Trapping of time-harmonic waves by freely floating structures consisting of multiple bodies (motionless and/or heaving)

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We study the coupled time-harmonic motion of the following mechanical system: infinitely deep water bounded above by a free surface and an immersed structure formed by a finite number $N \geqslant 2$ of surface-piercing bodies floating freely. The surface tension is neglected and the water motion is irrotational, whereas the motion of the whole system is of small amplitude near equilibrium. The latter assumption allows us to apply the linearized model proposed by John [1], which we write in the matrix form used in [3]. Our aim is to prove the existence of structures whose properties are as follows: (i) there exists a time-harmonic wave mode trapped by such a structure; (ii) some of the structure's bodies (may be none) are motionless, whereas the rest of the bodies (may be none) are heaving at the same frequency as water. Our construction is based on a generalization of the semi-inverse procedure (see [4] for a survey of this technique). The usual semi-inverse procedure was applied in [3] for obtaining infinitely many families such that every member of a certain family is a single *motionless* body floating freely; the corresponding trapped wave mode is common for each of these families. In the case of heaving bodies, a novel moment is that a subsidiary (with respect to the arising eigenvalue problem) condition guaranteeing the equilibrium is essential for verifying that the constructed structure floats freely.

1 Statement of the problem for multi-body structures

Let the Cartesian coordinates $(\boldsymbol{x}, y)$, $\boldsymbol{x} = (x_1, x_2)$, be such that the y-axis is directed upwards, whereas the $\boldsymbol{x}$-plane coincides with the mean free surface of water. By $\widehat{B}_k$, $k = 1, \dots, N$, we denote the domain occupied by the k-th body in its equilibrium position; its immersed part $B_k = \widehat{B}_k \cap \mathbb{R}^3_- \neq \emptyset$ can consist of several connected components (see fig. 1), $\mathbb{R}^3_- = \{\boldsymbol{x} \in \mathbb{R}^2, y < 0\}$. Let $B = \cup_{k=1}^{N} B_k$ and $W = \mathbb{R}^3_- \setminus \overline{B}$ be the structure's submerged part and the water domain, respectively. The latter is supposed to be simply connected, but B has at least $N \geqslant 2$ connected components (see fig. 2); their number is greater than N if bodies like that shown in fig. 1 are present. Furthermore, $S_k = \partial B_k \cap \mathbb{R}^3_-$, $D_k = \widehat{B}_k \cap \{y = 0\}$, $F = \{y = 0\} \setminus (\cup_{k=1}^{N} \overline{D}_k)$ (see fig. 2), and $\boldsymbol{n}$ is the unit normal to ∂W directed to the exterior of W.

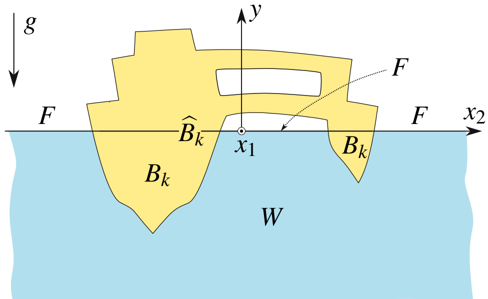

Figure 1: A body with two immersed parts.

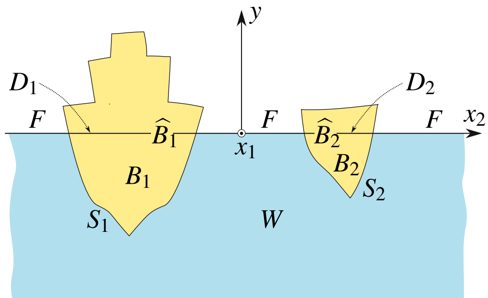

Figure 2: Definition sketch for two bodies.

