External Dynamics System of Twin Floating Bodies for Perfect Wave Absorption

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INTRODUCTION

Regular waves propagating in a two-dimensional waterway can be absorbed perfectly by a single floating body which oscillate in two modes of motion such as a couple of heave and sway motions or heave and roll motions. It is naturally required the symmetrical and asymmetrical motions for the perfect wave absorption.

In a specific situation we can absorb waves perfectly by either motion only of the floating body. For an instance, the floating body placed at the end of the experimental towing tank without gaps between the floating body and the tank wall enables it by the heave motion only. That is the result of canceling another mode because of the symmetrical condition of the free surface at the end of the tank. The wave tank having wave-makers and active wave absorbers is generally designed based on this principle. However, the situation of the experiment and the experimental tank make it difficult to place the wave-makers and absorbers at the end of the tank. For the multiple application of the experimental tank, it is desired to place the wave-makers and absorbers at an arbitrary place and time.

Symmetrical and asymmetrical motions of the floating body provide symmetrical and asymmetrical waves propagating from the floating body. This wave condition can be generated by heave motion only of two floating bodies. The mechanism of the wave-maker allowing only heave motion is simpler and more practical than that of two modes of motion.

This paper addresses the problem of the wave absorption by the twin floating bodies with the external dynamics system. In the beginning, the wave condition for the perfect wave absorption by the twin floating bodies is described and formulated based on the theory of the mutual interaction of waves. As a solution of these formulae, the characteristics of the external dynamics system attached on the wedge-shaped floating body and block-shaped floating body are obtained. These results give us the possibility and problem of wave absorption of this system.

WAVE CONDITION FOR PERFECT ABSORPTION

The arrangement of the twin floating bodies is shown in Fig. 1. The geometry of floating bodies is symmetric and each body has geometry. The heave motion is allowed only for these floating bodies. The external dynamics system attached on the floating body is modeled with a spring and a dash pot. Changing the restoring and damping factors of a spring and a dash pot depending on a wave period, this system enables to absorb incident waves perfectly. In a real system a spring and a dash pot are replaced with a mechanical actuator controlled by the displacement and velocity of the motion of the floating body. The wave and wave force acting on the floating body assume to be obtained by the linear potential theory.

Let s, $d_j^{\pm}$ and $r_j^{\pm}$ denote the complex amplitude of incident waves, diffraction waves and radiation waves. The incident waves is naturally

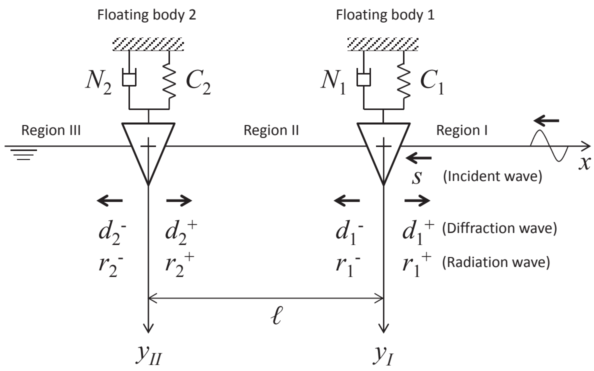

Fig. 1 Definition of waves and coordinate system.

expressed as

$$\zeta_a(x,t) = \mathrm{Re}\left[s e^{i(kx+\omega t)}\right].$$

Where k is a wave number and ω is an angular frequency of waves.

The incident wave coming from a far field into the floating body 1 is completely absorbed in a steady condition, then the complex amplitude of waves in the region I and III is described as

$$d_1^+ + r_1^+ + L(d_2^+ + r_2^+) = 0, \tag{1}$$

$$d_2^- + r_2^- + L(d_1^- + r_1^-) + Ls = 0. \tag{2}$$

Where, L denotes the transfer function of the waves progressing the distance ℓ.

$$L = e^{-ik\ell}.$$

Relation of the mutual interaction regarding the diffraction waves is described as

$$d_1^+ = iH_{14}^- L(d_2^+ + r_2^+) + iH_{14}^+ s, \tag{3}$$

$$d_1^- = iH_{14}^+ L(d_2^+ + r_2^+) + iH_{14}^- s, \tag{4}$$

$$d_2^+ = iH_{24}^+ L(d_1^- + r_1^- + s), \tag{5}$$

$$d_2^- = iH_{24}^- L(d_1^- + r_1^- + s). \tag{6}$$

Where, $H_{j4}^{\pm}$ is the Kochin function of diffraction waves, H_{j4}^+ indicates the progressive wave from the floating body j toward the counter direction as the incident wave and H_{j4}^- indicates the progressive wave from the floating body j toward the same direction as the incident wave.

