HYDRODYNAMIC INTERACTIONS AMONG MULTIPLE CYLINDRICAL OWC' DEVICES RESTRAINED IN REGULAR WAVES

by

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1 INTRODUCTION

In the present contribution, the diffraction and pressure-dependent-radiation problems for an array of OWC's devices restrained in regular waves with finite water depth is investigated in order to highlight the effect of the hydrodynamic interactions among the devices on the wave loading and wave energy extraction. The problem of the hydrodynamic interaction among neighbouring OWC devices may be important in evaluating the absorbed wave energy by each device in the array since the hydrodynamic characteristics of each member of a multi OWC's configuration may differ from the ones of a standalone, isolated device due to interaction phenomena. The latter are evaluated using the single OWC's hydrodynamic characteristics and the physical idea of method of multiple scattering (Twersky's [1952], Okhusu [1974], Mavrakos [1991]). Air volume flow rate, inner pressure and absorbed wave power are parametrically evaluated for several distances among devices in an array consisting of three devices.

2 FORMULATION OF THE HYDRODYNAMIC PROBLEM

We consider a stationary group of N rigid vertical axisymmetric oscillating water column devices excited by a plane periodic wave of amplitude $H/2$, frequency ω and wave number k propagating in water of finite water depth d. The outer and inner radii of each device q, q=1, 2,…, N, are a_q, b_q, respectively, whereas the distance between the bottom of the q device and sea bed is denoted by h_q and the distance between the devices by l (Figs. 1, 2). It is assumed small amplitude, inviscid, incompressible and irrotational flow so that linear potential theory can be employed. A global Cartiesian co–ordinate system O–XYZ with origin on the sea bed and its vertical axis OZ directed positive upwards is used. Moreover, N local cylindrical co-ordinate systems (r_q, θ_q, z_q), q = 1, 2, …, N are defined with origins on the sea bottom and their vertical axes pointing upwards and coinciding with the vertical axis of symmetry of the q device.

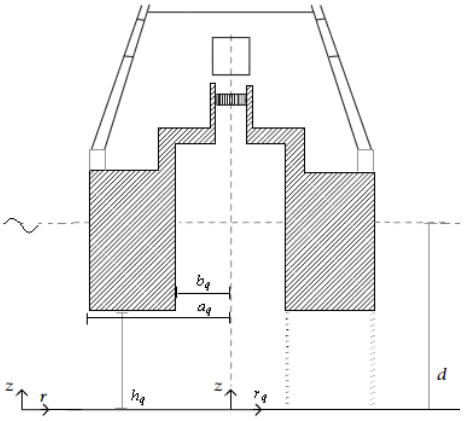

Fig.1. Schematic representation of a typical OWC device

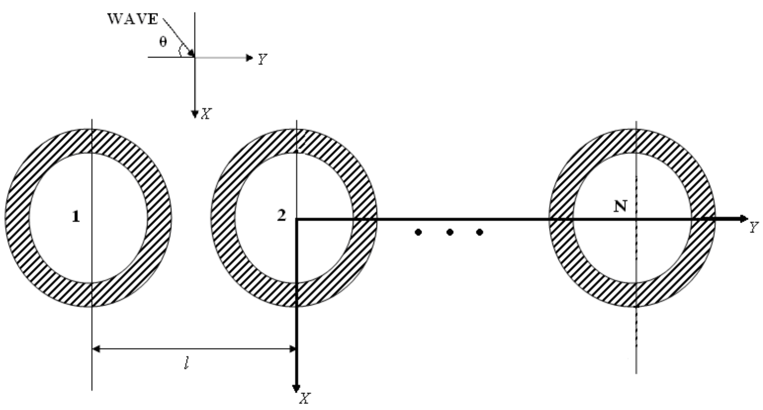

Fig.2. Stationary group of N identical OWC devices in a row

The fluid flow around the q=1,2,…,N device expressed in its own co-ordinate system can be described by the potential function: $\Phi^q(r_q, \theta_q, z_q; t) = \mathrm{Re}\{\phi^q(r_q, \theta_q, z_q) \cdot e^{-i\omega t}\}$ with

$$\phi^q\left(r_q,\theta_q,z_q\right)=\phi_0^q\left(r_q,\theta_q,z_q\right)+\phi_7^q\left(r_q,\theta_q,z_q\right)+\sum_{p=1}^{N}\phi_P^{qp}\left(r_q,\theta_q,z_q\right) \tag{1}$$

Here, ϕ_0^q is the undisturbed incident linear wave potential; ϕ_7^q is the scattered wave potential with the q device being considered fixed in the wave field with open duct and ϕ_P^{qp} is the radiation potential on body q due to time harmonic oscillating pressure head, $P_{in}^p=\mathrm{Re}\left\{p_{in0}^p\cdot e^{-i\omega t}\right\}$, in the chamber for the p device which is considered fixed in otherwise calm water.

