

# Simplified Formulation for Parametric Roll in Regular and Irregular Waves

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## 1. INTRODUCTION

The primary difficulty in quantifying parametric roll is that this phenomenon is a non-ergodic process. If the process is not ergodic, a large number of realizations are necessary to get a stable probability density function. Considering many sea states in wave scatter diagram, the total number of realizations to get a stable long-term prediction significantly increases. Therefore, very fast and effective numerical tool is needed for quantitative analysis under some simplification of real physics.

In most previous studies, the analytic or semi-analytic formulations have been developed to predict the occurrence of parametric roll. Particularly the formulations were based on 1-DOF Mathieu equation (e.g. Pauling, 1959) or approximated GZ variation with known pitch and/or heave-pitch motions. These studies were normally for regular excitation and they were limited to occurrence prediction, not suitable for quantitative analysis of nonlinear roll motion. Recently a few time-domain approaches such as panel, CFD, or impulse-response-function methods have been used to simulate nonlinear roll motions in regular and irregular waves. These can be used for quantitative analysis in irregular seaways, but a significant amount of computational effort is essential.

In this study, it is aimed to develop new formulae which are applicable to the quantitative analysis of parameters in irregular ways but heavy computation is not necessary. The primary idea of the study is to approximate the change of GM by using Fourier components and consider the first two components i.e. the mean and the first components, in the equation of roll motion. According to our study, the change of metacentre height GM, the main source of parametric roll, can be reasonably approximated as a linear system which is dictated by Gaussian distribution. In this study, new formulae are proposed for GZ variation and the equation of motion in regular and irregular waves.

## 2. APPROXIMATION OF GZ & GM

### 2.1 GZ Approximation

Let's consider the equation of motion s.t.

$$(I_{44} + A_{44})\ddot{\phi} + \delta_1\dot{\phi} + \delta_2\phi|\dot{\phi}| + \Delta \cdot GZ(z, \phi, \theta) = 0 \quad (1)$$

where  $\Delta \cdot GZ(z, \phi, \theta)$  indicates the restoring term which is due to ship displacement  $\Delta$  and nonlinear restoring arm  $GZ(z, \phi, \theta)$ . Belenky(2010) and Umeda(2010) proposed the approximated formulae of  $GZ(z, \phi, \theta)$  as follows:

$$GZ(\phi, t) = \frac{GM(t)}{GM_{still}} GZ_{still}(\phi) \text{ (Belenky)}, \quad GZ(\phi, t) = (GM_{still} + GM_0) \cdot \phi + \{GM_1 \cdot \cos(\omega t)\} \cdot \left\{1 - \left(\frac{\phi}{\pi}\right)^2\right\} \cdot \phi \text{ (Umeda)} \quad (2)$$

In this study, based on computational observation,  $GZ(z, \phi, \theta)$  is approximated as follows:

$$GZ(\phi, t) = GZ_{still}(\phi) + \{GM(t) - GM_{still}\} \{\sin(\phi) - \sin^3(\phi) / \sin^2(\phi_{max})\} \quad (3)$$

where subscript 'still' indicates the values at still water. Fig.1 shows the GM variations for 65,000 and 8,000 TEU containerships when wave length and ship length are the same. Three approximated formulae are compared with the GZ variation obtained from direct computation. Better agreement of Eq.(3) is obvious.