The linearized time-dependent setting of the problem for $N = 1$ was obtained in [1] (see [2] for its matrix form). In our case, the system's motion is described by the following first-order variables: the velocity potential $\Phi(\boldsymbol{x}, y; t)$ and N vectors $\boldsymbol{q}^{(k)}(t) \in \mathbb{R}^6$. The latter characterize the motion of the centre of mass of the k-th body about its given rest position $\big(\boldsymbol{x}_0^{(k)}, y_0^{(k)}\big)$; namely, the horizontal and vertical displacements are $q_1^{(k)}$, $q_2^{(k)}$ and $q_4^{(k)}$, respectively, whereas $q_3^{(k)}$ and $q_5^{(k)}$, $q_6^{(k)}$ are the angles of rotation about the axes that go through $\big(\boldsymbol{x}_0^{(k)}, y_0^{(k)}\big)$ parallel to the y- and x_1-, x_2-axes, respectively. For studying time-harmonic oscillations of the radian frequency $\omega > 0$ we put $\big(\Phi(\boldsymbol{x}, y, t), \boldsymbol{q}^{(1)}(t), \dots, \boldsymbol{q}^{(N)}(t)\big) = \operatorname{Re}\big\{e^{-i\omega t}\big(\varphi(\boldsymbol{x}, y), i\boldsymbol{\chi}^{(1)}, \dots, i\boldsymbol{\chi}^{(N)}\big)\big\}$. Then the bounded complex-valued function φ and the vectors $\boldsymbol{\chi}^{(k)} \in \mathbb{C}^6$, $k = 1, \dots, N$, must satisfy the following problem:

$$\nabla^2 \varphi = 0 \ \text{ in } W; \quad \partial_y \varphi - \nu \varphi = 0 \ \text{ on } F; \quad \int_{W \cap \{|\boldsymbol{x}| = a\}} \big|\partial_{|\boldsymbol{x}|} \varphi - i\nu\varphi\big|^2 \, ds = o(1) \ \text{ as } a \to \infty; \tag{1}$$

$$\partial_{\boldsymbol{n}} \varphi = \omega\, \boldsymbol{n}^{\mathsf{T}} \boldsymbol{D}_k \boldsymbol{\chi}^{(k)} \ \text{ on } S_k; \quad \omega^2 \boldsymbol{E}_k \boldsymbol{\chi}^{(k)} = -\omega \int_{S_k} \varphi\, \boldsymbol{D}_k^{\mathsf{T}} \boldsymbol{n} \, ds + g\, \boldsymbol{K}_k \boldsymbol{\chi}^{(k)} \quad \text{for every } k = 1, \dots, N. \tag{2}$$

Here $\nabla = (\partial_{x_1}, \partial_{x_2}, \partial_y)$; $\nu = \omega^2/g$ and g is the acceleration due to gravity (see fig. 1); $^{\mathsf{T}}$ denotes the matrix transposition and $\boldsymbol{D}_k = \boldsymbol{D}\big(\boldsymbol{x}-\boldsymbol{x}_0^{(k)}, y-y_0^{(k)}\big)$, where $\boldsymbol{D}(\boldsymbol{x},y) = \begin{bmatrix} 1 & 0 & x_2 & 0 & 0 & -y \\ 0 & 1 & -x_1 & 0 & y & 0 \\ 0 & 0 & 0 & 1 & -x_2 & x_1 \end{bmatrix}$. Two matrices in the equation of motion of the k-th body [the 2nd condition (2)] are as follows: $\boldsymbol{E}_k = \rho_0^{-1} \int_{\widehat{B}_k} \rho_k(\boldsymbol{x},y)\, \boldsymbol{D}_k^{\mathsf{T}}(\boldsymbol{x},y)\, \boldsymbol{D}_k(\boldsymbol{x},y)\, \mathrm{d}\boldsymbol{x}\, \mathrm{d}y$;

$$\boldsymbol{K}_k = \begin{bmatrix} \mathbb{O}_3 & \mathbb{O}_3 \\ \mathbb{O}_3 & \boldsymbol{K}'_k \end{bmatrix}, \text{ where } \boldsymbol{K}'_k = \begin{bmatrix} I^{D_k} & I_2^{D_k} & -I_1^{D_k} \\ I_2^{D_k} & I_{22}^{D_k}+I_y^{B_k} & -I_{12}^{D_k} \\ -I_1^{D_k} & -I_{12}^{D_k} & I_{11}^{D_k}+I_y^{B_k} \end{bmatrix} \text{ and } \mathbb{O}_3 \text{ is the } 3\times 3 \text{ null matrix; } I^{D_k} = \int_{D_k} \mathrm{d}\boldsymbol{x},$$

$$I_i^{D_k} = \int_{D_k} \big(x_i - x_{0i}^{(k)}\big)\, \mathrm{d}\boldsymbol{x},\ I_{ij}^{D_k} = \int_{D_k} \big(x_i - x_{0i}^{(k)}\big)\big(x_j - x_{0j}^{(k)}\big)\, \mathrm{d}\boldsymbol{x},\ I_y^{B_k} = \int_{B_k} \big(y - y_0^{(k)}\big)\, \mathrm{d}\boldsymbol{x}\, \mathrm{d}y,\ \ i,j = 1,2.$$

The elements of $\boldsymbol{E}_k$ are various moments of the whole body $\widehat{B}_k$ (see [3]); $\rho_k(\boldsymbol{x},y) \geqslant 0$ is the distribution of density within the k-th body and $\rho_0 > 0$ is the constant density of water. All these 6×6 matrices are symmetric and positive definite. Every matrix $\boldsymbol{K}_k$ (it is related to buoyancy of the k-th body; see [1]) is symmetric as well.

