# Second-order resonance among an array of two rows of vertical circular cylinders


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## 1. Introduction

Wave trapping in an array of floating bodies has been one of the main topics in the recent workshops. It is a scientifically interesting phenomenon, but also it has practically important implications in that wave trapping could entail large free-surface displacement and/or large forces on the bodies.

Other than first-order wave trapping, some of the recent works are focusing on the second-order free-surface dynamic elevations, e.g. Murakami and Yoshida (1994), Sanada et. al (1997), Malenica et. al (1999), Wang and Wu (2008), Teng and Cong (2012). The arrays subjected to these works are mostly composed of 2~4 cylinders and they all conclude that second-order free-surface displacement could sometimes be relevant and could not be neglected in the design of corresponding structures such as TLPs. Some of the above mentioned works compare their theory with experimental results (Murakami and Yoshida (1994), Sanada et al. (1997)), but they do not necessarily refer to the results from the viewpoint of wave trapping.

On the other hand, in theoretical works (Malenica et. al (1999), Teng and Cong (2012) ) it is suggested that second-order wave trapping takes place at half the wave frequency at which the corresponding linear near-trapped mode occurs.

In the present study, we show that the second-order resonance of dynamic free-surface displacement could in some cases be quite relevant among an array of two rows of vertical cylinders not only in theory but also in real phenomena. Discussion is also made if the second-order wave trapping is manifested at half the wave frequency at which the corresponding linear near-trapped mode occurs as suggested by the works mentioned above.

## 2. Experiment

Water-tank experiments were conducted using an array of two rows of vertical truncated circular cylinders as shown in Figure 1. Four kinds of cylinder arrangement, that is, 2 x 9 cylinders, 2 x 7 cylinders, 2 x 5 cylinders, 2 x 3 cylinders were used. (Figure 1 is the case of 2 x 9 cylinders.) The cylinders were fixed in regular head waves and the water-surface displacements at the origin of the coordinate system were measured.

## 3. Theory

The theory used in the present calculations is not a new one but that presented by Sanada et. al (1997), which extended the theory of Linton and Evans (1990) for the analysis of second-order wave diffraction. The theory is applicable only to bottom-mounted vertical cylinders and therefore, in a strict sense, can not be used for the present cases, in which the cylinders were truncated, but we confirmed through the comparison of first-order free-surface displacements obtained by the theory with those obtained by a theory applicable for truncated cylinders that the theory used in the present calculations give results with acceptable accuracy within the context of the present experiments.

## 4. Results and Discussion

Figures 2 show the experimental and theoretical results on the free-surface displacement at the origin of the coordinate system in 2 x 9, 2 x 7, 2 x 5, 2 x 3 arrays. The horizontal axes of the figures are the period of the incident wave. From these figures, the following facts are observed.

(1) Second-order free-surface displacement could be manifested significantly at certain wave frequency range.

(2) The theoretical results on the second-order surface elevation agree quite well with the experimental results. The second-order peak value can be really as high as that predicted by the theory.

(3) The peak value of the second-order free-surface displacement is enhanced as the number of the cylinders involved increases. It is also observed that the peak period of the second-order free-surface displacement becomes closer to one of the peak periods of the first-order free-surface displacement as the number of the cylinders increases.

In many of resonant phenomena, the peak value of the response could be quite large theoretically, while in reality the peak value is attenuated due to unaccounted damping forces, but in the present case it is surprising that the peak value in reality can be as high as that predicted theoretically, which only accounts for radiation damping as the damping.

As for the suggestion made by some of the former works that the second-order wave trapping is manifested at half the wave frequency at which the corresponding linear near-trapped mode occurs, it seems that it is not the case in the present cases.

The free-surface displacement at two other places (at (-0.165, 0.0), (-0.165, 0.165)) were calculated for the case of 2 x 9 cylinders. The distinct differences from those shown in Figures 2 are:

1. The peak of the first-order free-surface displacement observed in Figures 2 at around T=0.6sec totally disappears at (-0.165, 0.0), (-0.165, 0.165).

2. Though the peak of the second-order displacement observed in Figures 2 also appears at (-0.165, 0.0) and (-0.165, 0.165), the peak values at these two points are not as distinct as that observed in Figures 2.