Regarding the radiation waves, due to the symmetrical geometry of the floating body, we can obtain $r_1^+ = r_1^- = r_1$ and $r_2^+ = r_2^- = r_2$. Then the solution of the simultaneous equations 1-6 are shown as

$$d_1^+ = W^{-1}\left[sL^2(R_1 - T_1)(R_2 - T_2)(R_1 + Q_1) - sR_1\right], \tag{7}$$
$$d_1^- = W^{-1}\left[sL^2(R_1 - T_1)(R_2 - T_2)(R_1 + Q_1) - sQ_1\right], \tag{8}$$
$$d_2^+ = W^{-1}sLR_2(R_1 - T_1), \tag{9}$$
$$d_2^- = W^{-1}sLQ_2(R_1 - T_1), \tag{10}$$
$$r_1 = W^{-1}\left[-sL^2(R_1 - T_1)(R_2 - T_2)(R_1 + T_1) + sR_1\right], \tag{11}$$
$$r_2 = W^{-1}sLT_2(-R_1 + T_1). \tag{12}$$

Where W, R_j, Q_j, T_j are defined as

$$W = L^2(R_1 - T_1)(R_2 - T_2) - 1,$$
$$R_j = iH_{j4}^+,$$
$$Q_j = iH_{j4}^-.$$
$$T_j = 1 + Q_j.$$

MOTION OF FLOATING BODY

Let M_1 denote the total mass of the floating body and the external dynamics system. Let y_1 denote the displacement of the heave motion. The force acting on the floating body 1 consists of the mechanical force by the external dynamics system and the hydrodynamic force. Concerning the hydrodynamic force, we must take the exciting force induced by the reflected waves from the floating body 2. The linear equation of motion of the floating body 1 with respect to the angular frequency ω is given as

$$\begin{aligned} M_1\ddot{y}_1 &= -N_1\dot{y}_1 - C_1y_1 - a_1\ddot{y}_1 - b_1\dot{y}_1 - c_1y_1 \\ &\quad +\mathrm{Re}\left[\rho g H_{12}L(d_2^+ + r_2^+)e^{i\omega t}\right] + \mathrm{Re}\left[\rho g H_{12} s e^{i\omega t}\right]. \end{aligned} \tag{13}$$

Where, H_{12} is the Kochin function of the heave oscillation of the floating body 1. The symbols of a_1, b_1 and c_1 indicate the added mass, the wave damping coefficient and the restoring coefficient respectively. The complex amplitude of the heave oscillation expressed as Y_1 and A_1 is defined as

$$A_1 = -\omega^2(M_1 + a_1) + i\omega(N_1 + b_1) + C_1 + c_1, \tag{14}$$

then, the equation 13 is represented as

$$A_1Y_1 = \rho g H_2\left\{L(d_2^+ + r_2^+) + s\right\}. \tag{15}$$

The Kochin function gives the relation between r_1 and Y_1 as

$$r_1 = -ikH_{12}Y_1.$$

Applying this relation to the equation 15, we obtain

$$A_1 = \frac{-ik\rho g {H_{12}}^2}{r_1}\left\{L\left(d_2^+ + r_2^+\right) + s\right\}. \tag{16}$$

In a similar way, the equation for the floating body 2 is provided as

$$A_2 = \frac{-ik\rho g {H_{22}}^2}{r_2} L\left(d_1^- + r_1^- + s\right). \tag{17}$$

Where, H_{22} is the Kochin function of the heave oscillation of the floating body 2.

CHARACTERISTICS OF EXTERNAL DYNAMICS SYSTEMS

The characteristics of the external dynamics system is determined by substituting wave conditions for the equation of motion. Substituting Eqs. 8, 9, 11 and 12 for Eqs. 16 and 17, we obtain A_1 and A_2 for the perfect absorption. They are expressed as A_{p1}, A_{p2};

$$A_{p1} = ik\rho g {H_{12}}^2 \frac{2L^2(R_1 - T_1)(R_2 - T_2) - 1}{L^2(R_1 - T_1)(R_2 - T_2)(R_1 + T_1) - R_1}, \tag{18}$$
$$A_{p2} = ik\rho g {H_{22}}^2 \frac{1}{T_2}. \tag{19}$$