Problem (1)–(2) must be augmented by conditions guaranteeing equilibrium for each of N floating bodies: • $I_\rho^{\widehat{B}_k} = \rho_0^{-1} \int_{\widehat{B}_k} \rho_k(\boldsymbol{x},y)\, \mathrm{d}\boldsymbol{x}\, \mathrm{d}y = \int_{B_k} \mathrm{d}\boldsymbol{x}\, \mathrm{d}y$ (Archimedes' law for the k-th body); • $\int_{B_k} \big(x_i - x_{0i}^{(k)}\big)\, \mathrm{d}\boldsymbol{x}\, \mathrm{d}y = 0$, $i = 1,2$ (the centre of buoyancy of the k-th body and that of its mass lie on the same vertical line); • $\boldsymbol{K}'_k$ is a positive definite matrix (this implies the stability of the equilibrium position for the k-th body; see [1, § 2.4]).

The boundedness of φ implies that $\nabla\varphi$ decays as $y \to -\infty$; the radiation condition [the 3rd condition (1)] means that waves are outgoing at infinity. In the same way as in [3], one proves the following assertion about the energy of the coupled system.

Let $(\varphi, \chi^{(1)}, \dots, \chi^{(N)})$ *satisfy problem* (1)–(2). *Then* φ *belongs to the Sobolev space* $H^1(W)$ *and* $\int_F |\varphi|^2\, \mathrm{d}\boldsymbol{x} < \infty$, *that is, the kinetic and potential energy of the water motion is finite. Moreover, the equality*

$$\int_W |\nabla\varphi|^2\, \mathrm{d}\boldsymbol{x}\, \mathrm{d}y + \omega^2 \sum_{k=1}^N \overline{\chi}^{(k)\mathsf{T}} \boldsymbol{E}_k \chi^{(k)} = \nu \int_F |\varphi|^2\, \mathrm{d}\boldsymbol{x} + g \sum_{k=1}^N \overline{\chi}^{(k)\mathsf{T}} \boldsymbol{K}_k \chi^{(k)}$$

expresses the equipartition of energy of the system's motion.

In what follows, we suppose that φ is a real element of $H^1(W)$ and $\chi^{(k)} \in \mathbb{R}^6$, $k = 1, \dots, N$, because the real and imaginary part of any solution separately satisfy problem (1)–(2). Such a real solution $\big(\varphi, \chi^{(1)}, \dots, \chi^{(N)}\big)$ is called a *trapped mode*, whereas the corresponding value of ω is called a *trapping frequency*.

2 Trapped modes with axisymmetric velocity fields

Since we are interested in trapped modes with axisymmetric velocity fields, we consider structures consisting of axisymmetric bodies (motionless and/or heaving). In order to find them we apply a modification of the semi-inverse procedure applied in [3]; this modification uses not only a special choice of the velocity potential, but also a particular form of the vectors $\chi^{(k)}$. Namely, we fix $\omega > 0$ and seek a trapped mode in the form $\big(\omega\nu^{-2}\varphi_*, \boldsymbol{d}\chi_*^{(1)}, \dots, \boldsymbol{d}\chi_*^{(N)}\big)$, where $\boldsymbol{d}$ is the diagonal matrix $\mathrm{diag}\{\nu^{-1}, \nu^{-1}, 1, \nu^{-1}, 1, 1\}$ and φ_*, $\chi_*^{(k)}$ are dimensionless. We put

$$\varphi_*(\nu|\boldsymbol{x}|, \nu y) = 2\int_0^\infty (k\cos k\nu y + \sin k\nu y)\, P(k\nu|\boldsymbol{x}|)\, Q(k\nu r) \frac{k^2\, \mathrm{d}k}{k^2+1} - \pi^2 \mathrm{e}^{\nu y} L(\nu|\boldsymbol{x}|)\, M(\nu r), \quad y \leqslant 0. \tag{3}$$

