Wagner-type models of water impact with separation for a finite wedge

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Symmetric two-dimensional problem of a rigid wedge entering water at a constant speed is considered. The wedge is of finite height. The second stage of the flow, when the wedge is already completely wetted and a cavity is formed behind the wedge, is of concern. In particular, we need to calculate the hydrodynamic force acting on the wedge during this stage. It is known that the Wagner-type models quite well predict the force evolution during the impact stage, when the wedge is only partially wetted. The idea of the present approach is to introduce a fictitious body of infinite height, which consists of the wedge surface and the streamlines of the cavity behind the wedge computed for unbounded fluid region, and to calculate the hydrodynamic force by using the Wagner-type models applied to such a body.

1. Introduction

Wagner-type models include the Original Wagner Model (OWM), Modified Logvinovich Model (MLM) and the Generalised Wagner Model (GWM). There are also several other advanced models such as Second-order Wagner Model (Korobkin, 2007 and Oliver, 2007) and Combined Asymptotic Model, which are not considered in this study, thought they could be promising as it will be clear from the presented results.

Wagner-type models were derived for bodies with small deadrise angle. However, GWM and MLM can also predict the force $F(t)$ acting on entering bodies with moderate deadrise angles. Calculations by the Wagner-type models will be performed for the symmetric wedge with deadrise angle of 30 degrees. The wedge is of finite height with the upper base of length $B = 0.5$m. The wedge enters water of infinite depth at constant speed V. Gravity and surface tension are not included in the models. The results by the Wagner-type models are compared with the numerical results by S. Seng (2012) for the same wedge and conditions of the impact. Seng did his calculations by OpenFoam and presented the force coefficient $F_v = F/(\rho V^2)$ in terms of non-dimensional penetration depth h/H, where ρ is the water density, $h = Vt$ is the wedge displacement and $H = \frac{1}{2}B\tan(30^o)$ is the distance between the keel and the base of the wedge. It was known that the coefficient F_v is independent of the entry speed for the infinite wedge. The calculations by S. Seng showed that this coefficient is approximately independent of the wedge speed also during the stage with separation from the knuckles of the wedge. This result by Seng encouraged us to develop an approximate model of impact with separation where the body surface is continued with the cavity streamlines making a body of infinite length. The streamlines represent the free surface of the cavity behind a body placed in the uniform steady flow. These streamlines are treated as rigid within the present models.

2. Shape of the cavity behind symmetric wedge in uniform current

The analysis from the book by Gurevich (1978) was used to determine the shape of the cavity in the parametric form

$$x = \frac{B}{2} + \frac{B}{2æ(\gamma)}X(s), \quad y = H + \frac{B}{2æ(\gamma)}Y(s),$$

where s is the non-dimensional curvilinear coordinate along the free surface of the cavity, γ is the deadrise angle of the wedge and

$$æ(\gamma) = 8\cos\gamma \int_0^1 \frac{u^\sigma(1-u^2)\mathrm{d}u}{(1+u^2)^3}, \quad \sigma = \frac{2\gamma}{\pi}, \quad k = 1-\sigma.$$

The functions $X(s)$ and $Y(s)$ are obtained by integration of the following differential equations

$$\frac{\mathrm{d}X}{\mathrm{d}s} = \cos[k\phi(s)+\gamma], \qquad \frac{\mathrm{d}Y}{\mathrm{d}s} = \sin[k\phi(s)+\gamma], \quad \phi(s) = \arccos[(2s+1)^{-\frac{1}{2}}]$$

with the initial conditions $X(0) = 0$ and $Y(0) = 0$. The wedge and its continuation with the cavity surface are shown in Figure 1 for $x > 0$.

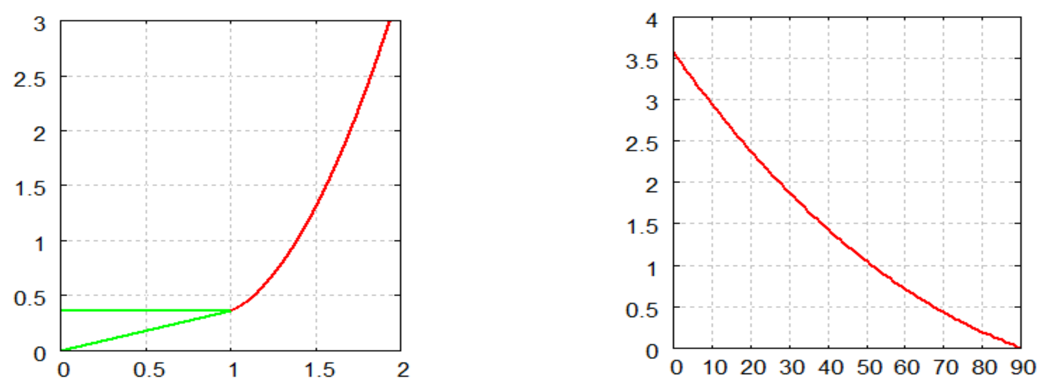

Figure 1. Shape of the wedge continued with the streamline in non-dimensional coordinates $x/(0.5B)$ and $y/(0.5B)$ (left) and the function $æ(\gamma)$, where γ is in degrees (right).

