Second order Wave Loads based on Second order TEBEM

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Highlights:

- Second order Taylor Expansion Boundary Element Method are newly developed for solving the direct boundary integral equation on no-smoothed floating body surface, free surface and control surface. High accuracy of 2nd order TEBEM is confirmed by comparing the numerical results with analytical solutions.
- 3-D wave radiation and diffraction problems up to 2nd order for vertical truncated circular cylinder and bottom mounted cylinder are solved in time domain by 2nd order TEBEM and compared with published results by other low order and high order BEM solutions. The difference of results between different methods are discussed.

1. Introduction

Development of efficient and accurate numerical method for solving the hydrodynamic problems related to wave interaction with arbitrary form floating bodies is continue demanding new routes. Although Boundary Element Method is most widely used in ship and ocean engineering analysis today, it is found by comparing study that some numerical inaccuracy or inconsistency phenomenon are exist for same class of problem with the characteristic that either the floating body surface is no-smoothed when solved by free-surface Green function method or the hydrodynamic problem definition boundary has corners when solved by simple Green function method. No matter which kind of BEM method is used, for no-smoothed floating body, the fluid velocity calculated on the body surface near the corners are not accurate and the resulting hydrodynamic forces are not easy to convergent due to the inaccurate contribution of the corner parts using the pressure integral procedure. Some studies assume this difficulty is due to singularity of hydrodynamic problems with corner boundaries. But the careful study show that the main reason is due to inaccurate numerical solution of the boundary integral equation by different BEM.

There exist two types of BEM from numerical view point, that is Low Order and High Order BEM. For Low Order direct BEM, after solving the velocity potential at each element, the induced velocity is not easy derived accurately by numerical difference scheme on the elements near the corners of boundary. For low order indirect BEM(source distribution method), after solving the source strength at each element, the induced velocity by source distribution has larger errors on the no-smoothed boundary elements comparing with accuracy on smoothed boundary elements. For direct HOBEM, there exist complex singular integral contribution to the coefficients of derived algebra equations. For boundary with no-regular corners, this singular integral can not derived analytically, so detailed numerical tremens routes directly influence the robust of accuracy of solved velocity on the boundary of corners. Even more complex problems are that, for hydrodynamic singularity problems with corner boundary, one can not exactly assume the singular order of velocity potential for complex no-regular corners in the HOBEM. Shao and Faltinsen(2010) proposed a body fixed formulation to avoid the difficulty in earth fixed formulation for radiation problem by HOBEM. On the contrary, in the LOBEM ,one can avoid this difficulty by take the control point locate on the element center which never reach but nearly towards the real limit value of the mathematical definition problem of velocity potential at the corner.

To avoid the difficulty of high order BEM in dealing with corner boundary and overcome the short comings of low order direct BEM for deriving the velocity as well as overcome the bad properties of low order indirect BEM for the velocity near the corner boundaries. Duan(2012) proposed the so called Taylor Expansion Boundary Element Method(TEBEM) for solving 2-D linear hydrodynamic problems, it is found that the accuracy of velocity at corner boundary improved much by 1st order TEBEM. For second order radiation and diffraction problems, there exists second order derivatives of first order velocity potential on the body boundary surface condition and free surface boundary condition, so the 2nd order TEBEM is developed.

This paper described the principle of the 2nd order TEBEM in part 2, and analytical validation example in part 3.The second order hydrodynamics problems of 3-D body are demonstrated in part 4.

2. Second order TEBEM

The potential flow hydrodynamic problem is considered for solving the wave-body interaction problems. Application of the Green's third formula to the domain of BVP leads to the following equation for the evaluation of φ in the fluid.

$$4\pi\varphi(p)=\iint_{s}\left(G(p,q)\cdot\varphi_{n}-\varphi\cdot G(p,q)_{n_q}\right)ds_{q} \tag{1}$$

where $G=1/r$ is the Rankin source Green function in the present paper, r represents the distance between the source point $q(X_q,Y_q,Z_q)$ and the field point $p(X_p,Y_p,Z_p)$. S is the boundary of the fluid domain of BVP. It is clear that φ is completely determined provided φ and φ_n on S are known.