The potentials φ_j^l ($l\equiv q,\ qp$; j=0, 7, P; p=1, 2,…, N) are solutions of Laplace's equation in the entire fluid domain and satisfy, besides the zero normal fluid velocity on the sea bottom (z = 0) and on the mean body's wetted surface, S_0^q, the following boundary conditions on the inner and outer free surface, $z = d$:

$$\omega^2\varphi_j^l-g\frac{\partial\varphi_j^l}{\partial z}=\begin{Bmatrix} 0 & \text{for} \quad r_q\geq a_q;\ l\equiv q \text{ or } qp;\ j=0,7,P \\ 0 & \text{for} \quad 0\leq r_q\leq b_q;\ l\equiv q;\ j=0,7 \\ -\delta_{qp}i\omega p_{in0}^q/\rho & \text{for} \quad 0\leq r_q\leq b_q;\ l\equiv qp;\ j=P \end{Bmatrix} \tag{2}$$

Finally, a radiation condition for ϕ^q has to be imposed which ensures that propagating disturbances are outgoing.

As mentioned in the introduction, the method for evaluating the fluid flow around the q device in the array relies on single device hydrodynamic characteristics and accounts for the hydrodynamic interactions among the devices using the physical idea of multiple scattering. Each device of the configuration scatters / radiates waves towards the others, which in turn scatter waves contributing to the excitation of the initial device and so on. In this way, by superposing the incident wave potential and various orders of propagating and evanescent modes that are scattered and radiated by the cylindrical array elements, Fourier – Bessel series representations of the total wave field around each device q in the array may be obtained in its own cylindrical co-ordinate system. In doing so, use is made of the Bessel function addition theorem to express the radiated /scattered wave field from the pth device in the reference co–ordinate system $\left(r_q,\theta_q,z\right)$. By the way of example, the wave field around the body q of the arrangement due to inner pressure variation in p device, φ_P^{qp}, expressed in q-th body co-ordinate system is given by:

$$\varphi_P^{qp}(r_q,\theta_q,z)=\frac{p_{in0}^p}{i\omega\rho}\sum_{m=-\infty}^{\infty}\Psi_{Pm,j}^{qp}(r_q,z)e^{im\theta_q} \tag{3}$$

Where for the outer fluid domain, $r_{q\geq}a_q$, , $0\leq z\leq d$ the function $\Psi_{Pm,j}^{qp}$ is given by:

$$\Psi_{Pm,j}^{qp}(r_q,z)=\delta_{pq}\sum_{j=0}^{\infty}F_{Pm,j}^{qq}\frac{K_m(a_jr_q)}{K_m(a_ja_q)}Z_j(z)+\sum_{j=0}^{\infty}\left[G_{Pm,j}^{qp}\frac{I_m(a_jr_q)}{I_m(a_ja_q)}+F_{Pm,j}^{qp}\frac{K_m(a_jr_q)}{K_m(a_ja_q)}\right]Z_j(z) \tag{4}$$

The first term in (4) represents the isolated device wave field around the q body due to its own internal pressure variation, the second term denotes the incident wave fields on body q emanating from the scattered fields of the remaining devices considered open and the last term is the scattered wave filed around the q –th device. In (4) I_m and K_m denotes the mth order modified Bessel function of first and second kind, respectively; $F_{Pm,j}^{qq}$ are unknown Fourier coefficients for the radiated waves form the isolated body q with internal pressure head variation, $F_{Pm,j}^{pq}$ Fourier coefficients for the scattered waves from remaining open duct devices; $Z_j(z)=\left\{0.5\left[1+\sin(2a_jd)/\left(2a_jd\right)\right]\right\}^{-1/2}\cos(a_jd)$

Moreover, it holds:

$$G_{Pm,j}^{qp}=\sum_{l=1}^{N}(1-\delta_{lq})\sum_{v=-\infty}^{\infty}i^{m+v}\frac{K_{v-m}(a_j\ell_{qp})I_m(a_ja_q)}{K_v(a_ja_q)}F_{Pv,j}^{lp}e^{i(v-m)\theta_{\ell q}} \tag{5}$$

