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A nonlinear calculations of interfacial waves generated by a moving ship and evaluation of the forces in the dead water problem

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Fridtjof Nansen (1897) was the first to give a sufficiently precise, physical description of *dead water* so that the phenomenon could be given a scientific explanation. The initial systematics of the additional resistance on a ship moving in layered waters is credited to V. Wagnfrid Ekman. He was, by his supervisor Vilhelm Bjerknes, put to perform laboratory experiments on ship generated internal waves and measure the resulting force, and was able to explain the observations made by Nansen, see Ekman (1904). Recently, Ekman's experiments have been reinvestigated using modern techniques, see Mercier et al. (2011).

Linear theories of internal waves made by a moving point source have been derived, see Hudimac (1961), Crapper (1967), Keller and Munk (1970) and the references cited in these papers. They all consider the supersonic case where the source is advancing so fast, that the internal wake behind the source is a narrow v-shaped pattern. Avital and Miloh (1999) studied linear internal waves trapped by a moving vessel, basically the dispersion relation of the wake. Waves generated by a moving source in a two-layer ocean of finite depth have been studied by Yeung and Nguyen (1999). The dead water problem was also discussed in Miloh and Tulin (1988). Miloh et al. (1993) gave numerical solutions for the case of a semi-submersible slender-body (prolate spheroid) moving steadily on the free-surface.

Nonlinear calculations of dead water are scarce. What exists are calculations of the motion caused by singularities or pressure distributions. Nonlinear calculations of the wake and force on a realistic body seem to be nonexistent. We note that a nonlinear theory of interfacial waves generated by moving pressure distributions in super and subcritical conditions has been explored by Parau et al. (2007) with results presented at the 21st Workshop in 2006.

We shall here present calculations of the dead water problem using a truncated version of a fully nonlinear method of a two-layer fluid. The interfacial motion is driven by a realistic body geometry and the force on the body is evaluated. Results are obtained for subcritical, critical and supercritical motion.

Nonlinear two-layer theory

In three dimensions, we consider fully nonlinear interfacial motion of a two-layer fluid. The interfacial waves are generated by the motion of a geometry moving along the upper boundary of the upper fluid. Rigid lids are assumed at the top and bottom of the fluid layer, so there is no coupling to eventual free surface motion, which, however, can be evaluated subsequently. We assume potential flow in each of the layers. Let horizontal coordinates be denoted by $\mathbf{x} = (x_1, x_2)$ and y be vertical coordinate with $y = 0$ at the interface at rest. Let the parameters and functions describing the upper fluid be indexed by 1 and the lower fluid indexed by 0, where ρ_1, h_1, ϕ_1 denote density, layer depth at rest and velocity potential in the upper fluid, respectively, and ρ_0, h_0, ϕ_0 the corresponding quantities in the lower fluid. Thus, the upper boundary of the upper fluid is located at $y = h_1$ and the lower boundary of the lower fluid located at $y = -h_0$. The potentials are assumed to be Laplacian.

The interfacial motion is driven by a body moving horizontally along the top of the upper fluid. The geometry is given by $y - h_1 - \delta(\mathbf{x}, t) = 0$. Assuming a forward motion of the body with speed $U(t)$ along the x_1-axis the kinematic boundary condition reads

$$V_B = \frac{\partial \phi_1}{\partial n}\sqrt{1 + |\nabla \delta|^2} = U\frac{\partial \delta}{\partial x_1}$$

where n denotes the normal pointing out of the body and $\nabla = (\partial/\partial x_1, \partial/\partial x_2)$ horizontal gradient.

The interfacial elevation is determined by $y - \eta(\mathbf{x}, t) = 0$ giving as kinematic condition at the interface I: $\eta_t = V_I = \partial\phi_1/\partial n\,\sqrt{1 + |\nabla\eta|^2}$ where n is the unit normal along the interface pointing into the upper fluid 1. Balance of the pressure along the interface provides the dynamic condition obtained by $(\phi_{0,I} - \mu\phi_{1,I})_t + g'\eta + n.l.t. = 0$ where $\phi_{0,I}$ denotes the value of the potential in the lower fluid at the position of the interface I and $\phi_{1,I}$ value of the potential in the upper fluid at I. Further, $g' = g(1 - \mu)$ denotes the reduced gravity and $\mu = \rho_1/\rho_0$. With $n.l.t.$ we denote all the nonlinear terms which appear from the Bernoulli equation.