In order to supplement the integral equations for the first order TEBEM, the first order partial derivatives of potential at the field points p along the two mutually orthogonal tangential directions(denoted as x_p and y_p separately ,as p located on S after take limit along the normal of point p).

$$4\pi\frac{\partial\varphi}{\partial x_p}=\frac{\partial}{\partial x_p}\iint_s\left(\varphi_n G-\varphi G_{n_q}\right)ds_q;\ 4\pi\frac{\partial\varphi}{\partial y_p}=\frac{\partial}{\partial y_p}\iint_s\left(\varphi_n G-\varphi G_{n_q}\right)ds_q \tag{2}$$

Be similar way, one evaluate the second order and mixed partial derivatives of the field points along with lp and tp, and obtain the following formulas.

$$\begin{aligned}&4\pi\frac{\partial^2\varphi}{\partial x_p{}^2}=\frac{\partial^2}{\partial x_p{}^2}\iint_s\left(\varphi_n G-\varphi G_{n_q}\right)ds_q;4\pi\frac{\partial^2\varphi}{\partial y_p{}^2}=\frac{\partial^2}{\partial y_p{}^2}\iint_s\left(\varphi_n G-\varphi G_{n_q}\right)ds_q\\&4\pi\frac{\partial^2\varphi}{\partial x_p\partial y_p}=\frac{\partial^2\varphi}{\partial x_p\partial y_p}\iint_s\left(\varphi_n G-\varphi G_{n_q}\right)ds_q\end{aligned} \tag{3}$$

Next by the same discretization way as general low order BEM, and then expand the diploes strength at the panel's centroid through the Taylor expansion, and reserve the second order derivatives. The expansion is following as:

$$\varphi(q)=\varphi(q_0)+\varphi(q_0)_\xi\xi+\varphi(q_0)_\eta\eta+\frac{1}{2}\varphi(q_0)_{\xi^2}\xi^2+\varphi(q_0)_{\xi\eta}\xi\eta+\frac{1}{2}\varphi(q_0)_{\eta^2}\eta^2 \tag{4}$$

The tangential first order derivatives at the centroid of each element are denoted as $\varphi_{\xi j}$ and $\varphi_{\eta j}$. The tangential second order and mixed derivatives at the centroid of each element are denoted as: $\varphi_{\xi^2 j}$, $\varphi_{\eta^2 j}$ and $\varphi_{\xi\eta j}$. By substituting formula (4) into the equation (1), (2) and (3), the second order TEBEM equations(5) is obtained when the field point p is located through limit along normal to the boundary S.

$$\begin{cases}\sum_{j=1}^{N}\left(D_{ij}\varphi_j+D_{ij}^{\xi}\varphi_{\xi j}+D_{ij}^{\eta}\varphi_{\eta j}+D_{ij}^{\xi^2}\varphi_{\xi^2 j}+D_{ij}^{\xi\eta}\varphi_{\xi\eta j}+D_{ij}^{\eta^2}\varphi_{\eta^2 j}\right)=\sum_{j=1}^{N}C_{ij}\varphi_{nj}\\ \sum_{j=1}^{N}\left(D_{xij}\varphi_j+D_{xij}^{\xi}\varphi_{\xi j}+D_{xij}^{\eta}\varphi_{\eta j}+D_{xij}^{\xi^2}\varphi_{\xi^2 j}+D_{xij}^{\xi\eta}\varphi_{\xi\eta j}+D_{xij}^{\eta^2}\varphi_{\eta^2 j}\right)=\sum_{j=1}^{N}C_{xij}\varphi_{nj}\\ \sum_{j=1}^{N}\left(D_{yij}\varphi_j+D_{yij}^{\xi}\varphi_{\xi j}+D_{yij}^{\eta}\varphi_{\eta j}+D_{yij}^{\xi^2}\varphi_{\xi^2 j}+D_{yij}^{\xi\eta}\varphi_{\xi\eta j}+D_{yij}^{\eta^2}\varphi_{\eta^2 j}\right)=\sum_{j=1}^{N}C_{yij}\varphi_{nj}\\ \sum_{j=1}^{N}\left(D_{x^2ij}\varphi_j+D_{x^2ij}^{\xi}\varphi_{\xi j}+D_{x^2ij}^{\eta}\varphi_{\eta j}+D_{x^2ij}^{\xi^2}\varphi_{\xi^2 j}+D_{x^2ij}^{\xi\eta}\varphi_{\xi\eta j}+D_{x^2ij}^{\eta^2}\varphi_{\eta^2 j}\right)=\sum_{j=1}^{N}C_{x^2ij}\varphi_{nj}\\ \sum_{j=1}^{N}\left(D_{xyij}\varphi_j+D_{xyij}^{\xi}\varphi_{\xi j}+D_{xyij}^{\eta}\varphi_{\eta j}+D_{xyij}^{\xi^2}\varphi_{\xi^2 j}+D_{xyij}^{\xi\eta}\varphi_{\xi\eta j}+D_{xyij}^{\eta^2}\varphi_{\eta^2 j}\right)=\sum_{j=1}^{N}C_{xyij}\varphi_{nj}\\ \sum_{j=1}^{N}\left(D_{y^2ij}\varphi_j+D_{y^2ij}^{\xi}\varphi_{\xi j}+D_{y^2ij}^{\eta}\varphi_{\eta j}+D_{y^2ij}^{\xi^2}\varphi_{\xi^2 j}+D_{y^2ij}^{\xi\eta}\varphi_{\xi\eta j}+D_{y^2ij}^{\eta^2}\varphi_{\eta^2 j}\right)=\sum_{j=1}^{N}C_{y^2ij}\varphi_{nj}\end{cases} \tag{5}$$