Here $P = I_0$, $Q = K_1$, $L = J_0$ and $M = Y_1$ for $|\boldsymbol{x}| < r$, whereas $P = K_0$, $Q = I_1$, $L = Y_0$ and $M = J_1$ for $|\boldsymbol{x}| > r$. The value $r > 0$ is specified below; Y_0, Y_1 are the Neumann functions; J_0, J_1 and I_0, I_1, K_0, K_1 denote the standard and modified Bessel functions of the indicated orders, respectively. It is easy to check that φ_* is a non-dimensional harmonic function in $\mathbb{R}^3_-$. Moreover, the boundary condition $\partial_y\varphi_* - \nu\varphi_* = 0$ holds on $\partial\mathbb{R}^3_- \setminus \{|\boldsymbol{x}| = r, y = 0\}$, and φ_* has a singularity on the excluded circumference. Putting $r = r_m = \nu^{-1} j_{1,m}$, where $j_{1,m}$ is the m-th positive zero of J_1, we get that the last term in (3) vanishes for $|\boldsymbol{x}| > r$. We denote φ_* with $r = r_m$ by φ_m and see that $\varphi_m \in H^1(W)$ when W is a domain obtained by removing some neighbourhood of $\{|\boldsymbol{x}| = r_m, y = 0\}$ from $\mathbb{R}^3_-$. Therefore, φ_m can serve as the first component of a trapped mode provided m and a water domain are chosen properly. It is essential that the choice will also depend on the N-tuple $\big(\chi_*^{(1)}, \dots, \chi_*^{(N)}\big)$. We take $\chi_*^{(k)}$ in the form $(0,0,0,H_k,0,0)^{\mathsf{T}}$, and so $H_k = 0$ for a motionless body and $H_k > 0$ for a heaving body.

In order to define a water domain we use the Stokes stream function ψ_m corresponding to φ_m through the following relations:

$$\partial_{|\boldsymbol{x}|}\varphi_m = -(\nu|\boldsymbol{x}|)^{-1}\partial_y\psi_m, \quad \partial_y\varphi_m = (\nu|\boldsymbol{x}|)^{-1}\partial_{|\boldsymbol{x}|}\psi_m, \tag{4}$$

thus getting $\psi_m(\nu|\boldsymbol{x}|,\nu y) = -\pi^2\nu|\boldsymbol{x}|e^{\nu y} J_1(\nu|\boldsymbol{x}|)Y_1(j_{1,m}) - 2\nu|\boldsymbol{x}|\Psi(\nu|\boldsymbol{x}|,\nu r_m,\nu y), \quad |\boldsymbol{x}| < r_m,\ y \leqslant 0,$ and

$$\psi_m(\nu|\boldsymbol{x}|,\nu y) = -2\nu|\boldsymbol{x}|\Psi(\nu r_m,\nu|\boldsymbol{x}|,\nu y), \quad |\boldsymbol{x}| > r_m,\ y \leqslant 0,$$

where $\Psi(\sigma,\tau,\eta) = \int_0^\infty (k\sin k\eta - \cos k\eta)\, I_1(k\sigma)\, K_1(k\tau)\, \frac{k^2\,\mathrm{d}k}{k^2+1}$ and the constant of integration is chosen so that $\psi_m(\nu|\boldsymbol{x}|,\nu y) \to 0$ as $\nu^2(|\boldsymbol{x}|^2+y^2) \to \infty$.

Relations (4) yield that $\partial_{\boldsymbol{n}}\varphi_m = 0$ on every surface in $\mathbb{R}^3_-$, where $\psi_m = \mathrm{const}$. Some properties of streamlines $\psi_m(\nu|\boldsymbol{x}|,\nu y) = v$ for various values of v will be used below. In particular, all these lines are smooth in $\{|\boldsymbol{x}| > 0,\, y < 0\}$; if $v \neq 0$, then the line's end-points belong to the boundary of this quadrant, whereas one end can be at infinity when $v = 0$; a streamline emanates from every point on the half-axis $\{\nu|\boldsymbol{x}| > 0, \nu y = 0\}$ and the only exceptions are $(j_{1,m}, 0)$ and those points, where $\psi_m(\nu|\boldsymbol{x}|,0)$ attains its local extrema. Moreover, $\psi_m(\nu|\boldsymbol{x}|,\nu y) \to +\infty$ as $\nu^2[(|\boldsymbol{x}|-r_m)^2+y^2] \to 0$ and $y \leqslant 0$.

