Focused wave impact on a vertical cylinder: Experiment, numerical reproduction and a note on higher harmonics

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Introduction

Wave impact from phase focused waves on a vertical cylinder is studied by means of laboratory experiments and numerical computations. The laboratory experiments were carried out at DHI, Denmark as a part of a Hydrolab project and have previously been presented here at the workshop [3, 5], where the focus was on higher-harmonic forces. Here we consider data from the same set of experiments but focus on numerical reproduction and the observed differences. Special attention is paid to the appearance of artificial higher harmonics in the measurements, which are shown to be a consequence of impulsive unset of structural vibrations.

The experimental and numerical setup

The laboratory experiments were carried out at DHI, Denmark and a detailed description can be found in [5, 6]. It shall be noticed that the force on the cylinder was measured by a force cell placed at the top of the cylinder. To ensure that the force on the cylinder was entirely captured by the force cell a small clearance between the cylinder and the basin floor existed. The force is thus really the reaction force between the test pile and its fixed mount. The natural frequency of the cylinder, when placed in the basin, was measured by a decay test and estimated to be $f_n = 3.8$ Hz.

For reproducing the experiments a numerical domain is setup. The numerical domain has a total length of 10 m and a width of 3 m corresponding to $12D$, where $D = 0.25$ m is the cylinder diameter. At the wave maker the water depth is $h_0 = 0.8$ m, which is then gradually reduced from $x = 2.0$ m by a bed slope of 1/20. The domain is enclosed by generation- and absorption zones at the inlet and outlet respectively. A two-dimensional sketch of the numerical domain is presented in figure 1.

For the numerical computations the incompressible Navier-Stokes equations for the two phase flow of water and air are solved in combination with a volume of fluid (VOF) scheme for tracking the free surface. The numerical model is established within the framework of the open-source CFD-toolbox OpenFOAM® in combination with the wave generation utility waves2Foam [4].

Impacts from a single phase focused wave group with an amplitude distribution given by a JONSWAP spectrum with $H_s = 0.40$ m and $T_p = 2.00$ s is considered. Since the exact paddle signal to the wave maker is unknown, a linear reconstruction of the incident wave group is carried out. The method follows the description in [1], and the wave reconstruction is based on measurements from a single wave gauge located at $x = 0.77$ m.

Results

In figure 2a the measured and computed free surface elevation at $\{x, y\} = \{7.20; 0.00\}$ m is presented in both time and frequency domains. In the time domain a fair agreement is seen until the passage of the two large waves which are both underestimated by the numerical model. Hereafter, the

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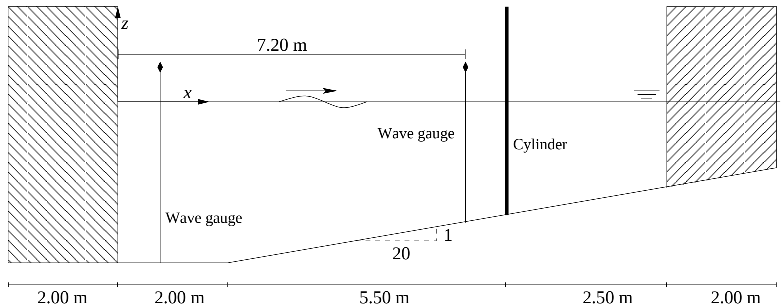


Figure 1: Two-dimensional sketch of the numerical domain. The generation and relaxation zone is indicated by oblique lines.

numerical model slightly underestimates the trough of the subsequent waves, whereas both the wave crest height and the phase is well captured. Considering the surface elevation in the frequency domain a good agreement is seen except for the second harmonic, which is slightly underestimated by the numerical model. The computations and experiments agree well for frequencies higher than the second harmonic.

In figure 2b the inline force on the cylinder is presented in both time and frequency domains. In the time domain the agreement between the measurements and the numerical computations is generally good. As seen from the figure, the experimentally measured inline force is occasionally traced by high frequency oscillations from the eigenmotion of the cylinder, excited by the two large waves. These oscillations are naturally not present in the computations where the cylinder is modelled as perfectly rigid. The peak force from the two largest waves seems to be underestimated by the numerical model. However, it is unclear to what extent the numerical model has underestimated the hydrodynamic force, as the eigenmotion of the cylinder also contributes to the large peak forces.

