Wave-power extraction from a finite array of Oscillating Wave Surge Converters

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Introduction and aim of the work

Deployment of wave energy converters (WECs) in large arrays is envisaged as a fundamental market acceleration strategy towards the commmercialisation of wave energy systems. When working together in an array, WECs can interfere either in a constructive or a destructive manner, depending on the distance between the elements (Budal, 1977; Falnes, 1980; Thomas & Evans, 1981; Mavrakos & McIver, 1997; Thomas, 2008; Garnaud & Mei, 2009; Babarit, 2010; Falnes & Hals, 2012). In this paper we shall consider an array of large devices known as Oscillating Wave Surge Converters (OWSCs). The latter are flap-type WECs hinged on a bottom foundation and pitching under the actions of incident waves in the nearshore (Whittaker & Folley, 2012). In order to investigate the behaviour of an in-line array of many identical WECs, four quantities are essential: the incident wave amplitude and wavenumber, A and k respectively, the characteristic width of the elements w and the spacing a. Several parameters can be formed from those quantities which are fundamental in defining the regime of the system: A/w, kw, ka. First, in this paper we shall restrict our analysis to small-amplitude waves, for which $A/w \ll 1$. Within this assumption, the behaviour of the system can be described by recurring to the linearised versions of the inviscid-irrotational equations of motion (potential-flow model, see for example Mei *et al.*, 2005). This hypothesis rules out the occurrence of vortex-shedding and nonlinear diffraction effects, which are currently being investigated with the aid of computational fluid dynamic models (Rafiee & Dias, 2012). Yet the linearised potential-flow model encompasses a number of cases of practical interest (Mei *et al.*, 2005) and is worth investigating. Another fundamental parameter to characterise the system regime is the product kw between the wavenumber of the incident wave and the characteristic width of a single device. Several existing analytical models are indeed applicable to the OWSC in the limiting cases $kw \ll 1$ and $kw \gg 1$. The first case corresponds to the so-called "point-absorber" approximation (Budal, 1977; Falnes, 1980), while the second one refers to the "line-absorber" limit (Falnes & Hals, 2012). However, considering a characteristic OWSC width $w \simeq 30\,\mathrm{m}$ and a characteristic wavelength $\lambda = 2\pi/k \simeq 100\,\mathrm{m}$, yields $kw = O(1)$, which falls outside the limits of applicability of the aforementioned theories. Recently, new models have been generated to investigate the behaviour of an OWSC in a channel (Henry *et al.*, 2010; Renzi & Dias, 2012), an infinite array of OWSCs (Renzi & Dias, 2013a) and a single OWSC in the open ocean (Babarit *et al.*, 2012; Renzi & Dias, 2013b). However, the analysis of a finite array of OWSCs seems to not have been undertaken yet. Indeed several theoretical models are available concerning the interactions in an array of floating bodies (see for example Kagemoto & Yue, 1986; Mei *et al.*, 1994; Mavrakos & McIver, 1997; Newman, 2001; Adamo & Mei, 2005; Siddorn & Eatock Taylor, 2008), some of them relying on simplifying assumptions on the parameter ka. For $ka \ll 1$, the spacing between the elements can be neglected without appreciable consequences, as shown by Adamo & Mei (2005) for an array of closely-spaced flaps designed to protect Venice from flooding. On the other hand, when $ka \gg 1$ the wide-spacing approximation can be applied, for which radially outgoing waves are approximated as plane waves (Mavrakos & McIver, 1997). In the intermediate case $ka = O(1)$, which corresponds to the situation investigated here, interference effects between the elements of the array must be appropriately accounted for.

In this paper, we devise a new semi-analytical model for a finite array of OWSCs by extending the theory of Adamo & Mei (2005) to the case $ka = O(1)$ and combining it with the semi-analytical approach of Renzi & Dias (2013b). Applications will be shown for the case of two in-line converters; investigations of a larger number of converters is not shown here for the sake of brevity, but will be presented at the

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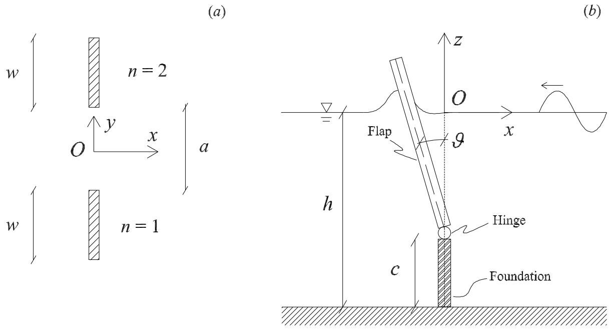

Figure 1: Geometry of the system.

Workshop.