The length of a small streamline element is $\delta l = B\delta s/(2æ)$. Calculations of the streamline for the wedge with deadrise angle 30 degrees were done with $\delta s = 0.01$ which corresponds to the length δl of elements equal approximately to 1mm. Coordinates of the points on the wedge surface, where $x < 0.25$m, and on the streamline, where $0.25\text{m} < x < 0.389$m, with the distance between points about 1mm were saved in a data file which was used in the following calculations by the Wagner-type model representing a fictitious symmetric body with the equation $y = y_b(x)$, which penetrates water at a constant speed. In total, 500 elements represent the shape.

3. Original Wagner Model

In the OWM, the rigid shape $y = y_b(x)$ is in contact with water over the interval $|x| < c(t)$, where $c(t)$ and $\dot{c}/\dot{h}$ are computed by using the Wagner equation

$$\int_0^c \frac{y_b(x)\mathrm{d}x}{\sqrt{c^2-x^2}} = \frac{\pi}{2}h(t), \qquad \frac{\dot{c}}{\dot{h}} = \frac{\pi c}{2S(c)}, \quad S(c) = \int_0^c \frac{y_b'(x)x\mathrm{d}x}{\sqrt{c^2-x^2}}. \tag{1}$$

The integrals are evaluated by using linear approximation of the shape function $y_b(x)$ between nodes of the elements. This approach can be used for any $h(t)$. In the present calculations,

$h = Vt$. The hydrodynamic pressure in the wetted area of the fictitious body is given by the linearised Bernoulli equation

$$p(x,t) = \rho V^2 \frac{\dot{c}}{\dot{h}} \frac{c}{\sqrt{c^2 - x^2}}.$$

The pressure is integrated over the wetted area, if $c < B/2$, and over the wedge surface, if $c > B/2$, with the result

$$F_v = \frac{2c\dot{c}}{\dot{h}} K(c),$$

where $K(c) = \frac{\pi}{2}$ for $c < B/2$ and $K(c) = \arcsin(B/2c)$ for $c > B/2$. The calculated force coefficient F_v is shown in Figure 2 with respect to the non-dimensional penetration depth Vt/H by the solid line. The dashed line shows the result by OpenFoam (Seng, 2012). It is seen that the OWM overpredicts the force during the impact stage, which is well known drawback of this model. However, OWM predicts correctly the duration of the impact stage and the force evolution during the separation stage. The latter result is less obvious.

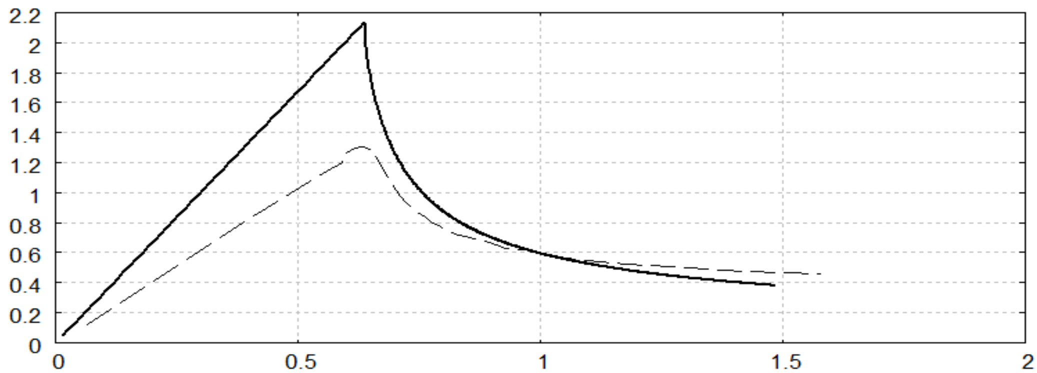

Figure 2. The force coefficient F_v as a function of non-dimensional penetration for the finite wedge of 30 degrees by the OWM (solid line) and by OpenFoam (dashed line).