In the frequency domain a good agreement is seen for the first harmonic, whereas a fair agreement is seen for the second harmonic. For frequencies higher than the second harmonic, a substantial amount of higher harmonic forcing is observed in the measurements, which is not present in the numerical computations. It shall be noticed that a similar deviation for the higher harmonics was not observed in the Fourier transform of the free surface elevation.

Discussion

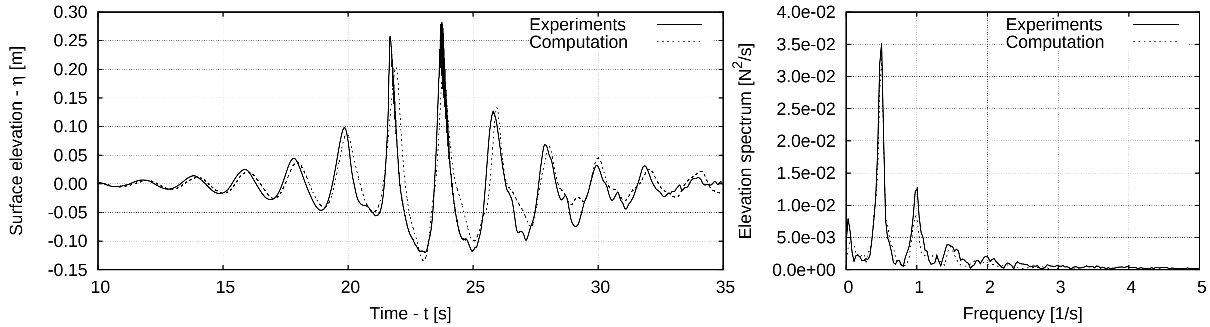
In general a good agreement between the numerical computations and the experiments is observed. The only exception is the substantial higher-harmonic forcing seen in the Fourier transform of the experimentally measured inline force. However, these higher-harmonic forces are not related to the hydrodynamic forcing but can be explained as an artefact of the intermittent eigenmotion of the cylinder.

The origin of the higher harmonics and why they are projected into the harmonics of the incident wave can be explained be considering the inline force on the cylinder written in the following form

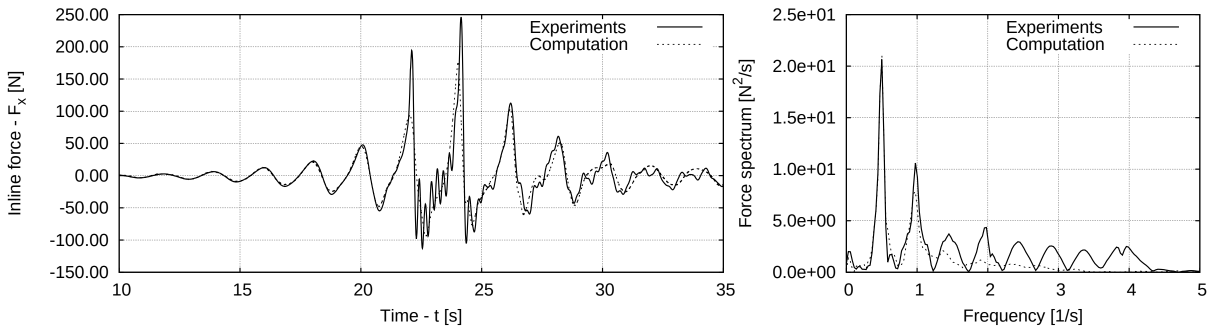
$$f(t) = f_{hydro}(t) + \cos(\omega_n t)\,\chi(t) \tag{1}$$

where the first term is the hydrodynamic forcing and the last term is a simplified representation of the eigenmotion, with radian frequency $\omega_n = 2\pi f_n$. Since the eigenmotion of the cylinder is only present in finite time intervals this term is modulated by a characteristic function, χ, of the following form

$$\chi(t) = \begin{cases} 1 & : x \in \mathbb{I} \\ 0 & : x \notin \mathbb{I} \end{cases} . \tag{2}$$



(a) Measured and computed surface elevation in front of the cylinder: $x = 7.2$m, $y = 0.0$m. Left panel; Time domain. Right panel; Frequency domain



(b) Measured and computed inline force on the cylinder. Left panel; Time domain. Right panel; Frequency domain.