In equations (5), N means the total elements on the boundary,the superscript means the influence coefficients are multiplied by this variable appeared in the Taylor expansion. The subscript i and j show the number of the element and others mean the space partial derivatives. All the coefficients in equation (5) can be analytical integrated.

where $D_{ii}=2\pi, D_{xii}^{\xi}=2\pi, D_{yii}^{\eta}=2\pi, D_{x^2ii}^{\xi^2}=2\pi, D_{xyii}^{\xi\eta}=2\pi, D_{y^2ii}^{\eta^2}=2\pi$

$D_{ij}=\iint_Q G_{n_qij}ds_q$, $D_{ij}^{\xi}=\iint_Q \xi G_{n_qij}ds_q$, $D_{ij}^{\eta}=\iint_Q \eta G_{n_qij}ds_q$, $D_{ij}^{\xi^2}=\frac{1}{2}\iint_Q \xi^2 G_{n_q,ij}ds_q$, $D_{ij}^{\xi\eta}=\iint_Q \xi\eta G_{n_q,ij}ds_q$,

$D_{ij}^{\eta^2}=\frac{1}{2}\iint_Q \eta^2 G_{n_q,ij}ds_q$, $C_{ij}=\iint_Q G_{ij}ds_q$, $\left(C_{xij},C_{yij},C_{zij}\right)=\nabla C_{ij}=\nabla_p\left(\iint_Q G_{ij}ds_q\right)$,

$\left(C_{x^2ij},C_{xyij},C_{xzij},C_{yxij},C_{y^2ij},C_{yzij},C_{zxij},C_{zyij},C_{z^2ij}\right)=\nabla\left(\nabla C_{ij}\right)=\nabla_p\left(\nabla_p\left(\iint_Q G_{ij}ds_q\right)\right)$,

$\left(D_{xij},D_{yij},D_{zij}\right)=\nabla D_{ij}=\nabla_p\left(\iint_Q G_{n_qij}ds_q\right)$, $\left(D_{xij}^{\xi},D_{yij}^{\xi},D_{zij}^{\xi}\right)=\nabla D_{ij}^{\xi}=\nabla_p\left(\iint_Q \xi G_{n_qij}ds_q\right)$,

$\left(D_{xij}^{\eta},D_{yij}^{\eta},D_{zij}^{\eta}\right)=\nabla D_{ij}^{\eta}=\nabla_p\left(\iint_Q \eta G_{n_qij}ds_q\right)$, $\left(D_{xij}^{\xi^2},D_{yij}^{\xi^2},D_{zij}^{\xi^2}\right)=\nabla D_{ij}^{\xi^2}=\frac{1}{2}\nabla_p\left(\iint_Q \xi^2 G_{n_qij}ds_q\right)$,

$$\left(D^{\xi\eta}_{xij}, D^{\xi\eta}_{yij}, D^{\xi\eta}_{zij}\right) = \nabla D^{\xi\eta}_{ij} = \nabla_p\left(\iint_Q \xi\eta G_{n_q ij} ds_q\right), \left(D^{\eta^2}_{xij}, D^{\eta^2}_{yij}, D^{\eta^2}_{zij}\right) = \nabla D^{\eta^2}_{ij} = \frac{1}{2}\nabla_p\left(\iint_Q \eta^2 G_{n_q ij} ds_q\right),$$

$$\left(D_{x^2ij}, D_{xyij}, D_{xzij}, D_{yxij}, D_{y^2ij}, D_{yzij}, D_{zxij}, D_{zyij}, D_{z^2ij}\right) = \nabla\left(\nabla D_{ij}\right) = \nabla_p\left(\nabla_p\left(\iint_Q G_{n_q ij} ds_q\right)\right),$$