Using the relations regarding the Kochin function:

$$H_{j2} = \bar{H}_{j2}(R_j + T_j), \tag{20}$$
$$b_j = \rho\omega H_{j2}\bar{H}_{j2}. \tag{21}$$

and the dispersion relation of wave: $\omega^2 = kg$, we obtain

$$A_{p1} = i\omega b_1 \frac{2L^2P - (R_1 + T_1)}{L^2P - R_1}, \tag{22}$$
$$A_{p2} = i\omega b_2 \left(\frac{R_2}{T_2} + 1\right). \tag{23}$$

Where, $\bar{H}_{j2}$ is a conjugate of H_{j2} and

$$P = (R_1 + T_1)(R_1 - T_1)(R_2 - T_2).$$

The characteristics of the external dynamics system at this moment are described as C_{pj} and N_{pj}. They independently consist of the real and imaginary part of A_{pj} as shown in Eq. 14. Therefore, comparing the real and imaginary part of (22) and (23) gives the characteristics of the external dynamics system.

$$C_{p1} = \mathrm{Re}\left[A_{p1}\right] + \omega^2(M_1 + a_1) - c_1 \tag{24}$$
$$N_{p1} = \frac{1}{\omega}\mathrm{Im}\left[A_{p1}\right] - b_1 \tag{25}$$
$$C_{p2} = i\omega b_2 \frac{R_2}{T_2} + \omega^2(M_2 + a_2) - c_2 \tag{26}$$
$$N_{p2} = 0 \tag{27}$$

Equations 26 and 27 are derived from the matter that R_j/T_j is a pure imaginary number. The wave power absorbed by each external dynamics system is expressed by

$$\text{Absorbed wave power} = \frac{1}{2}N_{pj}\omega^2\left|Y_j\right|^2.$$

The equation 27 naturally indicates that the external dynamics system of the floating body 2 does not absorb wave energy. Actually the characteristics expressed by Eq. 26 and 27 corresponds to the condition of the external dynamics system of a single floating body which perfectly reflects the incident waves. Namely, the wave energy is absorbed by the external dynamics system of the floating body 1 only. Moreover, this characteristics does not depend on the gap between the floating bodies.

Using a single symmetrical floating body with the external dynamics system, we can absorb a half of wave energy at a maximum efficiency. Then, the condition of the external dynamics system is well known as

$$C = \omega^2(M + a) - c, \tag{28}$$
$$N = b. \tag{29}$$

Equations 24-26 are regarded as the modified condition from Eqs. 28 and 29.

COMBINATION OF FLOATING BODY

The theoretical characteristics of the external dynamics system for the perfect absorption of incident waves are shown in several cases regarding the wedge-shaped and block-shape floating body and their combination.

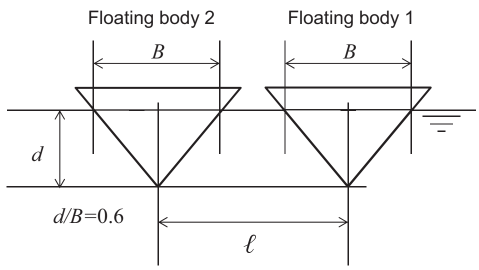

Fig. 2 Geometry of wedge-shaped floating bodies.

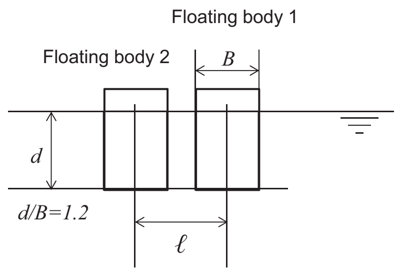

Fig. 5 Geometry of block-shaped floating bodies.

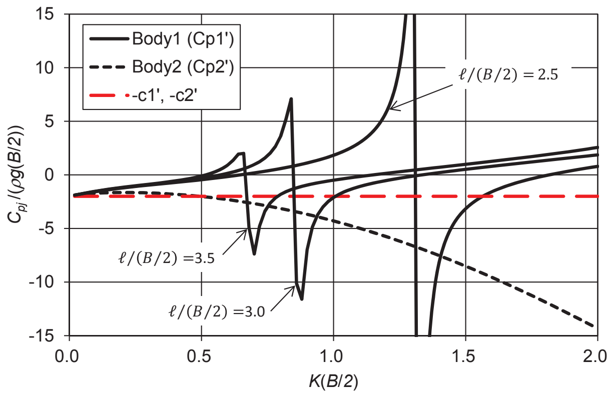

Fig. 3 Restoring coefficients of the external dynamic system of wedge-shaped floating bodies.