Mathematical model

Referring to figure 1, consider a finite number N of OWSCs in an ocean of depth h, all aligned along the y axis. Each converter is modelled as a rectangular box of width w, pitching about a hinge at depth $z = -h + c$. Following Adamo & Mei (2005) and Renzi & Dias (2012, 2013a,b), the thickness of the flap is assumed to be immaterial. Let the x axis denote the offshore coordinate and the centre of the system O be on the axis of symmetry of the system. Assume that all the converters have identical geometry and are separated by the same distance a (see again figure 1). Incident waves of small amplitude $A \ll w$ are incoming from the right and set the converters into motion, which is then transformed into useful energy by means of generators linked to each OWSC. Within the limits of a linearised potential-flow theory, there exists a velocity potential $\Phi(x, y, z, t)$ which satisfies the Laplace equation:

$$\nabla^2 \Phi = 0 \tag{1}$$

in the fluid domain, being $\nabla f = (f_{,x}, f_{,y}, f_{,z})$, where subscripts with commas denote differentiation with respect to the relevant variable. On the free surface, the kinematic-boundary condition

$$\Phi_{,tt} + g\Phi_{,z} = 0, \quad z = 0 \tag{2}$$

is applied, g being the acceleration due to gravity. In addition, absence of vertical flux is required at the bottom:

$$\Phi_{,z} = 0, \quad z = -h. \tag{3}$$

Finally, let $\theta_n(t)$ be the angle of rotation of the nth converter, positive if anticlockwise; then the boundary condition on each OWSC writes:

$$\Phi_{,x} = -\theta_{n,t}(t)(z + h - c)H(z + h - c), \quad \text{on } \mathcal{L}_n. \tag{4}$$

In the latter expression, $\mathcal{L}_n$ indicates the contour of the nth flap and the Heaviside step function H is used to assure absence of flux through the bottom foundation. For time-harmonic oscillations, the boundary-value problem (1)–(4) is solved in terms of the velocity potential Φ by applying Green's integral theorem in the fluid domain and by expanding the unknown jumps in potential across the flaps in terms of the Chebyshev polynomials of the second kind. Such method is an extension of that applied by Renzi & Dias (2012, 2013a,b) and will not be detailed here for the sake of brevity. Once the potential is known, the motion of the bodies can be fully characterised. The equation of motion of the nth body in the frequency domain is

$$\left[-\omega^2(I + \mu_{nn}) + C - i\omega\left(\nu_{nn} + \nu_{pto}\right)\right]\Theta_n - \sum_{m=1}^{N}{}'\left(\omega^2\mu_{mn} + i\omega\nu_{mn}\right)\Theta_m = F_n. \tag{5}$$

In the latter, I is the flap moment of inertia, C the flap buoyancy torque, ν_{pto} the damping coefficient of the generators, μ_{mn} and ν_{mn} are, respectively, the added inertia and radiation damping of body n when body m is moving, F_n is the exciting torque on body n, and finally Θ_n is the complex amplitude of rotation of body n. The prime on the sum indicates exclusion of the term $m = n$. Expression (5) is a linear system of equations for the unknowns Θ_n, $n = 1, \ldots, N$. Once the Θ_n are all known, the total generated power is determined as

$$P = \sum_{n=0}^{N} \frac{\omega^2}{2}\nu_{pto}\left|\Theta_n^2\right|. \tag{6}$$

Finally, the performance of the system is measured with the interaction factor $q = P/(NP_{iso})$ which is the ratio between the total power captured by the array and the power captured by N isolated elements (Budal, 1977), for given period of the incident wave. Noting that q hides the real amount of absorbed power, Babarit (2010) introduced a modified performance evaluator, defined as $q_n^{mod} = (P_n - P_{iso})/\max_T(P_{iso})$, where P_n is the power output by the nth body in the array and the maximum is taken over the selected period interval. When $q_n^{mod} > 0$, interference effects increase the absorbed power by the nth element with respect to the isolated case. In the following, an application will be shown for the case of two in-line converters.

Results

In this section, results are shown for a system of two in-line OWSCs. Parameters are: $w = 26\,\mathrm{m}$, $c = 4\,\mathrm{m}$, $h = 13\,\mathrm{m}$, $a = 30\,\mathrm{m}$. The damping coefficient has been optimised to yield the maximum power possible

