Two-dimensional unsteady motion of a smooth body over a water surface with multiple collisions and subsequent rebounds from this surface is considered. This problem is studied in connection with landing of aircraft on the water and flights of wing-in-ground-effect vehicles (WIG) in close proximity to the water surface. In the present analysis, the water is of small depth and its flow caused by the impacts is described by the shallow water model. Free body motions with subsequent water impacts and the corresponding hydrodynamic loads are of primary interest. In the past only a single impact between the water surface and a smooth body was studied [1-8]. These investigations revealed that the landing of a smooth body with high horizontal speed on water typically leads to the rebound of the body followed by skipping along the water surface. It was shown that the body rotation plays an important role in impacts and rebounds of a free body. This regime of the body motion with multiple impacts on the water surface may cause vibrations of the fuselage with significant bending stresses or even lead to the overturning of aircraft. It is interesting to note that regimes of the landing of a free body with a single impact on the water surface and the subsequent smooth planing along the water were not detected in the calculations. Motions of a free body are determined by its orientation and speeds before the impact, and the hydrodynamic loads acting on the body surface during its contact with the water. Between the subsequent impact, it is assumed that the body motions are governed by gravity only. Aerodynamic forces are not included in the model.

To find regimes with smooth landing on water, it is suggested to include restoring forces which reduce the vertical motion of the body and also its rotation. In addition, damping forces are included. Parameters of the restoring and damping forces can be considered as control parameters which could be selected to achieve smooth landing. It can be expected that wrong choice of the parameters may lead to instability of the body motions with increasing amplitudes of its vertical displacement and rotation. If the parameters do not vary in time, the control system should be considered as passive. Active control assumes that there are time-varying external forces which are changed by the pilot.
The configuration under investigation in this study is shown in Figure 1. A smooth body is represented by an elliptic cylinder whose center is connected by a vertical rod to the carriage of zero mass. The carriage can move only horizontally with the same horizontal speed as the speed of the body. The rod can move vertically with respect to the carriage. The distance between the carriage and the center of mass of the body is controlled by a linear spring with the rigidity coefficient $k_y$. In the equilibrium, the center of mass is at distance $y^*$ from the bottom of the liquid layer and the weight of the body is balanced by the restoring force of the spring. During the body motion along the water surface, the vertical restoring force acting on the body is $-k_y(y_0(t) - y_*)$, where $y_0(t)$ is the current vertical coordinate of the center of mass and $t$ is time. In addition, a vertical damping force, $-c_y\dot{y}_0(t)$, is included. This damping force could be due to a viscous friction between the carriage and the vertical rod. The rod is pivoted at the center of mass. The elliptic cylinder can rotate around this pivot with the angle between its axis and the horizontal being $\alpha(t)$. The axis of the cylinder is connected to the rod with a torsional spring of stiffness $k_\alpha$ and damping coefficient $c_\alpha$. The spring force is given by $-k_\alpha(\alpha(t) - \alpha_*) - c_\alpha\dot{\alpha}(t)$, where $\alpha_*$ is an equilibrium inclination angle of the cylinder. Initially, the carriage with the attached cylinder moves at a horizontal speed $\dot{x}_0(0)$ with the compressed vertical spring, $y_0 > y_*$, and equilibrium angle of inclination $\alpha_*$. Then the vertical spring is released, the cylinder moves down towards the water surface and touches it tangentially at $t = 0$.

Note that a body landing on water without control of its vertical motion ($k_y = 0$, $c_y = 0$) but with the restoring moment, $k_\alpha > 0$, can be considered as a body with internal gyroscopic stabilizer. Such stabilizers can be also used to reduce roll motions of ship. Experiments showed that a stone skips successfully over water if the stone’s rotational speed exceeds a certain value. The spinning stabilizes the stone much like an internal gyroscope stabilizes a body’s landing on water.

The body motions are governed by the equations

$$m\ddot{y}_0 = F_y(t) - k_y(y_0(t) - y_*)(t) - c_y\dot{y}_0(t), \quad m\ddot{x}_0 = F_x(t),$$

$$J\ddot{\alpha} = M(t) - k_\alpha(\alpha(t) - \alpha_*) - c_\alpha\dot{\alpha}(t),$$

where $m$ is the mass of the body, $J$ is the moment of inertia, $F_x(t)$ and $F_y(t)$ are the horizontal and vertical components of the hydrodynamic force acting on the body surface in the contact region, $M(t)$ is the moment of the hydrodynamic force. Dot stands for the time derivative. Note that the gravity force is balanced by the vertical spring and does not appear in the equations. The hydrodynamic forces and moment are calculated by integrating the pressure distribution along the wetted part of the body surface [4,5]. The hydrodynamic pressure is calculated within the shallow water approximation when the cylinder is in contact with water.