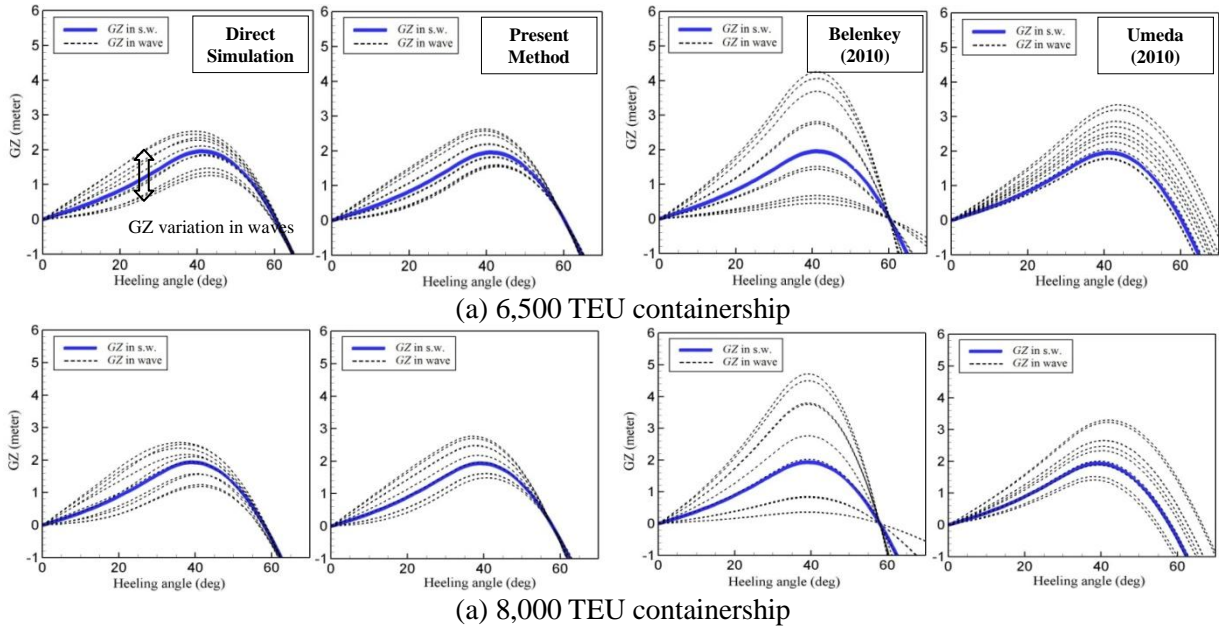


Fig.1 Comparison of GZ between direct computation and three approximated formulae for two containerships:  $kA_w=0.1$ ,  $\lambda = L$

Eq.(3) is nonlinear and GM is dependent on instantaneous displacement of ship motion. In the physical phenomenon of parametric roll, GM variation is a key mechanism. Therefore more simplification of Eq.(3) can be achieved when GM is properly approximated. In this study, GM is represented as a Fourier series.

$$GM(t) = GM_{still} + GM_f(t) = GM_{still} + GM_0 + \sum_{n=1}^{\infty} GM_n \cos(n\omega t + \alpha_n) \quad (4)$$

where  $GM_0$  is the mean of GM and  $GM_n$  is the  $n$ -th component of Fourier component. According to our study, the followings are found:

- (i)  $GM_0$  is always positive.
- (ii) The crests of  $GM_1$  occurs every  $1/n$ -th of effective length.
- (iii)  $GM_2$  is too small to generate parametric roll.

Fig.2 shows  $GM_n$  of four commercial ships when  $kA_w=0.1$  where  $A_w$  is wave amplitude. The three findings described above are clearly shown in these figures.

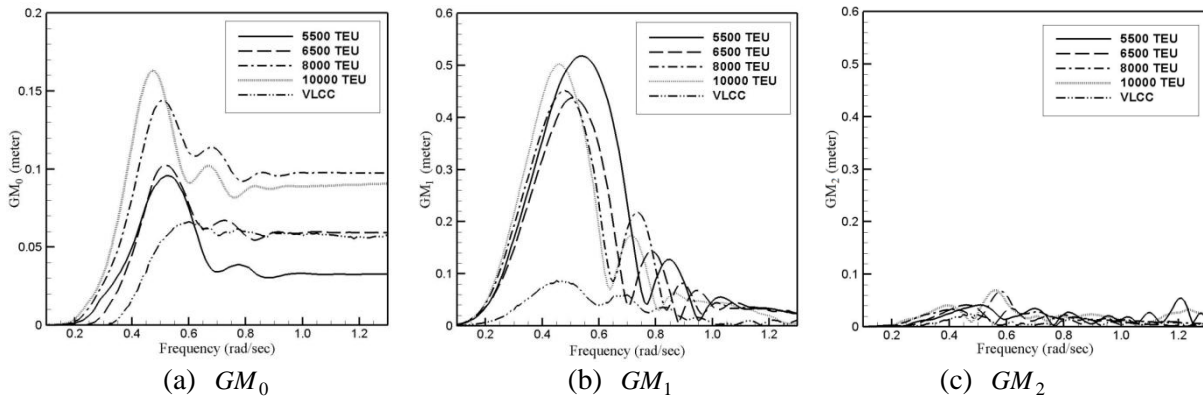


Fig.2  $GM_n$  normalized with respect to wave amplitude: four commercial ships:  $kA_w=0.1$