Integral equations

The Laplacian potentials are obtained by use of integral equations. In the upper layer, and for a position on the body surface B, we obtain the potential on the body surface, $\phi_B = \phi_1(\mathbf{x}, y = h_1 + \delta(\mathbf{x}, t), t)$,

$$\begin{aligned}&\int_B \left(\frac{1}{r} + \frac{1}{r_1}\right) V_B \mathrm{d}\mathbf{x} + \int_I \left(\frac{1}{r} + \frac{1}{r_1}\right) V_I \mathrm{d}\mathbf{x} \\ &= -2\pi\phi_B' + \int_B \phi_B \frac{\partial}{\partial n}\left(\frac{1}{r} + \frac{1}{r_1}\right)\sqrt{1 + |\nabla\delta|^2}\mathrm{d}\mathbf{x} + \int_I \phi_{1,I}\frac{\partial}{\partial n}\left(\frac{1}{r} + \frac{1}{r_1}\right)\sqrt{1 + |\nabla\eta|^2}\mathrm{d}\mathbf{x}, \quad (1)\end{aligned}$$

which connects ϕ_B, $\phi_{1,I}$, V_I and V_B where the latter is given by the body motion. In (1) a prime in ϕ_B' means $\phi_B(\mathbf{x}', y' = h_1 + \delta(\mathbf{x}', t), t)$. The distances r and r_1 are given by $r^2 = |\mathbf{x} - \mathbf{x}'|^2 + (y - y')^2$ and $r_1^2 = |\mathbf{x} - \mathbf{x}'|^2 + (y + y')^2$ where in (1) $(\mathbf{x}', y')$ is on B and $(\mathbf{x}, y)$ along I and B.

The function $1/r + 1/r_1$ is expanded in the vertical coordinate. For the integral over B we obtain $1/r + 1/r_1 = 1/R + 1/R_1 + (\delta' + \delta)(\partial/\partial(2h_1))(1/R_1) + ...$ where $R_1^2 = R^2 + (2h_1)^2$. The expanded integral equation is inverted by use of Fourier transform (Clamond and Grue, 2001, §6) where $\frac{1}{R_1} = \mathcal{F}^{-1}\left[\frac{2\pi}{k}e^{-\mathrm{i}\mathbf{k}\cdot\mathbf{x}' - 2kh_1}\right]$ is used, $\mathcal{F}$ denotes Fourier transform, $\mathcal{F}^{-1}$ inverse transform, $\mathbf{k}$ and $k = |\mathbf{k}|$ wave numbers. Following Fructus and Grue (2007) studying free surface motion we obtain the potential on the body geometry by successive approximations by $\phi_B = \phi_B^{(1)} + \phi_B^{(2)} + ...$ where

$$\mathcal{F}(\phi_B^{(1)}) = \frac{\mathcal{F}(\phi_{1,I})}{\cosh kh_1} - \frac{\tanh kh_1\,\mathcal{F}(V_B)}{k}$$

$$\mathcal{F}(\phi_B^{(2)}) = \frac{\mathcal{F}(\eta V_I^{(1)})}{\cosh kh_1} + \frac{\mathrm{i}\mathbf{k}\tanh kh_1}{k}\cdot\mathcal{F}(\delta\nabla\phi_B) - \mathcal{F}(\delta V_B)$$

The integral equation with the evaluation point on the interface I gives similar expressions for $V_I = V_I^{(1)} + V_I^{(2)} + ...$

$$\mathcal{F}(V_I^{(1)}) = -k\tanh kh_1\mathcal{F}(\phi_{1,I}) - \frac{\mathcal{F}(V_B)}{\cosh kh_1}$$

$$\mathcal{F}(V_I^{(2)}) = k\tanh kh_1\mathcal{F}(\eta V^{(1)}) - \mathrm{i}\mathbf{k}\cdot\mathcal{F}(\eta\nabla\phi_{1,I}) - \mathrm{i}\mathbf{k}\cdot\frac{\mathcal{F}(\delta\nabla\phi_B)}{\cosh kh_1}$$

Similar expressions are obtained for the normal velocity W_I in the lower layer along I, solving an integral equation for the potential $\phi_{0,I}$. The interfacial motion is obtained integrating the Fourier transformed kinematic and dynamic conditions at the interface, i.e. $\mathcal{F}(\eta)_t = \mathcal{F}(V_I)$ and $\mathcal{F}(\phi_{0,I} - \mu\phi_{1,I})_t + g'\mathcal{F}(\eta) = n.l.t.$ where right hand sides are obtained solving $V_I^{(1)} + V_I^{(2)} + ... = W_I^{(1)} + W_I^{(2)} + ...$ and using the Bernoulli equation. In the present calculations $\rho_1/\rho_0 = 1$ is used and all velocities are scaled by the linear long wave speed c_0.