The time dependent volume flow produced by the oscillating internal water surface in q OWC device, q=1, 2,…, N, is denoted by $Q\left(r_q,\theta_q,z,t\right)=\mathrm{Re}\left[q^q\left(r_q,\theta_q,z\right)\cdot e^{-i\omega t}\right]$

where:

$$q^q=\iint_{S_i^q}u_zdS_i=\iint_{S_i^q}u(r_q,\theta_q,z=d)r_qdr_qd\theta_q=\iint_{S_i^q}\frac{\partial\phi^q}{\partial z}r_qdr_qd\theta_q \tag{6}$$

Here u_z denotes the vertical velocity of the water surface, S_I^q the inner water surface of the *q* device and ϕ^q the velocity potential in *q* device's chamber. It proves convenient to decompose the total volume flow, q^q into two terms associated with the diffraction, q_D^q, and the pressure–dependent radiation problem, q_P^q, as follows:

$$q^q = q_D^q + p_{in0}^q q_P^q = q_D^q - p_{in0}^q \left(B^q - iC^q\right) \tag{7}$$

where B^q and C^q are the corresponding radiation conductance and susceptance, respectively. Assuming uniform pressure distribution inside the chamber, it can be shown that, even though all *m*-modes terms affect the values of the diffraction and radiation potentials, by substituting those potentials in Equation (3) only the modes with $m = 0$ contribute to q_P^q and q_D^q, as in an isolated device case [Mavrakos & Konispoliatis, 2011].
Assuming that the Wells turbine is placed in a duct between the chamber and the outer atmosphere, of the *q* device, and it is represented by a real valued pneumatic admittance g_T^q, then the total volume flow *Q* in *q* device, is equal to [Evans & Porter; 1997, Falnes; 2002, Falca˜o; 2002]:

$$Q(t) = g_T^q \cdot P_{in}^q(t) \tag{8}$$

The averaged value of the power absorbed from the waves over one wave period from the device *q* is obtained from [Evans & Porter; 1997]:

$$W^q = \frac{1}{2}\mathrm{Re}\left[\overline{q_D^q} \cdot p_{in0}^q\right] - \frac{1}{2}\overline{p_{in0}^q} \cdot B^q \cdot p_{in0}^q = \frac{1}{8}\overline{q_D^q}\left(B^q\right)^{-1} q_D^q \left\{1 - p_{in0}^q \left|g_T^q - \left(B^q + iC^q\right)\right|^2 \overline{p_{in0}^q} \Big/ p_{in0}^q \left|g_T^q + \left(B^q - iC^q\right)\right|^2 \overline{p_{in0}^q}\right\} \tag{9}$$

The capture width ℓ^q of the *q* device is the ratio of the power absorbed by the device to the available power per unit crest length of the incident wave [Martins-Rivas and Mei; 2009], i.e.

$$\ell^q = 2 \cdot W^q \Big/ \left(\rho g (H/2)^2 C_g\right) \tag{10}$$

C_g being the group velocity of the incident wave. The absorbed power (Eq.9) takes a maximum value of [Evans & Porter; 1997]:

$$W_{\max}^q = \overline{q_D^q}\left(B^q\right)^{-1} q_D^q \Big/ 8 \tag{11}$$

corresponding to an optimum inner pressure head $p_{in0_{optim}}^q = \left(B^q\right)^{-1} q_D^q \big/ 2$.

If g_T^q, from Eq.8, reaches an optimum value, $g_{T_{optim}}^q = \left|B^q - iC^q\right|$ then the maximum value of the absorbed energy by device *q*, is:

$$W_{\max}^q = \frac{1}{8}\overline{q_D^q}\left(B^q\right)^{-1} q_D^q \left[1 - p_{in0}^q \left(2\left(g_{T_{optim}}^q\right)^2 + i\left(B^q C^q - C^q B^q\right) - 2 g_{T_{optim}}^q B^q\right)\overline{p_{in0}^q} \Big/ p_{in0}^q \left(2\left(g_{T_{optim}}^q\right)^2 + i\left(B^q C^q - C^q B^q\right) + 2 g_{T_{optim}}^q B^q\right)\overline{p_{in0}^q}\right] \tag{12}$$

The resulting maximum capture width is then given by [Evans & Porter; 1997]:

$$\ell_{\max}^q = 2 \cdot W_{\max}^q \Big/ \left(\rho g (H/2)^2 C_g\right) \tag{13}$$

