Radiation and trapping behaviour of arrays of truncated cylinders

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1 Introduction

Trapped modes and the associated trapping structures are a topic which has excited some interest in hydrodynamics over the past two decades. The first trapped modes identified were local oscillations in the vicinity of fixed structures, now called sloshing trapped modes and sloshing trapping structures respectively. McIver and McIver [2006] describe such a trapped mode as 'a free oscillation of an unbounded fluid with a free surface that has finite energy, does not radiate waves to infinity, and in the absence of viscosity will persist for all time.' Sloshing trapping structures discovered include toroidal shapes derived using an indirect procedure and infinite arrays of bottom-mounted cylinders - or the equivalent case of a cylinder in a channel. True trapped modes cannot be excited by an incident wave but near-trapped modes, in which radiation (and hence decay rates) are low but non-zero, may be excited by incident waves. Finite arrays of bottom mounted cylinders in linear and circular configurations have been found to support near-trapped modes.

More recently the term motion-trapping structure has been introduced by McIver and McIver [2006] to describe structures which can support a local bounded oscillation (at the appropriate frequency) in the vicinity of the structure while the structure is free to move, with no radiation to infinity. Such a structure must, in addition to possessing a wave-free frequency, satisfy a resonance condition requiring the structural and fluid inertia terms to balance any restoring force terms at that frequency. Given an initial displacement from equilibrium, such a structure would eventually reach simple harmonic motion at the trapped mode frequency (Porter and Evans [2008]). The stiffness terms in the resonance condition could be hydrostatic and/or mooring terms and examples of each case have been studied. Once mooring terms are introduced, any wave-free structure can become a motion-trapping structure, so the more limited class of structure may be that in which only hydrostatic restoring forces are present. Motion-trapping structures without moooring forces were considered by McIver and McIver [2006], who constructed 2-dimensional motion trapping structures, while McIver and McIver [2007] extended this approach to 3-dimensions and Porter and Evans [2008] started with a pair of rectangular cylinders or a thick-walled cylindrical shell (in 2- and 3-dimensions respectively) and varied the geometry to find motion trapping structures. Motion-trapping structures with mooring restraints were studied by Evans and Porter [2007] who found that a moored submerged circular cylinder moving in heave or sway could be a motion trapping structure and Newman [2008] who analysed mooring stiffnesses on the continuum from $-\infty$ to ∞.

A third class at the intersection of these two, described by Fitzgerald and McIver [2010], is passive trapping structures, which are sloshing trapped modes which exert no force on the body, thereby allowing the body to be freely floating without any associated energy loss through radiation. These modes are not considered here.

Arrays of truncated cylinders are also able to support (sloshing) near-trapped modes; such a mode was identified in the array of four truncated cylinders investigated using a semi-analytical method by Siddorn and Eatock Taylor [2008] - the array is shown in Figure 1. In this study the cylinders were also allowed to move independently and the condition number of the damping matrix used to identify frequencies where almost-wave-free modes could occur. A paper presented by the current authors at an earlier workshop (Wolgamot et al. [2011]) dealt with the directionality of optimum power absorption from arrays of independently moving bodies. In that work radiation from the array and the inverse of the damping matrix for the array were of great importance. The condition number of the damping matrix is closely related, and in this paper we start by studying the modes that may be identified using this method.

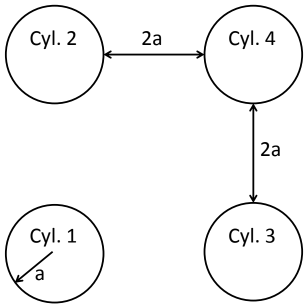

Figure 1: Plan view of four cylinder layout. Cylinders draft $d = 2a$, water depth $h = 4a$.

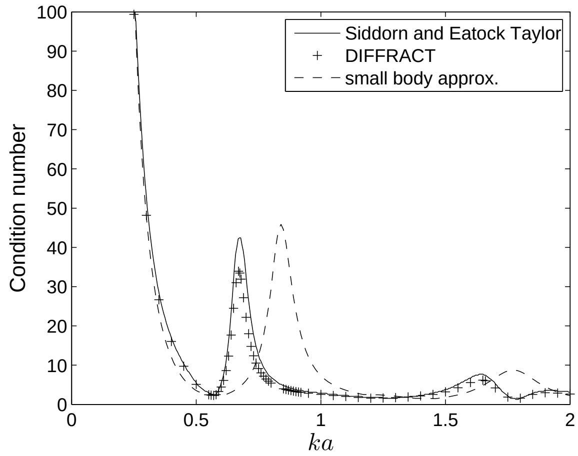

Figure 2: Condition number of heave damping matrix.

2 Four cylinders

The heave damping condition number plot of Siddorn and Eatock Taylor [2008] is shown in Figure 2. Also shown in this figure are results from the boundary element code DIFFRACT for the same array of independently moving cylinders and the condition number of the damping matrix when a small-body approximation is made; that is, the devices radiate but do not scatter. This latter information is interesting in assessing the importance of scattering on the behaviour of the radiating bodies.