3 Construction of freely floating multi-body structures

3.1. Motionless structures. The above-listed properties of ψ_m, $m = 1,2,\dots$, allow us to consider every bounded surface in $\mathbb{R}^3_-$ on which $\psi_m = \mathrm{const}$ (the only exceptions are unbounded nodal surfaces) as the rigid wetted boundary of a motionless axisymmetric body semi-immersed into water (see [3] for details in the case $N = 1$). Fig. 3 (b) shows the vertical radial cross-section of 2 such bodies defined by $\psi_2(\nu|\boldsymbol{x}|,\nu y)$; this function can also be used for defining 3 bodies like that located in the middle below the body with 2 immersed parts. We even have $m < N$ in this example, but m must be sufficiently large for guaranteeing $N-1$ changes of sign for $\psi_m(\nu|\boldsymbol{x}|,0)$ when $0 < \nu|\boldsymbol{x}| < j_{1,N}$ (this yields the existence of N bodies).

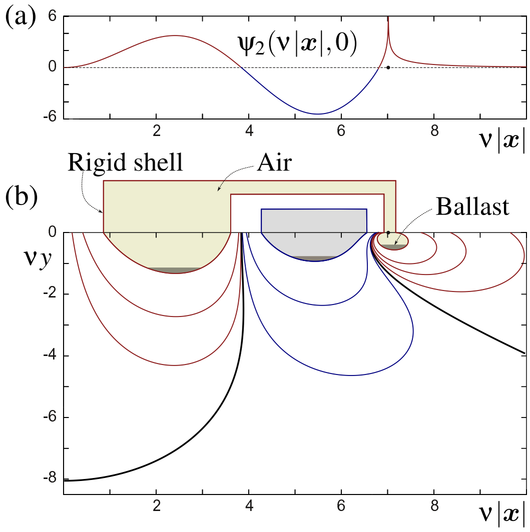

Figure 3: (a) The trace $\psi_2(\nu|\boldsymbol{x}|,0)$. (b) Streamlines $\psi_2(\nu|\boldsymbol{x}|,\nu y) = v$ for various values of v; nodal lines ($v = 0$) are bold. Straight segments show how wetted surfaces are connected above the free surface to form 2 bodies. Darkly shaded layers show the ballast guaranteeing that the bodies are in equilibrium.

Thus, using ψ_2 we defined a motionless structure and now it is necessary to check that it is freely floating. (In the general case, we have to show that $(\varphi_m, \mathbf{0}, \dots, \mathbf{0})$, where $\mathbf{0}$ (the zero element of $\mathbb{R}^6$) is repeated N times, is a trapped mode for a structure of this type.) For this purpose it remains to verify that the following $6N$ equalities hold [see the 2nd equation (2)]: $\int_{S_k} \varphi \boldsymbol{D}_k^{\mathsf{T}} \boldsymbol{n}\, \mathrm{d}s = 0$, $k = 1,\dots,N$. As in the case of a single body, $5N$ of these integrals vanish when all surfaces S_k are axisymmetric (see details in [3], Appendix). Therefore, it remains to show that $\int_{S_k} \varphi_m \partial_{\boldsymbol{n}} y\, \mathrm{d}s = 0$, $k = 1,\dots,N$, when $\partial_{\boldsymbol{n}}\varphi_m = 0$ on S_k. Let the constructed bodies be numbered so that $\widehat{B}_N$ contains the singularity circumference $\{|\boldsymbol{x}| = r_m, y = 0\}$ inside. (It is the doubly immersed body in fig. 3.) First, we apply the method used in [3], Appendix, for proving that $\int_{S_N} \varphi_m \partial_{\boldsymbol{n}} y\, \mathrm{d}s = 0$. It involves the 2nd Green's formula written for the functions φ_m and $y + \nu^{-1}$ in the domain $(\mathbb{R}^3_- \setminus \overline{B}_N) \cap C_{\rho,\delta}$; here $C_{\rho,\delta} = \{|\boldsymbol{x}| < \rho, -\delta < y < 0\}$ and $\rho, \delta > 0$ are taken so that $B_N \subset C_{\rho,\delta}$. Then the required result follows from the boundary conditions, the behaviour of φ_m as $y \to -\infty$ and the Riemann–Lebesgue lemma applied as $\delta \to +\infty$. Then the same procedure yields the result for every $k = 1,\dots,N-1$, but the domain must be changed to $(\mathbb{R}^3_- \setminus \overline{B_N \cup B_k}) \cap C_{\rho,\delta}$ and the fact obtained on the previous step must be taken into account.