4. Modified Logvinovich Model

Within MLM, we use the same shape of the fictitious body and equations (1) for the size of the wetted area and the speed of its expansion as in OWM. The force acting on the entering wedge is calculated by integrating the pressure only over the wedge surface. The formula for the force acting on an entering wedge by MLM was published by Korobkin (2004). This formula was updated to account for the cavity behind the wedge. First we calculate the coordinate $\tilde{x}(t)$ of the point on the wedge surface, at which the pressure is zero, if such a point exists. We introduce $\theta = \tilde{x}(t)/c(t)$ and χ such that $\theta = \sqrt{1-\chi^2}$. Then we consider the equation $\chi^2 \sin^2\gamma - 2\chi\dot{c}/\dot{h} + \cos^2\gamma = 0$ which is valid if the point $\tilde{x}(t)$ is on the wedge surface. If the latter equation has no real solution, it means that the point $\tilde{x}(t)$ is on the surface of the cavity behind the wedge. In this case we set $\tilde{x}(t) = B/2$. The force coefficient in MLM is given by

$$F_v = 2\frac{\dot{c}}{\dot{h}} \arcsin\theta - \frac{1}{2}\cos^2\gamma \log\left(\frac{1+\theta}{1-\theta}\right) - \theta\sin^2\gamma.$$

This force coefficient is shown in Figure 3 by the solid line. The dashed line shows the result by OpenFoam (Seng, 2012). It is seen that the MLM predicts the coefficient quite well during the impact stage but gives a wrong prediction at the separation stage. It is interesting to note that OWM predicts the force at the separation stage better than MLM.

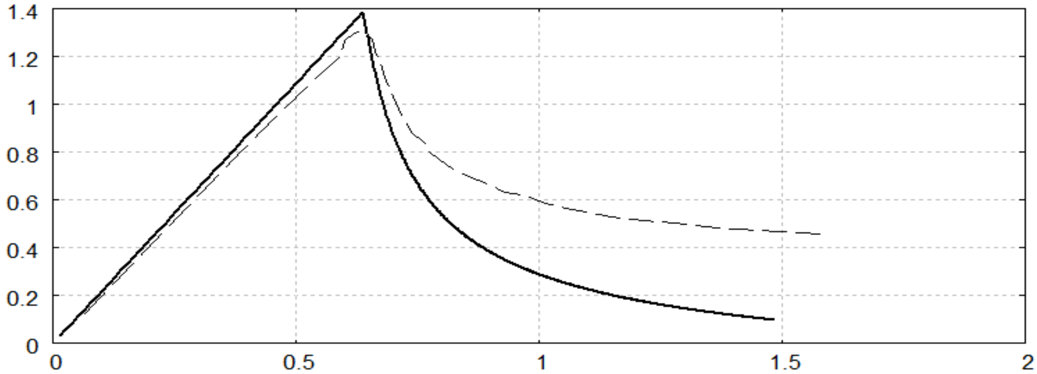

Figure 3. The force coefficient F_v as a function of non-dimensional penetration for the finite wedge of 30 degrees by the MLM (solid line) and by OpenFoam (dashed line).

5. Generalised Wagner Model

Calculations of the force acting on the wedge were performed by the GWM code (Korobkin, 2011) combined with the present model of separation (Figure 4).

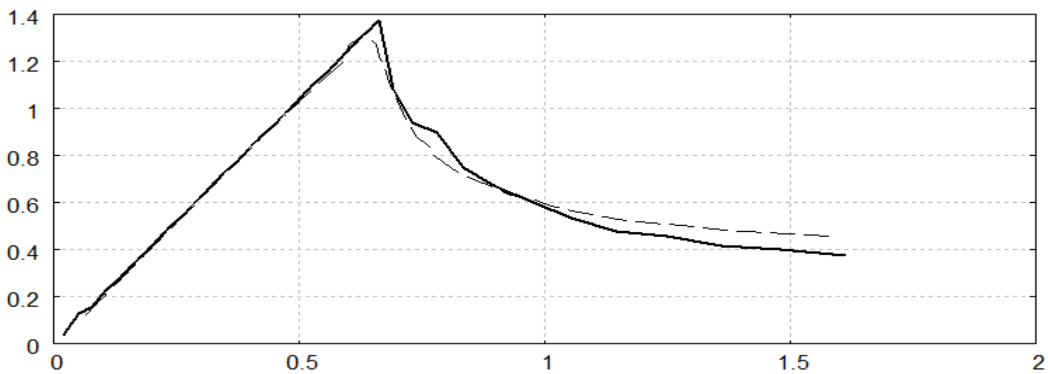

Figure 4. The force coefficient F_v as a function of non-dimensional penetration for the finite wedge of 30 degrees by the GWM (solid line) and by OpenFoam (dashed line).

It is seen that the GWM combined with the separation model suggested above predicts the force quite well during both the impact and separation stages. However, several problems with the GWM code were detected at the transition from the impact to separation stage.

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