

Towards Real Time Simulation of Ship-Ship Interaction - Part II: Double Body Flow Linearization and GPU Implementation*

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The marine industry uses full mission marine simulators for training naval officers and as a tool in marine engineering. The quality of the training and the engineering assessments relies on the realism and visual presentation of the simulated ocean waves, ship waves and ship-ship interaction forces. Ambient ocean waves are often calculated by FFT's of a wave spectrum. Realistic real time calculation of ship waves are not available in ship simulators today. Ship-ship interaction forces are calculated using boundary element models or based on model experiments in towing tanks and precomputed force tables. The present work is motivated by 1) The need for ship wave calculation and more accurate and reliable ship-ship interaction force calculations in full mission marine simulators, 2) The development of new numerically accurate and computationally efficient large scale ocean wave and ship wave models, and 3) The availability of affordable Graphical Processing Units (GPU's) ideal for high performance scientific computing. The main challenge is the real time constraint, which limits how complex the physical model can be and still have a computationally efficient and fast numerical solution. An ever changing simulated environment and human interaction are other challenges for the robustness and flexibility of the model. To meet these challenges, the physical model has to be simple, but still accurately representing the kinematic and dynamic effects of water waves and ship motions. The approach adopted is that described at last year's workshop Lindberg et al. [2012] and involves a simplified geometric representation of the ship hull by means of a dynamic pressure distribution applied on the free-surface; together with a GPU optimized fast Laplace solver based on high-order finite differences Engsig-Karup et al. [2009], Engsig-Karup et al. [2011].

Free Surface Flow and Ship Model

The mathematical model describing the flow around the ships and the wave motion is based on potential flow theory and the equations are presented in a ship fixed moving frame of reference

$$x = x_0 - Ut, \quad y = y_0, \quad z = z_0, \quad (1)$$

where (x_0, y_0, z_0) is the earth fixed frame of reference, t is time and U is the ship velocity positive x -direction. The water velocity is the gradient of the velocity potential $\mathbf{u} = (u, v, w)^T = \nabla\phi$. Continuity

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requires the velocity potential to satisfy the Laplace equation

$$\nabla^2 \phi = 0, \quad -h \leq z \leq \eta, \quad (2)$$

where $h = h(x, y)$ is the sea depth and $\eta = \eta(x, y)$ is the free surface elevation. The evolution of the free surface is described by the kinematic free surface boundary condition

$$\frac{\partial \eta}{\partial t} + (u - U) \frac{\partial \eta}{\partial x} + v \frac{\partial \eta}{\partial y} \phi = \frac{\partial \phi}{\partial z}, \quad z = \eta, \quad (3)$$

and the evolution of the free surface velocity potential is described by the dynamic free surface boundary condition

$$\frac{\partial \phi}{\partial t} - U \frac{\partial \phi}{\partial x} + \frac{1}{2} \mathbf{u} \cdot \mathbf{u} + g\eta + \frac{p}{\rho} = 0, \quad z = \eta, \quad (4)$$

where $g = 9.81 \text{ m/s}^2$ is the gravitational acceleration, p is the pressure and $\rho \approx 1000 \text{ kg/m}^3$ is the density. Note that the Bernoulli constant in the dynamic boundary condition is zero.

These equations comprise the fully nonlinear water wave problem and they are solved in a time dependent physical domain with a moving free surface boundary. Some complexity has to be removed to solve these equations in real time. The approximations start with perturbation expansions of the velocity potential and the free surface elevation

$$\phi = \phi_0 + \varepsilon \phi_1 + \varepsilon^2 \phi_2 + \dots, \quad \eta = \eta_0 + \varepsilon \eta_1 + \varepsilon^2 \eta_2 + \dots, \quad (5)$$

where the parameter ε is the ratio between the wave elevation and wave length $\varepsilon = H/\lambda$. The free surface velocity potential is further expanded in a Taylor series with respect to the still water level. The zero-order term is calculated either by a Neumann-Kelvin linearization $\phi_0 = Ux$ or a double body flow linearization

$$\nabla^2 \phi_0 = 0, \quad -h \leq z \leq 0, \quad (6)$$

$$\mathbf{n} \cdot \nabla \phi_0 = -n_x U, \quad (x, y) \in \Gamma_{ship}, \quad (7)$$