We now consider the Fourier transformation of the inline force, $(\mathcal{F}f)(\gamma)$, where γ is the frequency parameter in Fourier space. The characteristic function, χ, is assumed to be one in a finite interval of $I = [-a/2, a/2]$. The hydrodynamic force is in generally unknown, whereas the last term can be evaluated analytically. It follows from [2], that the Fourier transform of equation (1) is given by

$$(\mathcal{F}f)(\gamma) = (\mathcal{F}f_{hydro})(\gamma) + \frac{1}{2}\left(\frac{\sin \pi a\,(\gamma - \omega_n/2\pi)}{\pi\,(\gamma - \omega_n/2\pi)} + \frac{\sin \pi a\,(\gamma + \omega_n/2\pi)}{\pi\,(\gamma + \omega_n/2\pi)}\right) \quad (3)$$

From equation (3) it is seen that the characteristic function, χ, introduces harmonic oscillations in the Fourier transformed signal with a frequency proportional to the width of the characteristic function, a. In many cases it is reasonable to assume that the width, a, is related to the period of the incident waves. For instance, for the experimental measurements shown in figure 2b the typical duration is approximately $T/2$, where T is the wave period. So, since the eigenmotion of the cylinder is only present in finite time intervals the Fourier transform of this part of the signal could potentially introduce artificial higher harmonics.

For illustration purpose the following example is constructed (see, figure 2)

$$f(t) = [\sin \omega t + \cos(\omega_n t)\,\chi(t)]\,G(t) \quad (4)$$

Here the hydrodynamic forcing is assumed regular with radian frequency, ω, and the structure is assumed to oscillate with radian frequency ω_n. The characteristic function χ is equal to one in the time intervals $I \in \{[t_1, t_2]; [t_3, t_4]\}$, as indicated in figure 2, and with widths $a = \frac{2\pi}{\omega}\{0.57; 0.56\}$. Finally the signal is multiplied by a Gaussian, $G(t)$, to mimic the temporal behaviour of the phase focused group. Notice that the Gaussian modulation of the signal does not introduce any harmonic frequencies and only serves the purpose of increasing the visual similarity with the phase focused group. For the test function are the frequencies from the experiment, $\omega = \omega_{exp} = 2\pi 0.5$ rad/s and

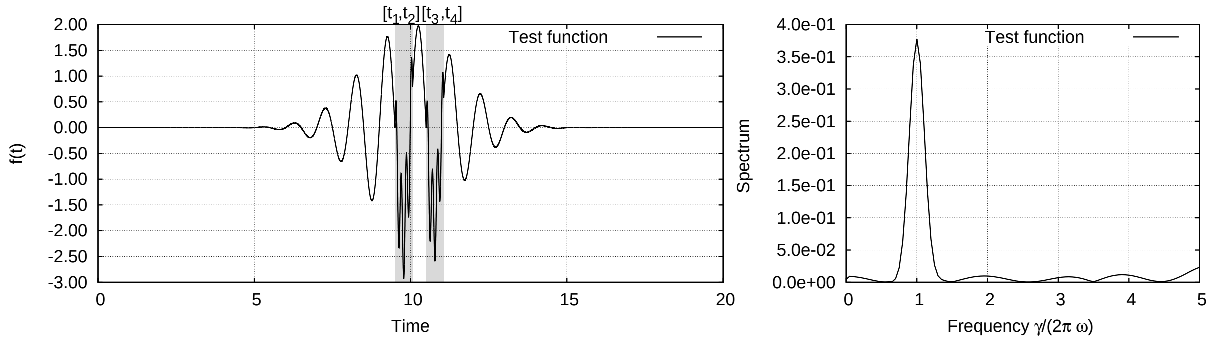


Figure 2: Time and frequency representation of test function. Areas of shaded grey indicates interavals where $\chi = 1$.

$\omega_n = 2\pi f_n$ rad/s, applied. In the Fourier transform of the test function artificial higher-harmonics, as the one observed in the experimental measurements, are clearly seen. Since the time intervals of $\chi = 1$ are not perfectly half the peak period is a perfect fit to the higher-harmonics not seen.

The example shows that the discontinuous onset of structural vibration can produce artificial higher harmonics over broad ranges of the spectrum.

From the analysis and the numerical example is it obvious that frequencies from the test setup are not easily filtered in Fourier space as a post process. The characteristic function, χ, and the time intervals a are in general unknown so there is no way of knowing at what frequencies energy from the structural motion has been project onto. A time domain filtering is not unproblematic either, as important details related to the hydrodynamic forcing is easily destroyed by the filtering process.

In the present work is numerical methods shown to be a vulnerable tool for investigating and understating experimental measurements and by that gaining further insight.

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