It appears that in heave there are two frequencies in the range shown at which near-wave-free modes occur - in fact, the plot for surge (not shown here) is very similar, except that the frequencies of each mode are slightly higher. The second (higher frequency) mode corresponds approximately to the sloshing near-trapping wavenumber of the array.

To determine the body motion modes corresponding to the respective peaks in the condition number plot the following method was employed, based on a similar application in Meylan and Eatock Taylor [2009]. Consider the radiation damping matrix $\boldsymbol{B}(k)$, a real, symmetric n x n matrix, and an associated vector of cylinder displacements, $\boldsymbol{z}$. An approximation to the wavenumber of peak condition number may be denoted k_0, and we seek to solve the problem $\boldsymbol{B}(k)\boldsymbol{z} = 0$ near this wavenumber. If we let the desired wavenumber $k = k_0 + \sigma$ then, using a Taylor series expansion, we may write:

$$(\boldsymbol{B}(k_0) + \sigma\frac{\mathrm{d}\boldsymbol{B}}{\mathrm{d}k})\boldsymbol{z} = 0 \tag{1}$$

which may be written as a generalised eigenvalue problem and solved for eigenvalue σ and corresponding eigenvector $\boldsymbol{z}$. Because DIFFRACT was used to calculate hydrodynamic coefficients we were unable to consider complex wavenumbers, but assume that the imaginary part of the wavenumber is small at the near-wave-free frequencies.

Using this method, the lowest-wavenumber peaks in the condition number plot for heave and surge radiation are found to give mode shapes with all bodies oscillating in phase; i.e. $z = [1, 1, 1, 1]$. The higher frequency peaks in heave and surge correspond to different modes, which will be discussed at the workshop. Thus the first peaks in each case correspond to a minimum of radiation from an oscillation of all cylinders as a single rigid body. Damping coefficients for the cylinder array heaving as a single body were calculated using DIFFRACT, and are shown in Figure 3, where a minimum may be observed at the expected frequency. In this Figure the damping of the multi-column structure is compared to four times the damping of a single heaving cylinder. This minimum damping frequency has been published previously in the results of Mavrakos [1991] and Kagemoto and Yue [1986], although no comment was made on the results. The minimum in damping is quite significant, with a reduction in damping from the reference case of four isolated cylinders by a factor of about 30. As identified by Siddorn and Eatock Taylor [2008], this motion is an almost-wave-free mode. Attaching a spring of the appropriate stiffness would make the structure a 'near-motion-trapping' structure, which would eventually experience a slowly decaying oscillation at this frequency if given an initial displacement. Surge results are similar and will be presented at the workshop.

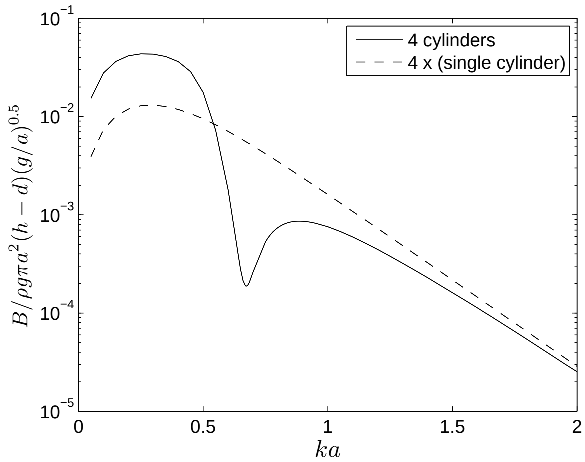

Figure 3: Heave damping of four rigidly connected cylinders.

The damping coefficient for the four heaving cylinders moving in a rigid body mode may be expressed as a combination of the elements of the heave damping matrix for the array, according to:

$$B = 4(B_{11} + 2B_{21} + B_{41}) \qquad (2)$$

using the symmetry of this problem. Some insight into what is occurring at the near-wave-free frequency may then be provided by using the small body approximation, in which damping coefficients are proportional to $J_0(ks)$ where J_0 is a Bessel function of the first kind and s is the spacing. Using this approximation, terms B_{21} and B_{41} will become negative when the s_{21} and s_{41} separations ($4a$ and $4\sqrt{2}a$) cause the argument ks of the Bessel function to pass beyond the first zero at $ks = 2.4048$ and into the negative region. The B_{41} term will become negative first, followed by the B_{21} term. At some middle point corresponding to optimum cancellation of the radiated waves from the respective cylinders the overall heave damping will reach its minimum. The effect of scattering shifts these behaviours in frequency, so that the minimum for the full problem occurs at lower frequency than the small body case.