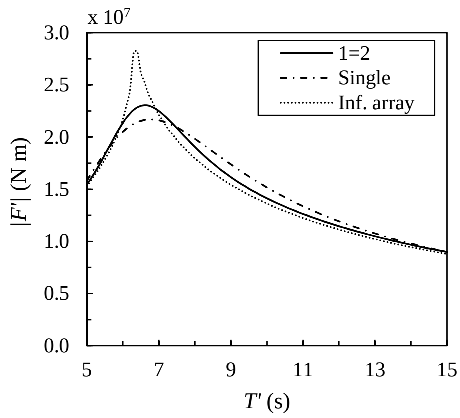

Figure 2: Exciting torque magnitude

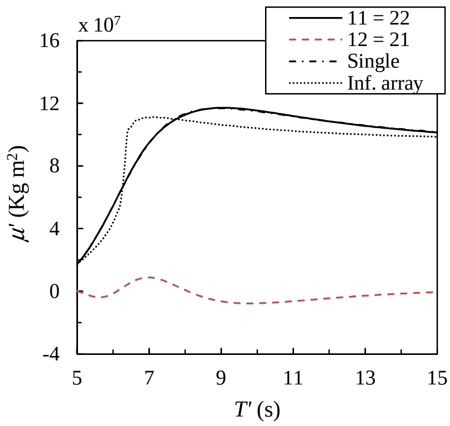

Figure 3: Reflexive ($\mu_{11} = \mu_{22}$) and mutual ($\mu_{12} = \mu_{21}$) added inertia torque.

with given I and C (note that this does not correspond to body resonance). Normally incident sinusoidal waves are considered. Since the configuration is symmetric with respect to $y = 0$, both flaps have the same hydrodynamic parameters. In figures 2–5, the exciting torque, the added inertia torque, the radiation damping, the q and q^{mod} factors are shown versus the period of the incident wave, together with the relevant curves for an infinite array (Renzi & Dias, 2013a) and a single flap (Renzi & Dias, 2013b).

In addition to observing that, as expected, the values of the exciting torque are in between those of a single flap and an infinite array, the following comments can be made:

- A *near-resonant* mechanism is identified: F attains its maximum value near the first resonant period of the infinite array system (see Renzi & Dias, 2013a). Due to damping associated to wave spreading towards infinity, the peak exciting torque for the finite array is lowered and shifted towards larger periods (see also Sammarco & Renzi, 2007, for a similar mechanism).

- The curves of the reflexive added inertia and damping terms, μ_{nn} and ν_{nn} respectively, superimpose to those of a single flap in the open ocean. The mutual terms μ_{nm} and ν_{nm} are larger in short waves and decay in long waves (Babarit, 2010).

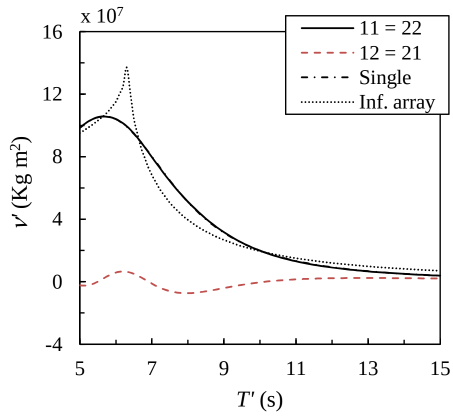

Figure 4: Reflexive ($\nu_{11} = \nu_{22}$) and mutual ($\nu_{12} = \nu_{21}$) radiation damping.

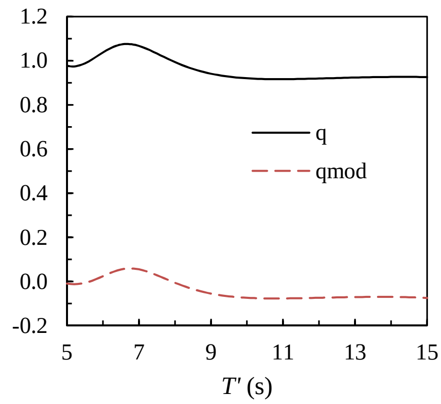

Figure 5: q and q_{mod} factors.

- As expected (Falnes, 1980; Mavrakos & McIver, 1997; Babarit, 2010), the q factor curve shows regions of constructive interaction ($q > 1$), as well as regions of destructive interaction ($q < 1$). The behaviour of the curve resembles that of the exciting torque. This shows that the OWSC hydrodynamics is governed by diffraction (Renzi & Dias, 2012, 2013a,b).

- Regions where $q > 1$ correspond to $q^{mod} > 0$. This means that for two in-line OWSCs, constructive interference is usually accompanied by an actual increase of absorbed power. Such result is not obvious and does not yield for systems of other converters (see Babarit, 2010).

- $\max q \simeq 10\%$ for $T \simeq 7\,\mathrm{s}$, indicating that the power output of each converter is $\simeq 10\%$ larger in the array then when acting isolated (i.e. "1+1 > 2").

Final Remarks

We provide a new semi-analytical solution for a finite array of in-line Oscillating Wave Surge Converters. Calculations for a system of two flaps show that constructive interference is possible for certain periods of the incident wave field. We also show that in an array of OWSCs the strongest constructive interaction is accompanied by the largest system efficiency. This does not yield in general for systems of other devices (see Babarit, 2010). Future applications will consider more populated systems and will aim to investigate the occurrence of trapped modes similar to those found by Mei *et al.* (1994) for a system of flap-type gates.

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