![Fig. 2](image-url) Four typical positions of the body during its impact on shallow water with high horizontal speed.
Hydrodynamic loads are set zero during the time intervals when the body is above the water surface. Interaction stages are separated by these time intervals. It is assumed that each impact occurs on the undisturbed water surface. Each interaction stage is subdivided into the impact phase (Fig. 2a), when the body enters the water layer and the wetted area expands in both directions with jetting at both edges of the wetted region [1], and the planing phase (Fig. 2b-d), when the jetting occurs only at the leading edge and the water surface separates smoothly from the body surface at the trailing edge [5]. The position of the separation point is determined by using the Brillouin-Villat condition which requires that both the pressure and its tangential derivative are equal to zero at this point. The position of the leading edge is determined by using the matching conditions which come from the conditions of mass, momentum and energy conservations at this point [2-6]. The equations of the body motions, the conditions at both the leading and trailing edges of the wetted region and the equations of the shallow water flow are reduced to the system of nine nonlinear differential equations and one algebraic equation for the position of the separation point. These equations are integrated numerically in time with the initial conditions specifying the body motions before each impact.

Fig. 3 Subsequent positions of the elliptic cylinder at different time instants are shown by thick lines when the cylinder is in contact with water and by thin lines when it is above water. The time step is 20ms.

Fig. 4 Inclination angle $\alpha(t)$ of the elliptic cylinder (a), the vertical (b) and horizontal (c) components of the body velocity are shown as functions of time. The time is in seconds, the speed in m/s and the inclination angle is in degrees. Thick lines are for $\alpha_0 = 6^\circ$, $k_\alpha = 10000$ N/rad and thin lines are for $k_\alpha = 0$. Dashed lines in both cases show motion of the cylinder without interaction with the fluid.
Calculations were performed in the dimensional variables for the elliptic cylinder with semi-axis \( a = 0.5 \) m and \( b = 0.125 \) m. The results of the calculations presented in Figures 3 and 4 are for the cylinder of mass 150 kg, water density \( \rho = 1000 \text{ kg/m}^3 \) and the depth of the water layer \( h = 0.05 \) m. The initial values of the vertical speed is \(-1 \) m/s, horizontal speed 10 m/s. The initial inclination angle of the cylinder is 6°. The rigidity of the vertical spring \( k_y \) is zero, the coefficient of the torsional spring \( k_\alpha \) is 10000 N/rad, and the equilibrium inclination angle \( \alpha^* \) of the cylinder is 6°. The damping coefficients \( c_y \) and \( c_\alpha \) are set zero. The time step of integration is \( 10^{-4} \) s.

The calculations were performed for the time interval of 1s duration. During this short interval the cylinder impacts on water and bounces back five times. Positions of the body with time step 20 ms are shown in Figure 3. Note the different vertical and horizontal scales of the axis. In Figure 4, the results for the inclination angle, vertical speed of the body and its horizontal speed without the restoring moment are shown with thin lines, and with this moment by thick lines. It is seen that without the restoring moment the inclination angle changes significantly during the first second of the calculations varying from the initial 6° to zero and then to 24°. The restoring moment reduces the variation of the angle to 3° around the equilibrium value. Note that the restoring moment does not change the motions of the body during initial 0.1s. The restoring moment makes the interaction stages shorter and the motions rather periodic with the period about 0.2 s. This implies that this moment governs the motions. The restoring moment does not change significantly the reduction rate of the horizontal speed of the body. This speed is reduced by 15% during the first second of the body interaction with the liquid layer.

The results of the calculations revealed that a smooth landing on water can be achieved by reducing both the horizontal and vertical speeds of the aircraft and keeping its inclination angle small. In the case of emergency landing the horizontal speed cannot be reduced but the vertical speed and the aircraft angle can be controlled. The horizontal speed can be as high as 50 m/s which makes the landing aircraft skip over the water surface. Hydrodynamic loads can be used to reduce the horizontal speed. To increase the horizontal component of the loads, the penetration depth should be big. This can be achieved by increasing the inclination angle and the time interval during which the fuselage is in contact with water. The rebound of the fuselage should be reduced by controlling the vertical motion and the inclination angle.

To gain the experience with effects of the control parameters on the aircraft emergency landing, a model of multiple impacts has been developed. This model can be used to find optimal control regimes of smooth landing. As a first attempt, it is suggested to determine the control parameters \( k_y(t) \) and \( k_\alpha(t) \) which maximize the duration of the first impact on water for a given set of landing conditions.

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References