$$\frac{\partial \phi_0}{\partial z} = 0, \quad (x, y) \notin \Gamma_{ship}, \quad z = 0, \quad (8)$$

$$\frac{\partial \phi_0}{\partial z} = 0, \quad z = -h. \quad (9)$$

where $\mathbf{n} = (n_x, n_y, n_z)$ is the normal vector of the ship hull surface Γ_{ship} . Currently we are working with two approximations of the body boundary condition: a flat ship approximation and a pressure distribution representation. The first order solution is calculated from the kinematic and dynamic boundary conditions which are linearized around the steady solution

$$\frac{\partial \eta}{\partial t} + \left(\frac{\partial \phi_0}{\partial x} - U \right) \frac{\partial \eta}{\partial x} + \frac{\partial \phi_0}{\partial y} \frac{\partial \eta}{\partial y} = \eta \frac{\partial^2 \phi_0}{\partial z^2} + \frac{\partial \phi_1}{\partial z}, \quad (10)$$

$$\frac{\partial \phi_1}{\partial t} + \left(\frac{\partial \phi_0}{\partial x} - U \right) \frac{\partial \phi_1}{\partial x} + \frac{\partial \phi_0}{\partial y} \frac{\partial \phi_1}{\partial y} + g\eta + \frac{p}{\rho} = 0, \quad z = 0, \quad (11)$$

where the pressure is zero on the free surface $(x, y) \in \Gamma_{FS}$ and determined by the following quasistatic approximation on the ship hull

$$p = -\rho \left(\left(\frac{\partial \phi_0}{\partial x} - U \right) \frac{\partial \phi_1}{\partial x} + \frac{\partial \phi_0}{\partial y} \frac{\partial \phi_1}{\partial y} + g\eta_0 \right), \quad (x, y) \in \Gamma_{ship}, \quad z = 0. \quad (12)$$

Here it is assumed that the ship hull can be represented by a single valued function of the horizontal coordinates $\eta_0 = \eta_0(x, y)$.

Finally the radiation boundary condition

$$\phi = \begin{cases} \mathcal{O}((x^2 + y^2)^{-1/2}) \\ \mathcal{O}(1) \end{cases} \quad \text{as } x^2 + y^2 \rightarrow \infty \quad \text{for } \begin{cases} x > 0 \\ x < 0 \end{cases} \quad (13)$$

is approximated by the zero'th order absorbing boundary condition

$$\frac{\partial \phi}{\partial t} + (Cn_x - U) \frac{\partial \phi}{\partial x} + Cn_y \frac{\partial \phi}{\partial y} = 0, \quad (x, y, z) \in \Gamma_A, \quad (14)$$

where $C = \sqrt{gh}$ is the wave celerity of absorbed wave component and Γ_A is the artificial physical domain boundary. This boundary condition is most efficient for shallow water waves and a wave direction normal to the boundary. Efficient absorbing boundary conditions for dispersive and possibly non-linear water waves at large angles is an advanced and open question for the water wave community.

Numerical Approximations and Solvers

The Laplace equation (2) for the potential is approximated by higher-order finite differences and solved by a multigrid preconditioned defect correction (PDC) method, Engsig-Karup et al. [2011]. The PDC enables efficient solution due to linear scaling in both computational work and memory. An explicit higher-order Runge-Kutta method is used for the temporal discretization of the linear kinematic free surface boundary condition (10) and dynamic free surface boundary condition (11) and the absorbing boundary condition (14). The functions for the free surface elevation η and the free surface potential ϕ have kinks at the waterlines of the ships and steep gradients in the vicinity of the ship hulls, which can give spurious oscillation in the numerical approximations of these functions and their derivatives. The derivatives are therefore approximated by the higher-order Weighted Essentially Non-Oscillatory (WENO) method and the advective terms in the kinematic (10) and (11) are evaluated by the Hamilton-Jacobi WENO method with Lax-Friedrics flux Osher and Shu [1991].

