y,z)e^{-i\omega t})$, where the angular frequency ω is related to the wave number k through the dispersion relation $\omega^2 = gk\tanh(kh)$. Due to the linearity of the problem, the fluid potential will be a linear superposition of incoming waves, scattered waves among the fixed cylinders, and radiated waves from the bodies own oscillations, $\phi = \phi_{\text{in}} + \phi_S + \phi_R$. A general solution to the Laplace equation and the boundary conditions in the exterior domain can be found by separation of variables. In local cylindrical coordinates (r_j, θ_j, z) with origin in the center of cylinder j, the solution takes the form

$$\phi^{j,\text{ext}} = \sum_{n=-\infty}^{\infty}\left[\sum_{m=0}^{\infty}\psi_m(z)\left(A^j_{nm}\frac{K_n(k_m r_j)}{K_n(k_m R)} + B^j_{nm}\frac{I_n(k_m r_j)}{I_n(k_m R)}\right)\right]e^{in\theta_j}, \tag{2}$$

where $\psi_m(z)$ are normalized vertical eigenfunctions. The wave number $k_0 = -ik$ is a root to the dispersion relation above and $K_n(k_0 r) \propto H_n{}^{(1)}(kr)$ and $I_n(k_0 r) \propto J_n(kr)$ correspond to propagating modes. The wave numbers k_m, $m > 0$ are roots to the dispersion relation $\omega^2 = -gk\tan(kh)$ and correspond to evanescent modes.

Diffraction problem The diffraction problem is the solution to the scattering among fixed cylinders. Consider an incoming wave with amplitude A and angle χ against the x-axis, and write the Bessel functions in terms of the exponential function as $\phi^j_{\text{in}} = -iAg\psi_0(z)e^{ikr_j\cos(\theta_j-\chi)}/\omega$. The diffracted wave in the exterior domain will be a superposition of ϕ^j_{in} and the scattered waves from the remaining cylinders. Defining $\lambda_n = \pi n/(h-d)$, a general potential in the interior domain underneath the cylinder can be written on the form

$$\phi_D^{j,\text{int}} = \sum_{n=-\infty}^{\infty}\left[\frac{1}{2}\gamma^j_{n0}\left(\frac{r_j}{R}\right)^{|n|} + \sum_{m=1}^{\infty}\gamma^j_{nm}\cos(\lambda_m(z+h))\frac{I_n(\lambda_m r_j)}{I_n(\lambda_m R)}\right]e^{in\theta_j} = \sum_{n=-\infty}^{\infty}\varphi^{j,\text{int}}_{D,n}(r,z)\,e^{in\theta}. \tag{3}$$

Radiation problem The radiation potential ϕ_R is the solution to the problem of heaving cylinders and no incoming waves ϕ_{in}. In the interior domain underneath the cylinder, the general solution is given as a sum of a particular and a homogeneous solution, similar to the interior diffraction potential in equation (3), with unknown coefficients a_{mn}. The general solution in the exterior domain is given on the form (2) with only outgoing waves, i.e., $B_{mn} = 0$.

Dynamical equations The force of the waves on the floating bodies is given by the pressure integrated along the wetted surface of the cylinder. In the frequency domain, this is proportional to the wave potentials,

$$\bar{F} = \iint_S p d\bar{S} = -\rho\iint_S \frac{\partial\phi}{\partial t}d\bar{S} = i\omega\rho\iint_S \phi d\bar{S} = i\omega\rho\iint_S[\phi_{\text{in}} + \phi_S]d\bar{S} + i\omega\rho\iint_S \phi_R d\bar{S}. \tag{4}$$

The first term resulting from the incoming and scattered waves, is the excitation force factor f_{exc}; the second term originating from the radiated waves is the radiated force f_{rad}, with real and imaginary parts proportional to the added mass and damping coefficients, respectively.