Of course, the ballast density within each body is to be chosen so that the conditions guaranteeing the body's equilibrium are fulfilled (see [3], § 3.3, for details).

3.2. Heaving structures. Let us turn to constructing N-body trapping structures heaving so that $H_k = H > 0$ in $\chi_*^{(k)}$ for all k. For this purpose we modify our method outlined in sect. 3.1 as follows. We require a structure to be formed by bodies for which every S_k is a surface of constant level

$$\psi_m^{(H)}(\nu|\boldsymbol{x}|,\nu y) = \mathrm{const}, \quad \text{where} \quad \psi_m^{(H)}(\nu|\boldsymbol{x}|,\nu y) = \psi_m(\nu|\boldsymbol{x}|,\nu y) - (\nu|\boldsymbol{x}|)^2 H/2. \tag{5}$$

If H is sufficiently small, then these level surfaces have properties similar to those listed at the end of sect. 2. Hence relations (4) imply that $\partial_{\boldsymbol{n}}(\varphi_m - H\nu y) = 0$ on every $S_k \subset \mathbb{R}^3_-$, $k = 1,\dots,N$, and so the 1st condition (2) describes the heave motion of the whole structure. Again we choose S_N so that it separates the singularity of $\psi_m^{(H)}$ from infinity.

Fig. 4 (b) shows the vertical radial cross-section of 2 such bodies defined by $\psi_1^{(H)}(\nu|\boldsymbol{x}|,\nu y)$ with $H = 0.1$. In this example of a heaving structure defined with the help of $\psi_1^{(H)}$, we have $m < N$ (cf. sect. 3.1); however, generally, m must be sufficiently large and H must be sufficiently small to guarantee the existence of N bodies.

It remains to verify the equations of motion [the 2nd condition (2)] for the constructed heaving structure which are as follows for axisymmetric bodies:

$$\nu H I_\rho^{\widehat{B}_k} = -\int_{S_k} \varphi_1 \, \partial_{\boldsymbol{n}} y \, \mathrm{d}s + H I^{D_k}, \quad k = 1,2; \qquad (6)$$

we restrict ourselves by the case shown in fig. 4 (b). In order to check (6) we begin with applying the 2nd Green's formula to φ_1 and $y + \nu^{-1}$ in $(\mathbb{R}^3_- \setminus \overline{B}_2) \cap C_{\rho,\delta}$, where positive ρ, δ are taken so that $B_2 \subset C_{\rho,\delta}$. As in sect. 3.1, considerations based on the boundary conditions, the behaviour of φ_1 as $y \to -\infty$ and the Riemann–Lebesgue lemma with $\delta \to +\infty$ are applicable, but now we obtain that

$$\int_{S_2} \varphi_1 \partial_{\boldsymbol{n}} y \, \mathrm{d}s = H\nu \int_{S_2} (y + \nu^{-1}) \, \partial_{\boldsymbol{n}} y \, \mathrm{d}s = -H\nu \int_{B_2} \mathrm{d}\boldsymbol{x}\mathrm{d}y + H I^{D_2}.$$

Substituting this into (6) with $k = 2$, we see that (6) is true because it reduces to Archimedes' law for $\widehat{B}_2$. Then the same procedure yields the result for $k = 1$, but the domain must be changed to $(\mathbb{R}^3_- \setminus \overline{B_1 \cup B_2}) \cap C_{\rho,\delta}$ and the fact obtained on the previous step must be taken into account as well as Archimedes' law for $\widehat{B}_1$.