$$\left(D^{\xi}_{x^2ij}, D^{\xi}_{xyij}, D^{\xi}_{xzij}, D^{\xi}_{yxij}, D^{\xi}_{y^2ij}, D^{\xi}_{yzij}, D^{\xi}_{zxij}, D^{\xi}_{zyij}, D^{\xi}_{z^2ij}\right) = \nabla\left(\nabla D^{\xi}_{ij}\right) = \nabla_p\left(\nabla_p\left(\iint_Q \xi G_{n_q ij} ds_q\right)\right),$$

$$\left(D^{\eta}_{x^2ij}, D^{\eta}_{xyij}, D^{\eta}_{xzij}, D^{\eta}_{yxij}, D^{\eta}_{y^2ij}, D^{\eta}_{yzij}, D^{\eta}_{zxij}, D^{\eta}_{zyij}, D^{\eta}_{z^2ij}\right) = \nabla\left(\nabla D^{\eta}_{ij}\right) = \nabla_p\left(\nabla_p\left(\iint_Q \eta G_{n_q ij} ds_q\right)\right),$$

$$\left(D^{\xi^2}_{x^2ij}, D^{\xi^2}_{xyij}, D^{\xi^2}_{xzij}, D^{\xi^2}_{yxij}, D^{\xi^2}_{y^2ij}, D^{\xi^2}_{yzij}, D^{\xi^2}_{zxij}, D^{\xi^2}_{zyij}, D^{\xi^2}_{z^2ij}\right) = \nabla\left(\nabla D^{\xi^2}_{ij}\right) = \frac{1}{2}\nabla_p\left(\nabla_p\left(\iint_Q \xi^2 G_{n_q ij} ds_q\right)\right),$$

$$\left(D^{\xi\eta}_{x^2ij}, D^{\xi\eta}_{xyij}, D^{\xi\eta}_{xzij}, D^{\xi\eta}_{yxij}, D^{\xi\eta}_{y^2ij}, D^{\xi\eta}_{yzij}, D^{\xi\eta}_{zxij}, D^{\xi\eta}_{zyij}, D^{\xi\eta}_{z^2ij}\right) = \nabla\left(\nabla D^{\xi\eta}_{ij}\right) = \nabla_p\left(\nabla_p\left(\iint_Q \xi\eta G_{n_q ij} ds_q\right)\right),$$

$$\left(D^{\eta^2}_{x^2ij}, D^{\eta^2}_{xyij}, D^{\eta^2}_{xzij}, D^{\eta^2}_{yxij}, D^{\eta^2}_{y^2ij}, D^{\eta^2}_{yzij}, D^{\eta^2}_{zxij}, D^{\eta^2}_{zyij}, D^{\eta^2}_{z^2ij}\right) = \nabla\left(\nabla D^{\eta^2}_{ij}\right) = \frac{1}{2}\nabla_p\left(\nabla_p\left(\iint_Q \eta^2 G_{n_q ij} ds_q\right)\right).$$

It is obvious that the low order direct BEM with $N \times N$ equation systems is extended to the $6N \times 6N$ equations system of the second order TEBEM to calculate the potential and first order tangential derivatives, namely induced velocity, and second order tangential derivatives of velocity potential simultaneously.

3. Numerical validation with analytical experiment solution

To validate the proposed method, the boundary value problem of Laplace equation, where the boundary is not smoothed for a cube , is constructed with velocity potential given on the bottom surface and normal derivatives given on other 5 surfaces as example. The side length of cube is 20 meters. Choose the centroid of each element of the shadow in figure 1 as comparing illustration. Apply the low order indirect BEM(source distribution), the first order TEBEM and second order TEBEM to calculate the tangential velocity on the boundary respectively. Different panel numbers are used for almost same amount of numerical calculation between each BEM. The results are shown in figure 2. It is found the relative errors of the first order and second order TEBEM are almost equal to zero and are much smaller than the results of the low order indirect BEM, so the TEBEM has a clear advantage over the low order indirect BEM in calculating the induced velocity on the no-smooth boundary.

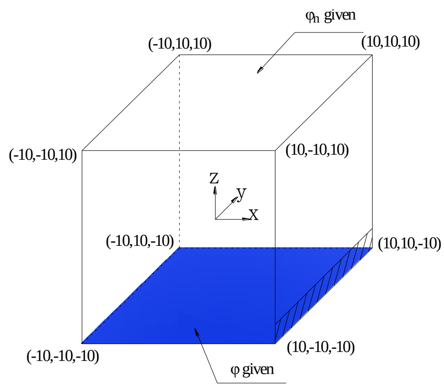

Fig. 1. locations of calculation points

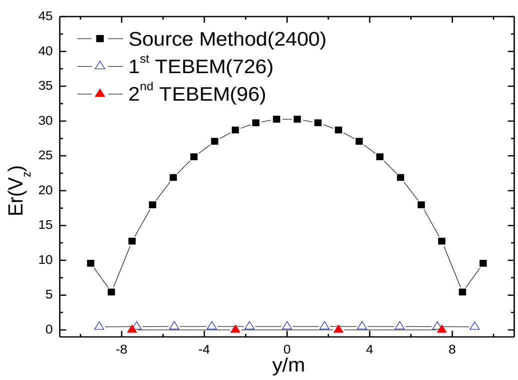

Fig. 2. relative error of the tangential velocity V_z at the selected points of shadow at $x = 5m$