The dynamics of each buoy is determined by Newton's second law $F_{\text{tot}}(t) = m\ddot{z}(t)$, where the total force is a sum of the exciting force from incoming waves, the damping force from radiated waves, the statical restoring force for submerged bodies and power take-off force. In the frequency domain, the equation of motion can be solved for the vertical coordinate $z(\omega)$ as

$$z(\omega) = \frac{f_{\text{exc}}(\omega)\,\eta_{\text{in}}(\omega)}{-(m + m_{\text{add}}(\omega))\omega^2 - i(B(\omega)+\gamma)\omega + \rho g\pi R^2 + k_s} = H(\omega)\eta_{\text{in}}(\omega), \tag{5}$$

where $H(\omega)$ is the transfer function (response amplitude operator) and m is the total mass of the translator and the submerged buoy. The vertical position of the buoy is then obtained in the time-domain by inverse Fourier transform, $z(t) = \widehat{z(\omega)} = (h * \eta_{\text{in}})(t)$, where $h(t)$ is the transfer function in the time-domain. With the position of the buoy in time determined, the absorbed power of the WEC can be calculated as $P(t) = \gamma\dot{z}(t)^2$.

Variance As described above, one of the most important effects of park interactions is the reduction of power fluctuations. The fluctuations in a park with N WECs can be measured in terms of the normalized variance of the total power, $v(N) = \sigma^2(P_{\text{tot}})/\overline{P_{\text{tot}}}^2$, where σ is the standard deviation and $\overline{P_{\text{tot}}}$ the time-averaged power.

3 Method

In this paper, we study different parameters affecting the performance of wave energy parks, with the goal to optimize the power quality. The simulations are performed using a numerical approach as well as a semi-analytical method. Both consider non-steep waves based on linearized potential flow theory. The sea state used in the simulations is characterized by energy period $T_e = 5.01$ s and significant wave height $H_s = 1.53$ m. The simulations presented here all consider arrays with square geometries.

3.1 Numerical approach

In the numerical approach, the hydrodynamical coefficients in equation (4) are calculated using the commercial boundary element potential flow solver WAMIT. The output from WAMIT is used in a time-domain model in Matlab calculating the dynamical equation (5), and the power and variance are calculated as described above.

3.2 Semi-analytical approach

In a first approach, we neglect multiple scattering among the buoys but include single-body diffraction and interaction by radiated waves among all cylinders. In order to calculate the excitation and radiation forces (4) in the z-direction, the potentials in the interior domains must be integrated along the bottom of the cylinders. The wave numbers k_n are solutions of the dispersion relation and the unknown coefficients γ_{mn} and a_{mn} in the interior potentials can be found by requiring continuity between the exterior and interior domains and no-slip condition on the cylinder surfaces. The constraints imply a system of equations that can be solved in terms of the unknown coefficients. Due to axisymmetry only the $n = 0$ mode gives non-zero contribution to integration over the cylinder bottom, and the exciting force factor can then be calculated as $f_{\text{exc}} = 2\pi i\omega\rho \int_0^R dr\, r\varphi_{D,0}^{j,\text{int}}(r, -d)$. Similarly, the added mass and damping coefficients can be found from the radiation force as

$$m_{\text{add}}(\omega) + \frac{i}{\omega}B(\omega) = \pi\rho(h-d)R^2\left(\frac{1}{2} + a_0(\omega) - \frac{R^2}{8(h-d)^2}\right) + 4\pi\rho(h-d)R\sum_{n=1}^{\infty} a_n(\omega)\frac{I_1(\lambda_n R)}{\lambda_n I_0(\lambda_n R)}. \qquad (6)$$

With the obtained hydrodynamical coefficients, the dynamics as well as the power and variance for all the devices can be calculated in the time-domain as described in the previous section.

4 Results

In [11], rectangular and circular arrays with 32 WECs were studied, and the circular geometry was found to have roughly three times lower variance as compared to the rectangular geometry. In [12], the variance was found to reduce strictly with the number of units along the wave direction if the separating distance between adjacent units is kept constant, but not otherwise. Hence, including more devices in a wave energy park with fixed area will not necessarily result in lowered power fluctuations, which is of relevance for design of full-scale wave energy parks. In the two papers [11, 12], each array configuration was studied separately by calculating the hydrodynamical coefficients in WAMIT; a robust and reliable, but rather time-consuming method. Here, we have presented a new method based on hydrodynamical coefficients calculated semi-analytically using expansions in eigenfunctions. The semi-analytical method is very fast and enables studies of parameters that can be varied continuously.