The free surface elevations for the heave mode show that the mode is a pumping mode, where the free surface inside the array is displaced in the opposite direction to the cylinders. A transition from preferential radiation along the diagonals of the square array at frequencies below the near-wave-free frequency, to radiation perpendicular to the faces of the square at higher frequencies is observed, with minimum radiation at some optimum intermediate frequency. The free surface at this near-wave-free frequency is shown in Figure 4 and the radiation pattern may be readily identified as intermediate between the two cases discussed above.

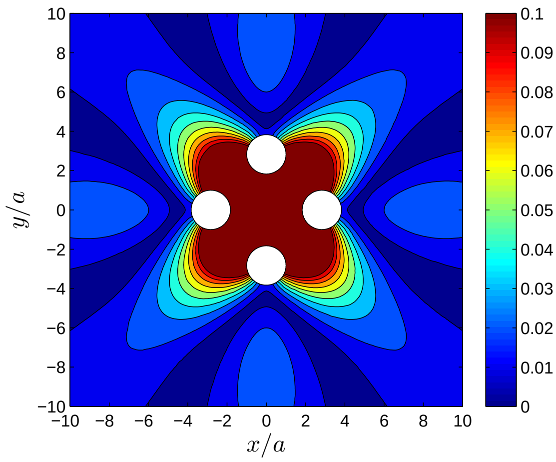

Figure 4: Free surface amplitudes at the first near-wave-free frequency around four rigidly connected cylinders oscillating in heave with unit amplitude. The colour axis is limited such that the radiation pattern external to the array may be seen.

This near-wave-free frequency has some interesting consequences. The Haskind relations relate the directionally averaged exciting forces to the damping coefficients. Therefore, a non-axisymmetric structure such as this will have different ratios of exciting force to damping (i.e. resonant response) at different incoming wave directions. For an optimum incoming wave direction this may lead to a very large resonant response, should the conditions for resonance be satisfied.

3 Eight cylinders

Porter and Evans [2008] found motion-trapping modes in axisymmetric shells of rectangular cross-section and many wave-free structures with an internal free surface have been reported. The simple array of four cylinders considered above is a poor approximation to a shell-like structure, but exhibits a near-wave-free mode. This suggests that a symmetric array with more cylinders could more closely approximate a shell structure, and exhibit more complete wave-free behaviour. To this end, an array of 8 cylinders, moving as a rigid body, was analysed using DIFFRACT. The cylinders, of the same radius and draft as those considered in Section 2, were placed with centres evenly distributed around the circumference of a circle of radius $5a$. Note that this geometry falls outside the range considered by Porter and Evans, but was considered a convenient starting point. The heave damping coefficient for this structure, plotted on a logarithmic scale, is shown in Figure 5 and it is evident that there is strong wave-free behaviour at a frequency of around $ka = 0.42$. Indeed, the damping coefficients calculated by DIFFRACT at frequencies around the minimum (more accurately located at $ka = 0.4216$) were very small negative numbers, though monotonically increasing toward zero with increasing mesh refinement. (Such very small negative damping at a wave-free frequency was also observed by Newman [2008] where calculations were performed using WAMIT. These results would appear to be an artefact of the boundary element approximation in both codes.)

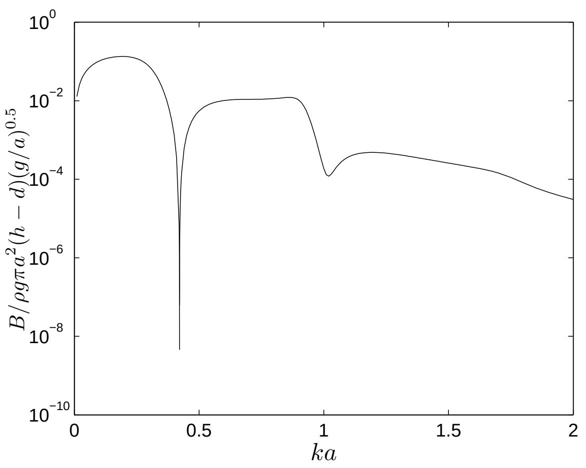

Figure 5: Heave damping of 8 rigidly connected cylinders, water depth $h = 4a$ as above.

This structure does not satisfy the resonance condition necessary for motion trapping in the geometry given, but the position of the resonance can be adjusted by changing the draft of the cylinders. Calculations indicate that an increase in draft of approximately 3.5% is sufficient to cause the resonance condition to be satisfied at the wave-free frequency. To the accuracy of the method used, the structure with the adjusted draft appears to be a motion-trapping structure, and can be constructed quite simply using this approach. The difficulty emphasised by Porter and Evans [2008] in finding a motion-trapping frequency due to the blow-up of the hydrodynamic coefficients at frequencies surrounding the wave-free frequency was not encountered, and may be due to their motion trapping being associated with higher modes than the pumping mode considered here.

Further comparisons of this structure with other known motion-trapping structures and related observations will be presented at the workshop.

Acknowledgement

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