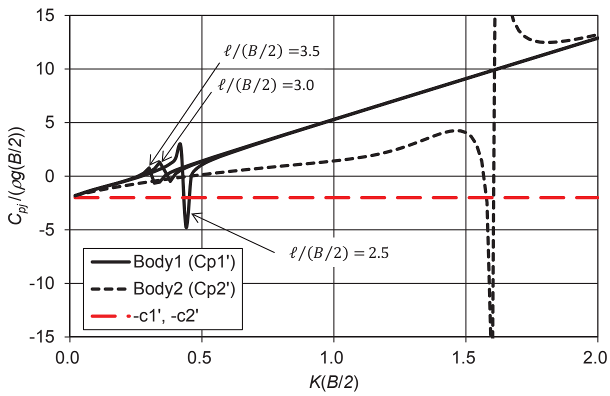

Fig. 6 Restoring coefficients of the external dynamic system of block-shaped floating bodies.

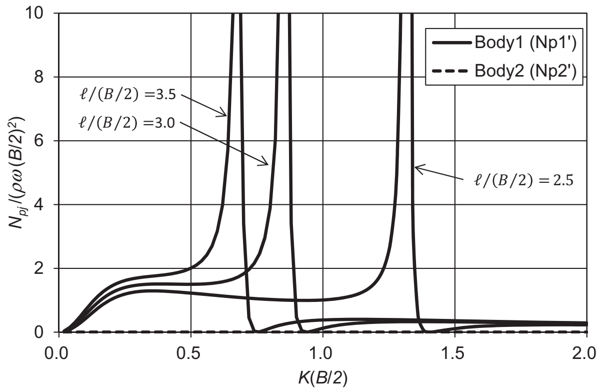

Fig. 4 Damping coefficients of the external dynamic system of wedge-shaped floating bodies.

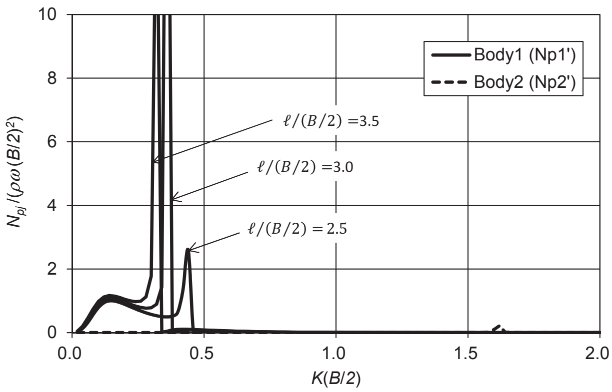

Fig. 7 Damping coefficients of the external dynamic system of block-shaped floating bodies.

The arrangement of the wedge-shaped floating body is shown in Fig. 2 and the theoretical characteristics of the external dynamics system are shown in Figs. 3 and 4. These were calculated by Eq. 24-27 and the Kochin functions were obtained by the boundary element method based on the linear potential theory. The aspect ratio of the draft:d and the breadth:B at the water line is $d/B = 0.6$. The gap between the floating bodies is changed from $\ell/(B/2) = 2.5$ to 3.5. The nondimensional frequency $K(B/2) < 2$ is appropriate for a real wave-maker and absorber of an experimental tank/basin. The mass inside the external dynamics system is disregarded in the calculation.

According to the motion of equation of the floating body, the total restoring coefficient must be positive in a steady oscillation. Its condition is provided as $C_{pj} + c_j > 0$. Therefore, the restoring coefficient of the external dynamics system must satisfy

$$C_{pj} > -c_j. \tag{30}$$

However, this condition is not applicable to C_{p2} in the wider range of wave frequency more than $K(B/2) > 0.5$. The snap point of C_{p1} depends on R_j, T_j and the gap between the floating bodies. In this point a steady oscillation is not obtained due to the condition of Eq. 30 and the wave absorption is impossible due to $N_{p1} = 0$ as shown in Fig. 4. As a result, the range of the wave frequency is quite restricted for the steady oscillation of this system.

The arrangement of the block-shaped floating body is shown in Fig. 5 and the theoretical characteristics of the external dynamics system are shown in Figs. 6 and 7. The restoring coefficients satisfy the condition Eq. 30 in the wider range of wave frequency than that of the wedge-shaped floating body as shown in Fig. 6. However, the damping coefficients are much less(not zero) in the range of $K(B/2) > 0.5$ as shown in Fig. 7. This matter indicates that a large amplitude of the heave motion

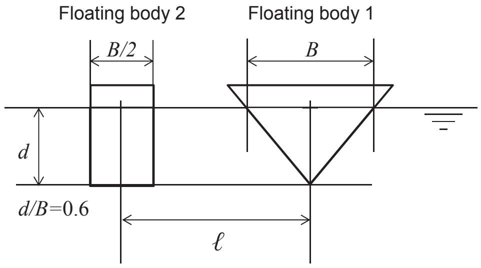

Fig. 8 Combination of wedge-shaped and block-shaped floating bodies.