4. Second order hydrodynamic problems by 2nd order TEBEM

To show the effects of the 2nd order TEBEM, the second order radiation boundary value problem of a truncated cylinder in finite water depth as shown in fig.5 and diffraction problem of a bottom mounted cylinder, are solved by the time domain solution method through 2nd order TEBEM. Fig.3 shows the diffraction induced velocity at the panels near the waterline. The agreement is again quite good between the second order TEBEM and the analytical solution, while the results of low order indirect BEM have noticeable difference . Fig.4 shows the first and second order wave elevation around the waterline. Good agreement is obtained between the present results and those of the reference [2].

For second order radiation problem,Fig.6 shows that the traditional HOBEM is neither able to capture the amplitude nor the phase of the second-order force, while good agreement can be achieved between the TEBEM and the HOBEM under the body-fixed coordinate system which can avoid the calculation of second order derivatives.

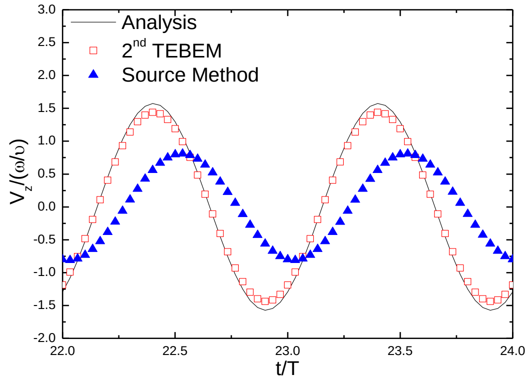

Fig. 3. dimensionless induced velocity along the z-axis direction at $(a,\pi,0.05a)$, $\nu a=2.0$

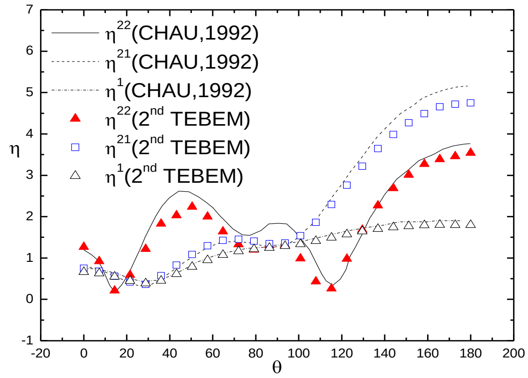

Fig. 4. dimensionless wave elevation around the circumference of a cylinder($\nu a=2.0$)

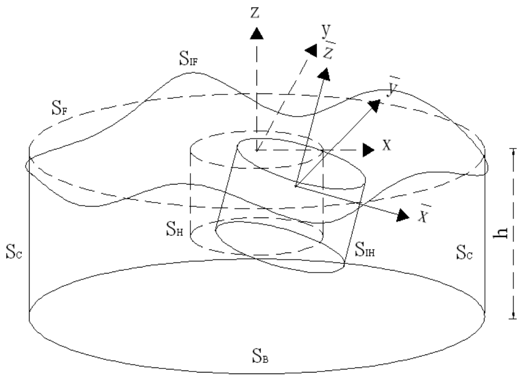

Fig. 5. the schematic of coordinate system and computational domain

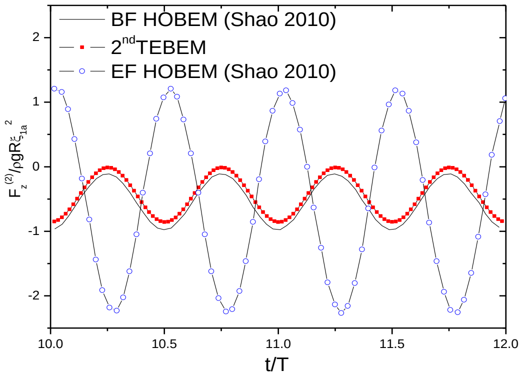

Fig. 6. the vertical sum-frequency force with forced surge motion

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