3 NUMERICAL RESULTS

In figure 3 we examined an array of three same oscillating water column devices in a row. The variations of the value of the dimensionless parameter $v = \left|q_P^q\right|$, for $a_q / d = 1/2$, $b_q / a_q = 3/4$, $h_q / d = 3/4$, $l / a_q = 4$, 8, 12, for the middle device of the array, are being plotted. Next, we investigate how the direction of the wave propagation affects the value of the inner pressure in each of the above OWC devices. In figures 4 and 5 the dimensionless modulus of the inner air pressure is plotted in each device of the above configuration, $q = 1,2,3$, for incident wave angles $\theta = 0$, π/2, $l / a_q = 4$ and $g_T^q = 0.1 m^5/(Ns)$. Finally, in figure 6 the value of $\left(\ell_{\max}^1 + \ell_{\max}^2 + \ell_{\max}^3\right) / \left(3\ell_{\max}^{isolated}\right)$ for the three devices of the above configurations is presented, for $l / a_q = 4$, 8, 12. Here $\ell_{\max}^j$, *j*=1, 2, 3 denotes the maximum capture width in the *j*–th device for optimum inner pressure head, $p_{in0_{optim}}^q$, whereas $\ell_{\max}^{isolated}$ is its single device counterpart for optimum inner pressure value. From the depicted results the effect of the hydrodynamic interactions on the alteration of the single OWC device characteristics is shown.

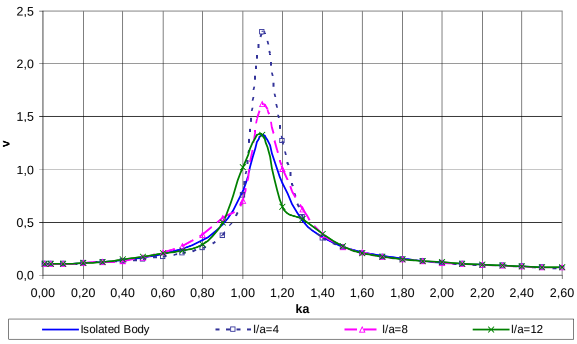

Fig.3. Modulus of $v = q_p^q \big/ (10\cdot\omega\cdot b_q^2/(\rho\cdot g))$ versus $k\cdot a_q$, for $l = 4a_q, 8a_q, 12a_q$

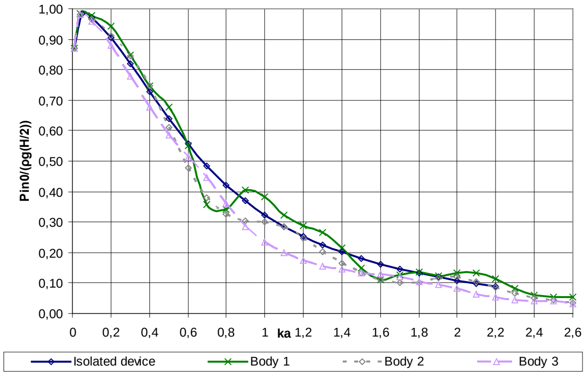

Fig.4. Modulus of inner air pressure versus $k\cdot a_q$, for θ=0

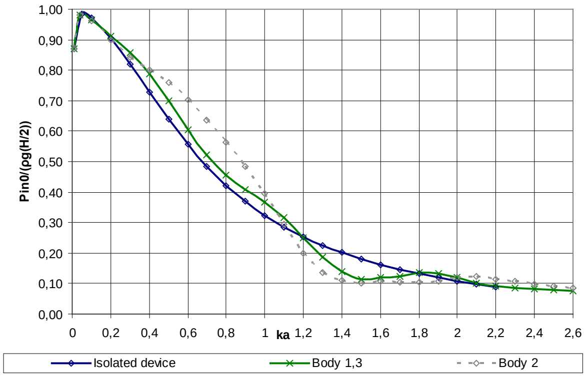

Fig.5. Modulus of inner air pressure versus $k\cdot a_q$, for θ=π/2

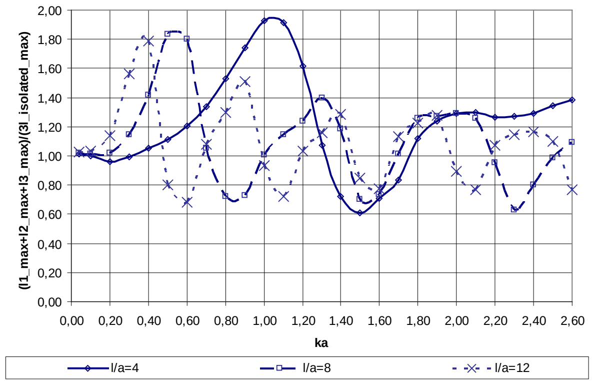

Fig.6. The value of $\left(\ell^1_{max} + \ell^2_{max} + \ell^3_{max}\right)/\left(3\ell^{isolated}_{max}\right)$ versus $k\cdot a$ for $l = 4a_q, 8a_q, 12a_q$ and $\theta = 0$ and optimum value of inner pressure head

4 ACKNOWLEDGEMENTS

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