(Abstract submitted to the 29th IWWWFB, Osaka, Japan, 2014)

A high-order finite-difference solver for the linearised potential flow wave resistance problem on curvilinear overset grids*

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January 13, 2014

1 Introduction

The potential flow, wave resistance problem has been solved using the boundary element method by a number of researchers, while relatively few have applied the finite-difference method [4], [8]. The high-order finite-difference method is attractive however, due to the relative ease of obtaining a linear scaling of the solution effort with the total number of unknowns [3]. The method also achieves high efficiency on massively parallel computer architectures [5]. The challenge for finite-difference methods is to represent the complex body geometries associated with ships and marine structures, and at the same time maintain these efficient solution properties. Ongoing work at DTU is focused on developing a solver capable of predicting the added resistance of the slow steaming ships based on the high order finite-difference method on overlapping, curvilinear grids building on the work of [9]. This abstract presents the results which have been obtained so far regarding the wave resistance of two simple geometries: a floating 2D cylinder and a hemisphere.

2 Mathematical formulation

We define a moving Cartesian coordinate system O-xyz which is in steady translation with the ship's forward speed U. The governing equation is the continuity equation:

$$\nabla^2\Phi = \frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} + \frac{\partial^2\Phi}{\partial z^2} = 0 \tag{1}$$

By assuming $\Phi = -Ux + \phi_u$, the Neumann Kelvin linearisation of the dynamic and kinematic free-surface boundary conditions is obtained:

$$\frac{\partial\phi_u}{\partial t} = -g\eta_u + U\frac{\partial\phi_u}{\partial x} \qquad \text{on } z = 0 \tag{2}$$

$$\frac{\partial\eta_u}{\partial t} = \frac{\partial\phi_u}{\partial z} + U\frac{\partial\phi_u}{\partial x} \qquad \text{on } z = 0 \tag{3}$$

Accordingly the linearised body boundary condition for the wave resistance problem is:

$$\frac{\partial\phi_u}{\partial n} = \vec{u}\cdot\vec{n} \qquad \text{on } S(0) \tag{4}$$

where $\vec{u} = (U, 0, 0)$ and $S(0)$ is the mean position of the body. To solve for the steady wave resistance, a solution is sought to the unsteady problem formulated above, where the body boundary condition is applied using a ramp function.

*The authors wish to thank the EU 7th Framework Programme (grant # 266030, ULYSSES) for funding and the Danish Center for Scientific Computing for supercomputing resources..

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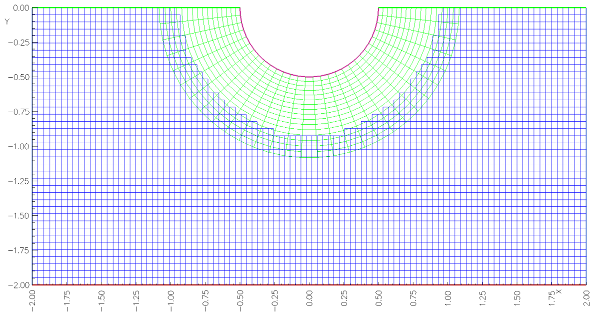

Figure 1: floating cylinder

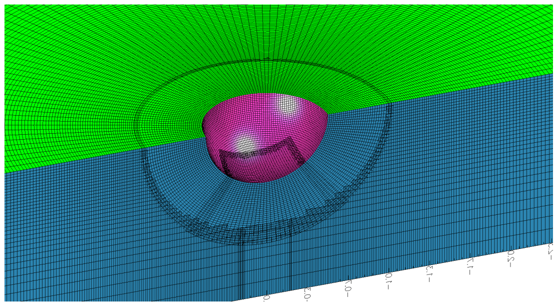

Figure 2: floating hemisphere (half geometry)

3 Numerical method

In this project we make use of an object-oriented code framework, Overture [2], which contains C++ libraries for solving partial differential equations on overset grids. The overlapping grid generator, Ogen [6], has been used to generate the grids for the cylinder and hemisphere geometries, as shown in Figures 1 and 2. The numerical solution is obtained by first solving the continuity equation on the domain, where a dirichlet boundary condition is applied on the free surface, and body boundary condition is satisfied using a non-homogeneous neumann condition on the body. In the next step, the velocity potential and elevation at the free surface are updated by time marching the free-surface boundary conditions (2) and (3) using the fourth order Runge-Kutta time integration scheme. All spatial derivatives in the equations are discretised by a centred 4th order finite-difference scheme. The boundary points at the end of the computational domain and around the body are handled by defining two layers of ghost points. On the free surface we define a bow region where we apply the extrapolation, and a stern region where we apply the following boundary condition to assign ϕ_u and η_u on the ghost layers for the free surface spatial derivatives:

$$\frac{\partial \eta_u}{\partial n} = -\frac{U}{g}\frac{\partial \phi_u}{\partial x} \quad \text{and} \quad \frac{\partial \phi_u}{\partial n} = \vec{u}\cdot\vec{n} \quad \text{where } \vec{\nabla}\cdot\vec{u} > 0 \tag{5}$$