Fig.3 shows  $GM_0$  and  $GM_1$  for different wave amplitude. In this study, it is assumed that  $GM_0$  and  $GM_1$  are linearly varied with respect to wave amplitude. Particularly, as Fig.3 shows, this approximation is very reasonable for  $GM_1$ . In the case of  $GM_0$ , the linear variation seems to have some error. However, a linear approximation can provide the dominant component of  $GM_0$ . The linear

approximation of  $GM_n$  ( $n=0,1$ ) is important particularly since the concept of response-amplitude-operator (RAO) is applicable for these components, i.e.

$$RAO \text{ of } GM_n = \frac{GM_n}{A_w} . \quad (5)$$

When this is the case, the following GM approximation can be proposed:

$$GM(t) = GM_{still} + a \cdot RAO_{GM_0}(\omega) + a \cdot RAO_{GM_1}(\omega) \cdot \cos(\omega t + \alpha) \text{ for regular waves} \quad (6)$$

$$GM(t) = GM_{still} + GM_0(Hs, Tz) + \sum A_{w,i} \cdot RAO_{GM_1}(\omega_i) \cdot \cos(\omega_i t + \alpha_i) \text{ for irregular waves} \quad (7)$$

$RAO_{GM_1}(\omega_i)$  indicates the RAO of  $GM_1$ , and

$$GM_0(Hs, Tz) = \sqrt{\int S(\omega) RAO_{GM_0}^2(\omega) d\omega} . \quad (8)$$

It should be mentioned that Eq.(8) is valid when Gaussian distribution is valid for  $GM_0$ .

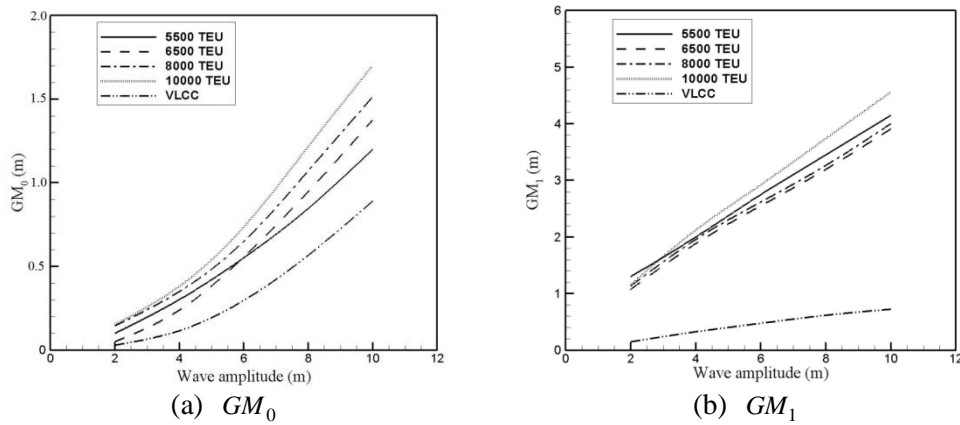


Fig.3  $GM_0$  and  $GM_1$  for different wave amplitude: four commercial ships

Fig.4 shows the spectra of  $GM_1$  of three large containerships which are under heave-pitch-roll motions in irregular wave conditions. The spectra of  $GM_1$  are compared between direct numerical simulation and the approximated  $GM_1$  by using Eq.(7). The overall agreement is very acceptable, proving the present approximation is quite reasonable.

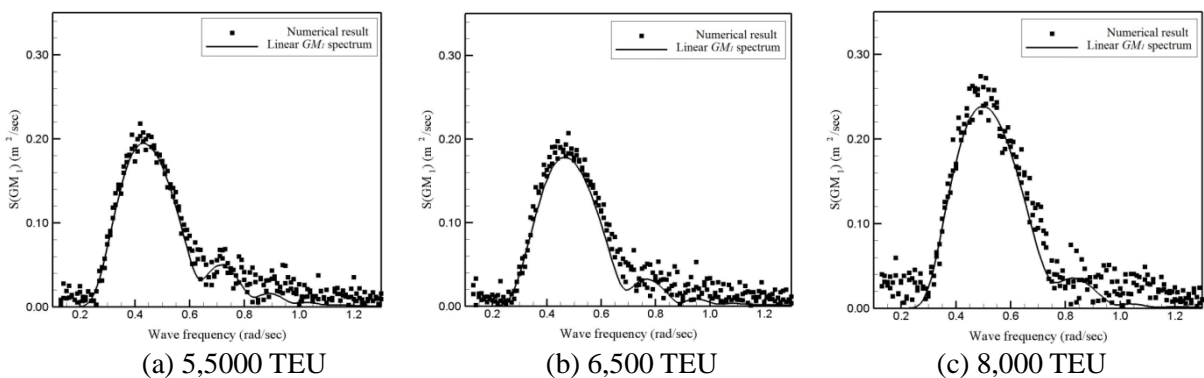


Fig.4 Comparison of  $GM_1$  spectra in irregular wave condition between direct computation and present approximation for three containerships:  $Hs = 6m$ ,  $\lambda_{modal} / L = 1.0$