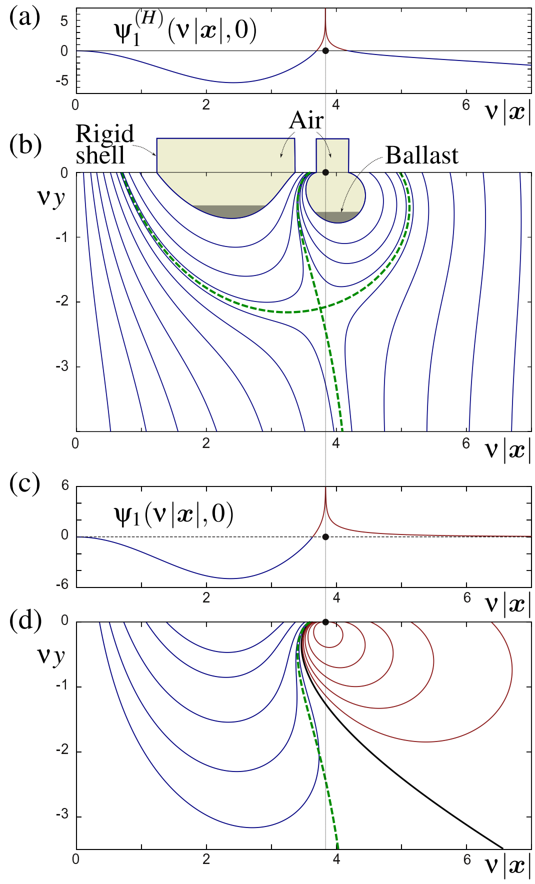

Figure 4: (a) The trace $\psi_1^{(H)}(\nu|\boldsymbol{x}|,0)$ with $H = 0.1$. (b) Streamlines $\psi_1^{(H)}(\nu|\boldsymbol{x}|,\nu y) = v$ are plotted for $H = 0.1$ and various $v \leqslant 0$; the nodal line ($v = 0$) serves as S_2 (the wetted boundary of the right body); straight segments connect wetted surfaces above the free surface to form two bodies; two dashed lines ($v \approx -0.9464$) separate four different families of level lines. (c) The trace $\psi_1(\nu|\boldsymbol{x}|,0)$. (d) The dashed line is taken from (b); streamlines $\psi_1(\nu|\boldsymbol{x}|,\nu y) = v$ are plotted for various values of v; the nodal line ($v = 0$) is bold.

3.3. Structures formed by motionless and heaving bodies. First, we consider $(\omega\nu^{-2}\varphi_1, \mathbf{0}, \boldsymbol{d}\boldsymbol{\chi}_2)$, where $\boldsymbol{\chi}_2 = (0,0,0,H,0,0)^\mathsf{T}$. It is clear that this triplet serves as a mode trapped by an axisymmetric structure formed by the heaving body $\widehat{B}_2$ defined in sect. 3.2 [the right body in fig. 4 (b)] and any motionless body obtained by rotation of the following domain: it is the union of the rectangle adjacent to the domain enclosed between the $\nu|\boldsymbol{x}|$-axis and a streamline of ψ_1 located to the left of the dashed curve in fig. 4 (d).

Now we take $(\omega\nu^{-2}\varphi_1, \boldsymbol{d}\boldsymbol{\chi}_1, \mathbf{0})$, where $\boldsymbol{\chi}_1$ is the same as $\boldsymbol{\chi}_2$ above. Hence this triplet serves as the mode trapped by an axisymmetric structure formed by the heaving body $\widehat{B}_1$ defined in sect. 3.2 [the left body in fig. 4 (b)] and any motionless body obtained by rotation of the following domain: it is the union of the rectangle adjacent to the domain enclosed between the $\nu|\boldsymbol{x}|$-axis and a streamline of ψ_1 located to the right of the dashed curve in fig. 4 (d).

4 Conclusion Generalizing considerations of sect. 3, one obtains the following

Theorem. *For any $\omega > 0$ and arbitrary integers $N \geqslant 2$ and n, $0 \leqslant n \leqslant N$, there exists an axisymmetric trapping structure that consists of N bodies and has the following properties:* (i) $\big(\omega\nu^{-2}\varphi_m, \boldsymbol{d}\boldsymbol{\chi}_*^{(1)}, \dots, \boldsymbol{d}\boldsymbol{\chi}_*^{(N)}\big)$ *is the corresponding trapped mode (see its description in sect. 2), where m is sufficiently large and H_k in $\boldsymbol{\chi}_*^{(k)}$ is an arbitrary non-negative number lesser than $2\pi^2 |Y_1(j_{1,m}) J_1(j'_{1,N})| / j'_{1,N-1}$ ($j'_{1,\ell}$ is the ℓ-th positive zero of J'_1);* (ii) *all bodies float freely, n of them are motionless and described by properly chosen level lines of ψ_m, the remaining $N - n$ bodies are heaving and described by chosen in a special way level lines of functions* (5).

References:

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