References

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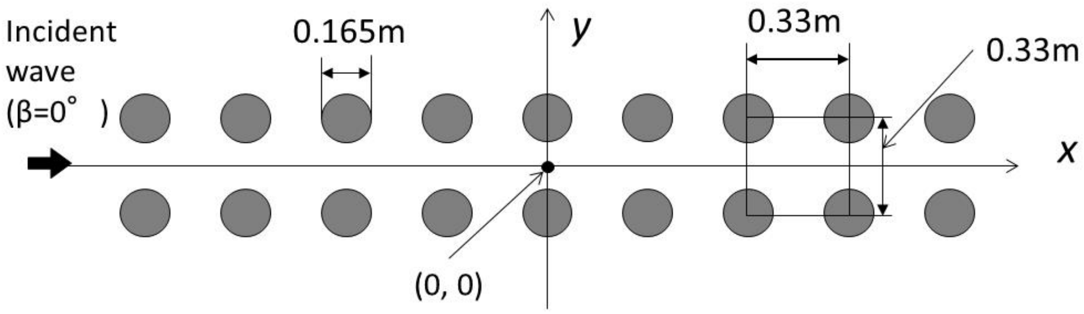


Figure 1 A 2 x 9 array of cylinders

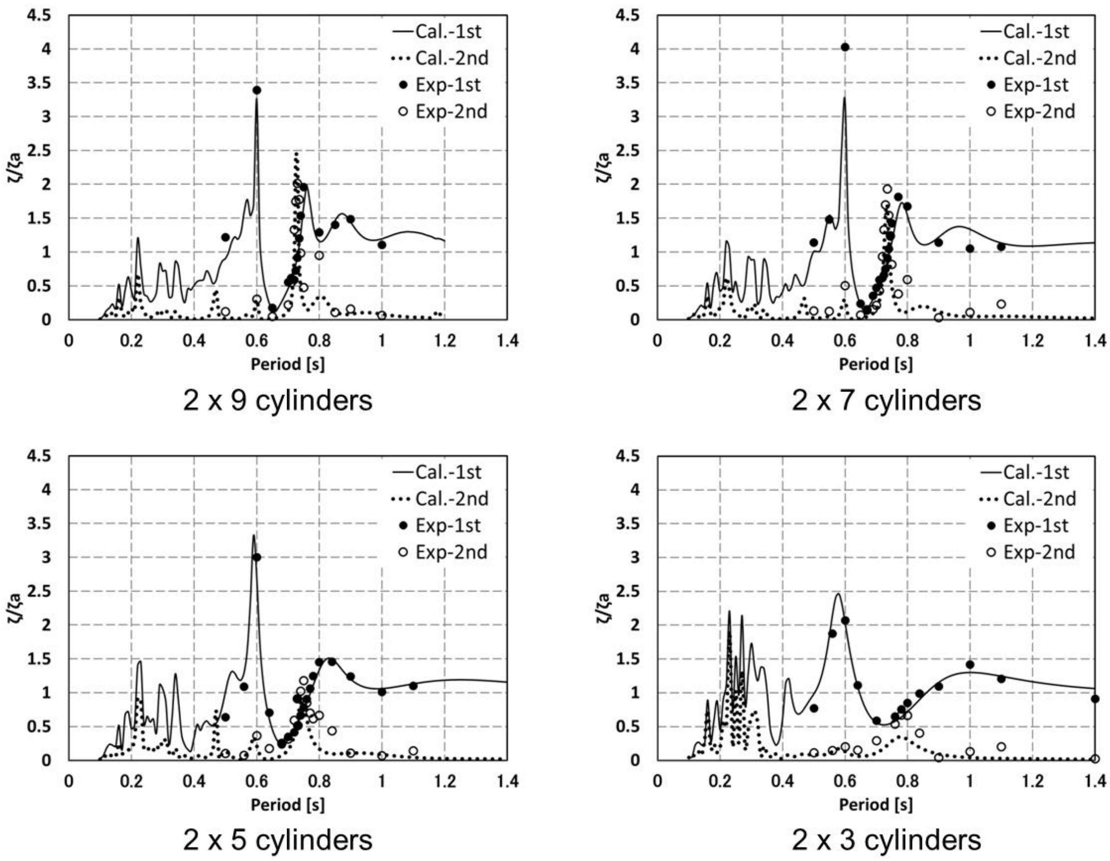


Figure 2 Numerical and experimental results on the 1-st and 2-nd order free-surface displacement among cylinder arrays at (0,0)