Calculations

The body is represented by the submerged part of an ellipsoid given by $(\delta/b)^2 + (x_1/a_1)^2 + (x_2/a_2)^2 = 1$ with $\delta < 0$. Calculations of the interface are obtained with a horizontal resolution of 500 by 500 nodes in the critical condition ($U/c_0 = 1$) and 1000 by 250 when $U/c_0 = 6$ (figures 1 and 2). All computations show a region of depression ahead of the body, an uplift of the interface at the aft of the body and then a wake of waves. Calculations of the wave resistance are obtained by integrating the pressure force obtained from the Bernoulli equation over the body surface. A drag coefficient C_R is obtained dividing by $\frac{1}{2}\rho SU^2$ with $S = \pi a_1 a_2$. Figure 2 (right) shows a C_R of about 4 times the skin friction (with $Re \sim 2\times 10^7$).

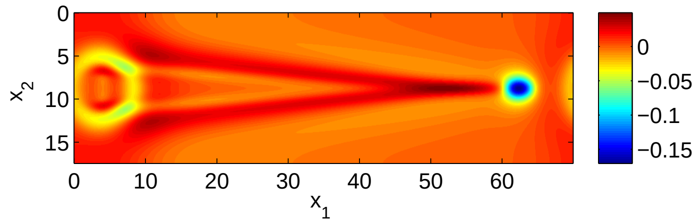

Figure 1: Interfacial elevation η/h_1 with $a_1 = 2.1h_1$, $a_2 = 0.7h_1$, $b = 0.7h_1$, $U/c_0 = 6$, $h_0/h_1 = 6$, horizontal domain of $70h_1$ by $17.5h_1$; final time $10h_1/c_0$. Final body position at $x_1/h_1 = 60.6 \pm 4$.

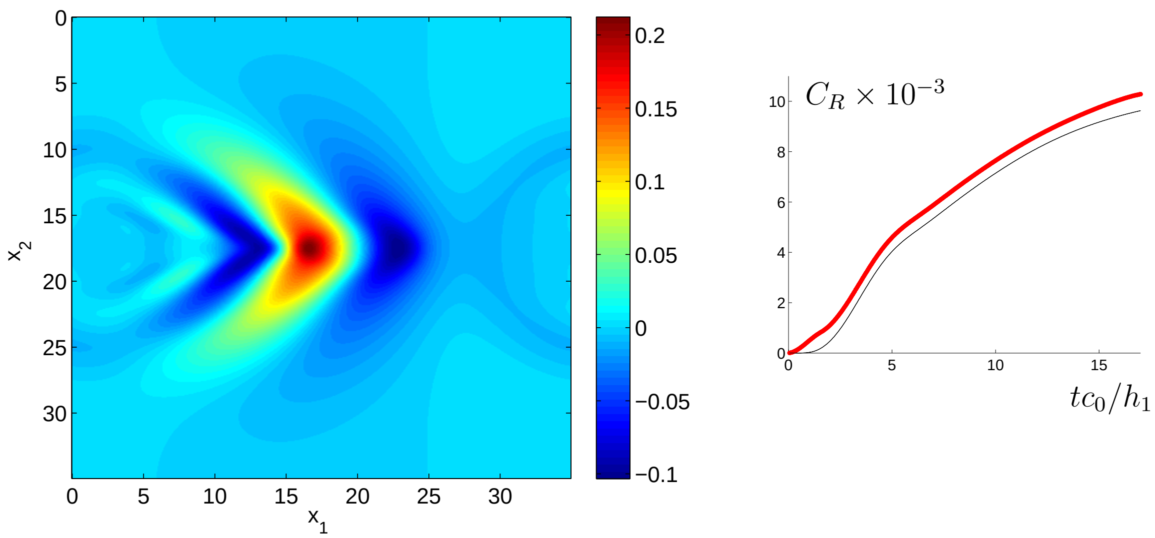

Figure 2: Interfacial elevation η/h_1 (left) and wave resistance (right) with $a_1 = 4h_1$, $a_2 = 1.5h_1$, $b = 0.3h_1$, $U/c_0 = 1$, $h_0/h_1 = 6$, horizontal domain of $35h_1$ by $35h_1$; final time $17h_1/c_0$. Final body position at $x_1/h_1 = 20.4 \pm 4$. Black curve in right plot: domain $50h_1$ by $50h_1$.

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