Parallel GPU Implementation

The development of massively parallel GPUs within recent years has made GPUs attractive for a broad range of computationally intense applications within scientific computing and engineering. We find that GPUs are ideal for computing the ship wave and ship-ship interactions, not only because the numerical model fits the parallel architecture well, but also because the ocean waves should eventually be rendered in the marine simulator by the GPU itself. We have used an in-house developed library, based on the CUDA programming model, for assembling the solver. All finite difference operators are implemented as flexible-order matrix-free operators, in order to exploit that only a few different stencil coefficients are required. In this way we are able to keep memory consumption and memory access low, two important properties for obtaining good performance on GPU systems. Preliminary results indicate reasonable performance speedups compared to similar CPU-based implementations in the order of one to two magnitudes. Future work is still to prove if the real time calculations are possible given the currently proposed numerical model and hardware.

Tests, Validation and Applications

The model needs to calculate ship waves and ship forces and moments accurately. The kinematic and dynamic properties of the linearizations (Kelvin-Neumann and double body flow) and approximation of ship hull boundary conditions (flat ship and pressure distribution) are investigated. The convergence of the numerical approximations are tested to determine the optimal order of finite difference and WENO approximations and the required mesh resolution. Different types of finite difference meshes are investigated. In particular meshes with clustering of points close to the free surface, close to the ships and dynamic adaption based on error indicators.

An example of a convergence study is seen in figure 1 where the wave resistance is calculated on a Series 60 CB 0.6 model for Froude numbers in the range $Fr = 0.1, \dots, 0.35$, Toda et al. [1992]. The number of points in the vertical direction are increased $N_z = 9, 10, 13, 17$ while the number of points in the horizontal directions are fixed at $(N_x, N_y) = (513, 385)$. Other numerical parameters are: 4th order finite difference, 2nd/4th order WENO and 4th order Runge-Kutta. The mesh points are clustered close to the free surface and around the ship. It is seen that at least 17 points are needed in the vertical direction and that the best agreement with the experimental data is found in the range $Fr = 0.1, \dots, 0.25$. At the workshop more result on convergence and computational efficiency will be presented and most importantly a ship-ship interaction validation case with a tug besides a tanker Simonsen et al. [2012].

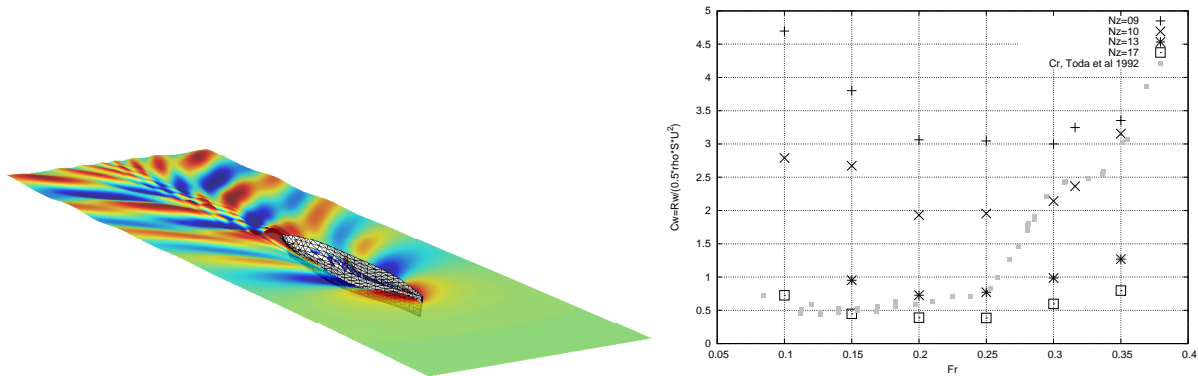


Figure 1: Left: A Series 60 CB 0.6 model at steady forward speed with $Fr = 0.316$. Right: Wave resistance for as a function of the Froude number and different N_z .

Conclusion

We have presented a linear free surface flow model for real time calculation of ship waves and ship-ship interaction in a full mission marine simulator. The model is currently being implemented for parallel execution on GPU's. In the time ahead it will be verified, tested and optimized. Validation test on forward speed and ship-ship interaction cases are carried out to determine the required numerical resolution and the model accuracy. An example of a convergence study on Series 60 CB 0.6 has been presented and it is seen that at least $(N_x, N_y, N_z) = (513, 385, 17)$ points are required. At the workshop we will present more convergence and performance results along with a ship-ship interaction case between a tug and a tanker.

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