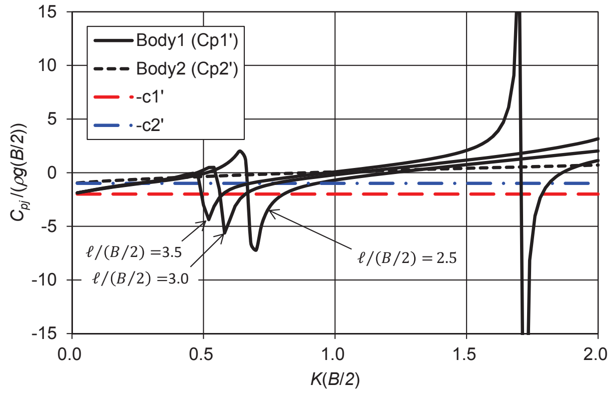

Fig. 9 Restoring coefficients of the external dynamic system of the combination of wedge-shaped and block-shaped floating bodies.

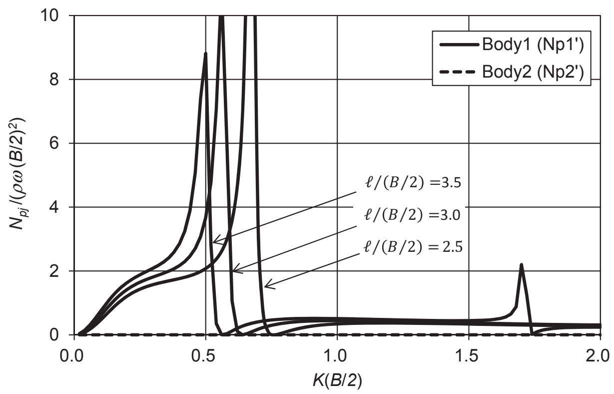

Fig. 10 Damping coefficients of the external dynamic system of the combination of wedge-shaped and block-shaped floating bodies.

of the floating body 1 appears for the wave energy absorption.

The combination of the wedge-shaped and block-shaped floating body is shown in Fig. 8 and the theoretical characteristics of the external dynamics system are shown in Figs. 9 and 10. The displacements of both floating bodies are same. The restoring coefficient C_{p2} satisfies the condition Eq. 30 in all range, however C_{p1} is not acceptable for this condition at the snap point. The snap points appear in the lower wave frequency: $0.5 < K(B/2) < 0.7$ than the combination of the wedge-shaped and wedge-shaped floating bodies because the gap between the floating bodies is relatively larger.

The combination of the block-shaped and block-shaped floating bodies seems to be the best for wave absorption, however the efficiency of wave generation of the block-shaped floating body is not higher. To apply the wedge-shaped floating body having the higher efficiency of wave generation, we need to avoid the snap point of C_{p1}. The relation between R_j, T_j and the gap of the floating bodies must be investigated because the snap point of C_{p1} is determined by the denominator of Eq. 23.

CONCLUSIONS

The characteristics of the external dynamics system attached on the twin floating bodies for the perfect wave absorption are shown theoretically with the mutual interaction theory of waves. The external dynamics system is modeled by a linear spring and dash pot. As a result, the following knowledge is obtained.

- The external dynamics system of the floating body located at the lee side for incident waves must reflect waves perfectly. The damping coefficient of this external dynamics system must be zero. All wave energy is consequently absorbed by the external dynamics system located at the weather side.
- The total of the mechanical and hydrodynamic restoring coefficients must be more than zero for the steady oscillation of the floating bodies. This condition is not satisfied in the wider wave frequency for the system of the wedge-shaped and wedge-shaped floating bodies. However, replacing the floating body by the block-shaped one, we can satisfy this condition.
- This system is also expected to have the function of a wave-maker, namely has to absorb and generate waves simultaneously. Considering the efficiency of wave generation, the wedge-shaped floating body is better at the weather side. The restoring coefficient of the external dynamics system attached on the wedge-shaped floating body snaps at a specific wave frequency. At this frequency, the condition for the steady oscillation is not satisfied. To apply the wedge-shaped floating body having the higher efficiency of wave generation, we need to avoid this snap point.

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