### 3. APPROXIMATED ROLL EQUATION FOR PARAMETRIC ROLL

Adopting the approximated GZ and GM, the following formulae are proposed for parametric roll in

regular and irregular waves:

$$(I_{44} + A_{44})\ddot{\phi} + \delta_1\dot{\phi} + \delta_2\phi + \Delta \cdot [GZ_{still}(\phi) + A_w \cdot \{RAO_{GM_0}(\omega) + RAO_{GM_1}(\omega) \cdot \cos(\omega t + \alpha)\} \cdot \{\sin(\phi) - \sin^3(\phi) / \sin^2(\phi_{max})\}] = 0 \quad \text{for regular waves} \quad (9)$$

$$(I_{44} + A_{44})\ddot{\phi} + \delta_1\dot{\phi} + \delta_2\phi + \Delta \cdot [GZ_{still}(\phi) + \{GM_0(H_s, T_z) + \sum A_{w,i} \cdot RAO_{GM_1}(\omega_i) \cdot \cos(\omega_i t + \alpha_i)\} \cdot \{\sin(\phi) - \sin^3(\phi) / \sin^2(\phi_{max})\}] = 0 \quad \text{for irregular waves} \quad (10)$$

These equations are much more practical than formulae based on Mathieu equation, since these are applicable for quantitative analysis of parameter roll, particularly including in irregular waves. In the case of Mathieu-equation-based formulae, those are not appropriate for quantitative analysis and valid for only for regular waves. Furthermore, Eqs.(9) and (10) are much more efficient compared with direct simulation programs with compatible solution accuracy.

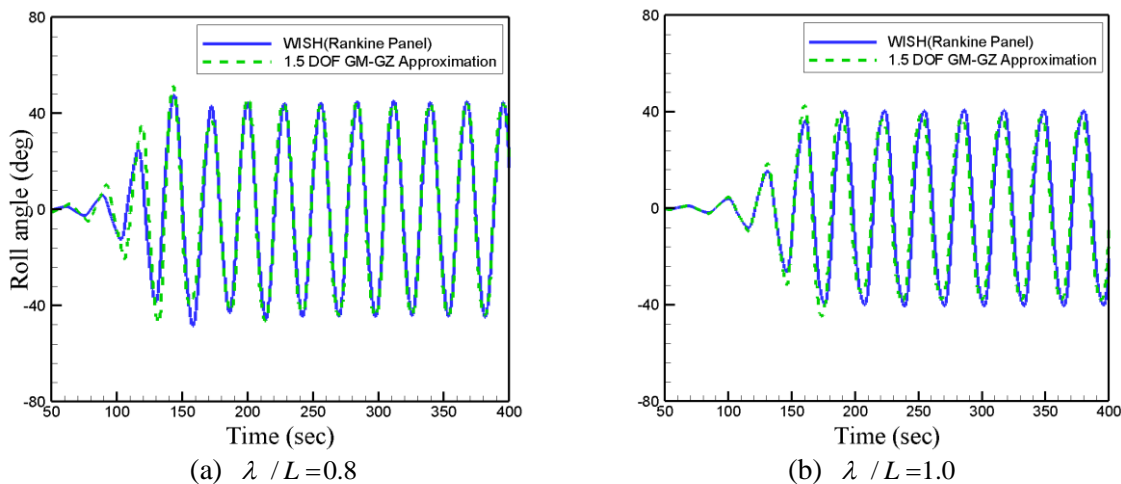


Fig.5 Comparison of roll motion between 3D panel method and present approximation: 6,500 TEU containership:  $kA_w=0.1$

Fig.5 shows the comparison of roll motion in two regular wave cases between WISH, a 3D panel method program, and the present approximation, i.e. Eq.(9). The agreement between two solutions is excellent.

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