The first term is obtained by taking x-derivative of the Bernoulli equation, and removing the time dependent part. For the ease of calculation the non-homogeneous condition for η_u has been approximated to first order by a homogeneous condition. It is known that the centred schemes are non-dissipative [10]. This fact in the case of the overlapping grid can cause trouble with regard to the instability due to grid-to-grid oscillation, grid irregularities or approximation of the boundary conditions, as there is no mechanism to damp out the high frequency modes. We introduce a Savitzky-Golay least-square filter to the solution, which enables us to apply a controllable amount of dissipation by defining the order and width of the stencil. The filter is expressed as [1]:

$$f^d(x_i) - \sigma \sum_{j=-p}^{Q} d_j f(x_i + j\Delta x) \tag{6}$$

The strength of the filter is defined by σ and d_j is the filter coefficient. At the boundaries where $P \neq Q$, the off-centred filter coefficients are used that are proposed by [1].

4 Results

We have applied the numerical model to solve the wave resistance problem for a two-dimensional floating cylinder with $Fr = 0.45$, and a floating hemisphere with $Fr = 0.9$. ($Fr = \frac{U}{\sqrt{gR}}$) R is the radius of the geometry. For the case of floating cylinder, an analytical solution can be obtained by conservation of energy and wave energy flux, [7], which is equal to $\frac{1}{4}\rho g A^2$. (Here A is the wave

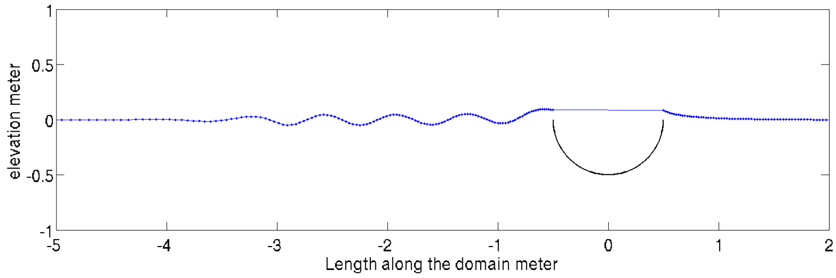

Figure 3: floating cylinder

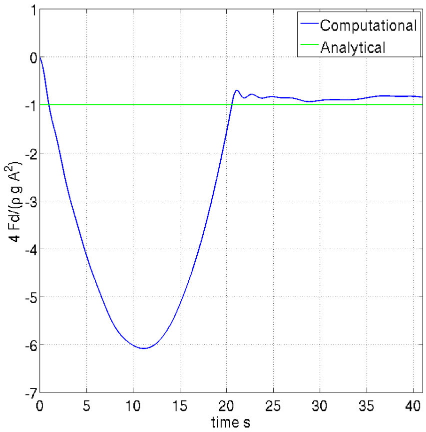

Figure 4: Wave resistance for floating cylinder

amplitude). The numerical wave resistance has been calculated by the near-field method, which is the integration of the pressure force on the mean position of the body:

$$Fd = -\rho\Big(\frac{\partial \phi_u}{\partial t} - U\frac{\partial \phi_u}{\partial x}\Big) \tag{7}$$

5 Conclusions

A high-order finite-difference linearised potential flow model has been implemented for the forward speed problem on overlapping grids. The numerical scheme for time stepping the free surface equations, needs to have a mechanism for dissipation of the high frequency modes in the solution due to irregularities in grid spacing between component grids, grid streteching and approximation of the boundary conditions on the body (5). A high order and very mild Savitzky-Golay least square filter is appropriate to keep the numerical solution stable. This work is in progress, and will be continued with more validation tests. The solver will be further developed to solve other unsteady problems and to calculate the added resistance of slow steaming ships.

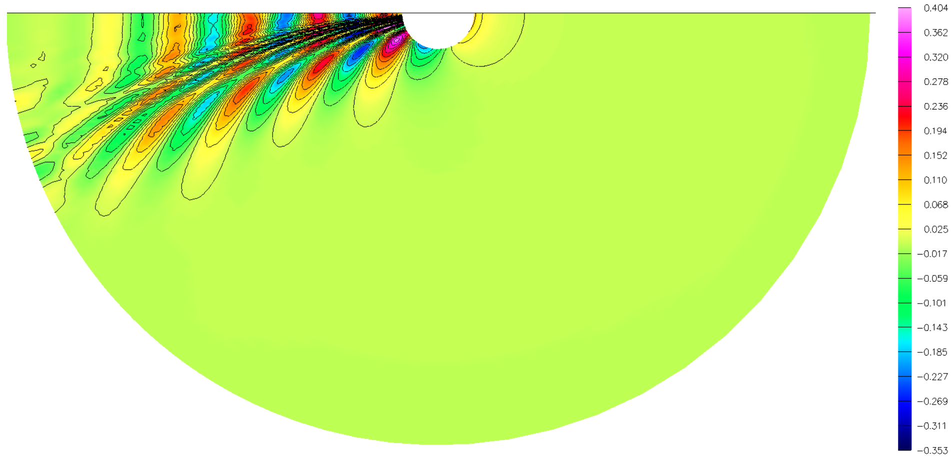

Figure 5: Kelvin wave patterns for a floating hemisphere

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