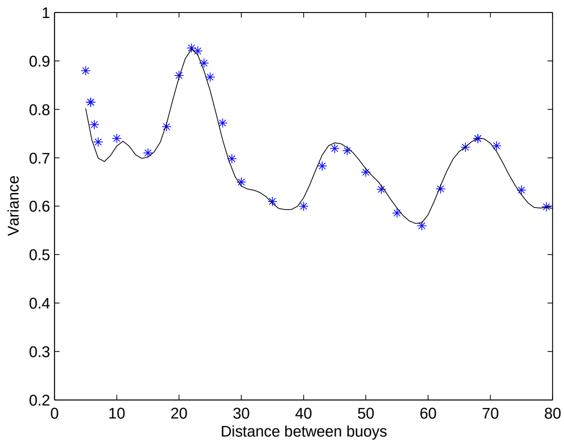

Figure 1: Comparison between the numerical (asterisks) and the semi-analytical (solid line) methods for power variance as a function of distance between adjacent units in an array of 9 WECs.

In figure 1, square parks of $9 = 3 \times 3$ WECs are studied. As seen in the figure, the power variance fluctuates as a function of the separating distance between adjacent units. The semi-analytical model (solid line) resembles the results from the numerical method (asterisks) remarkably well. Different sea states will display different fluctuating behaviour, as was discussed in [12].

The variance and average power per WEC as functions of the number of WECs in an array are plotted in figure 2. The distance between adjacent units is kept constant to 10 m for all arrays. As seen in the figures, the power fluctuations decrease strongly with the number of units. The average power per WEC also decreases, but

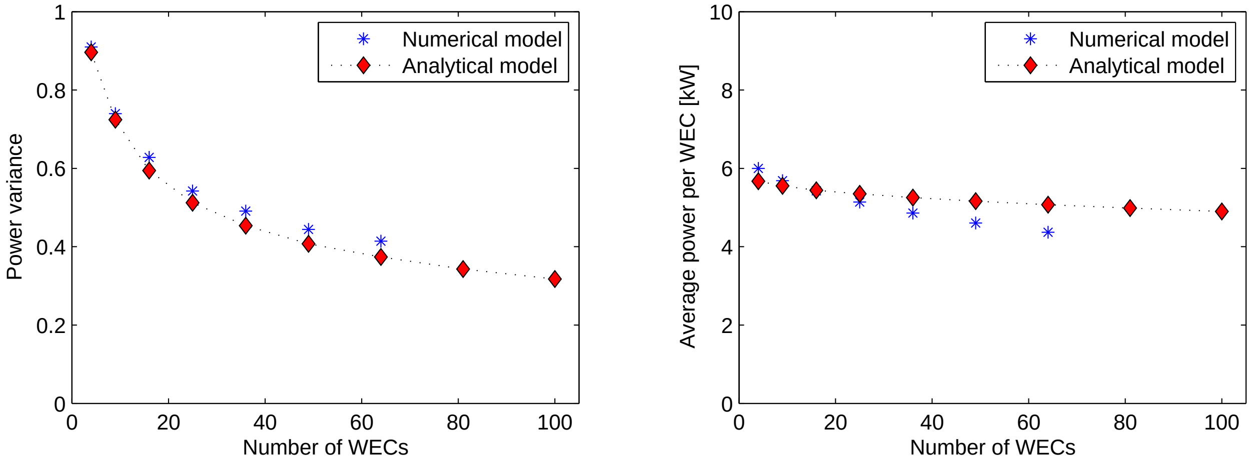

Figure 2: Power variance and average power per WEC as functions of number of WECs.

to a smaller extent. Due to the high computational costs in the numerical model, the simulations are limited to $64 = 8 \times 8$ WECs. As a contrast, the analytical model allows for simulations of much larger arrays.

One of the main advantages of the analytical model is the short computational time. Whereas a simulation of 50-100 WECs in the numerical method takes up to a week on a standard desktop computer, the analytical method requires only a fraction of an hour. To be more precise, the simulations of parks with 4, 49 and 100 WECs required 0.4, 7.9 and 20.8 minutes, respectively.

5 Discussion

For the design of economically viable and effective wave energy parks, parameters affecting the power fluctuations and the total output power must be considered carefully. Simulations with standard numerical software tend to be very heavy when the number of interacting units in the park grows. Here, we have presented a semi-analytical model that enables fast parameter studies of arrays. At this stage, the model includes interaction by radiated waves but neglects scattering between buoys. The difference between the numerical and analytical model in figure 2 is probably a result of this assumption; it is likely that the impact of scattered waves grows with number of units. We intend to continue developing the analytical method in future publications.

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