Capture width for arrays of wave energy converters

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In this paper I use the ”optical theorem for wave power” to calculate the capture width for arrays of wave energy converters. The optical theorem (in analogy with the optical theorem in atomic physics [1]) relates the capture width (CW) for a wave energy converter (WEC) to the polar diagram $f(\theta)$ of the total wave generated by the device due to its unmoving presence in the sea together with its motions and the forces it exerts,

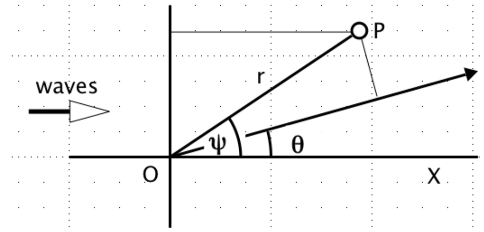
$$CW = \lambda \frac{|\Re f(0)|^2}{\int_{-\pi}^{\pi} |f(\theta)|^2 d\theta} \tag{1}$$

Here $f(\theta)$ gives the complex amplitude of the wave generated by the device as a function of the angle θ relative to the propagation direction of the incident waves, (the forward direction). Equation (1) has been derived independently by many authors and reviewed recently by Farley [2] with references to previous work. In [2] it is established that the relevant angular distribution $f(\theta)$ is for the total wave generated by the device and the forward amplitude $f(0)$ is required in the numerator of (1) rather than the backward amplitude $f(\pi)$. (Not to be confused with the backward amplitude $f(\pi)$ required in the theorem of Newman [3] using waves from the device motions alone omitting the waves generated by the device's unmoving presence). The formula gives the capture width in the best case, when the motions of the WEC have been optimised in amplitude and phase.

It was pointed out in [2] that equation (1) can be applied not only to a single machine but also to calculate the capture by any combination of reasonably localised WEC's in any pattern or array. One only needs to compute the overall polar diagram $f(\theta)$ and then apply (1).

In an array of identical WECs there are two effects to be considered:
a) The wave generated by each WEC propagates to the neighbouring WECs and adds to the incident wave. This is important if the array is closely packed. For an array of heaving point absorbers Mei [4] shows that the radiation damping is increased and the resonance is broadened.
b) At a distance the waves generated by each WEC combine to give the overall polar diagram $f(\theta)$ which determines the capture width. If the array spacing is large compared with the size of the individual elements this is the main effect and the direct interactions (a) can be neglected, as argued for example by Garnaud and Mei [5] who consider a long linear array parallel to the direction of wave propagation.

Here I assume that the components of the array are widely spaced so the modification of the incident wave by radiation from neighbouring elements can be neglected. All effects on the capture width arise from (b). In computing the overall angular distribution the polar diagram of each element should be included. The frequency response of the array will be just the same as the frequency response of the individual elements with no broadening. If all the elements are close together the polar diagram will be the same as that of a single element and the capture width will be unchanged. As the components are moved further apart the polar diagram changes and the capture width rises.



Array theory

Consider an array which is symmetric about a central point O and compute $f(\theta)$ as from this origin. The incident monochromatic waves propagate in the direction OX and θ

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is the angle of the generated wave relative to OX. The wave generated by a WEC element at point P, distance r from O at angle ψ, has a relative phase advance

$$\phi = kr\{cos(\psi - \theta) - cos(\psi)\} = 2\ k\ r\ sin(\psi - \theta/2).\ sin(\theta/2) \tag{2}$$

As the array is symmetric about O, imaginary parts of the generated waves will cancel. The overall amplitude at angle θ is given by the sum over the array of

$$f(\theta) = \Sigma\ cos(\phi) \tag{3}$$

For an array symmetric about O the overall phase will be the same as that from a WEC element at O. In the forward direction with $\theta = 0$, $\phi = 0$ for all elements, so their contributions all add up.

Linear arrays

Results for heaving buoys in a linear array perpendicular to the wave direction, buoy spacing d, are shown in figure 1 for a line of 11 boys and a line of 41 buoys. The CW rises to a peak when $kd = 5.4$ followed by a steep drop when the buoy spacing is one wavelength !! The best result gives a factor 2.57 improvement over the simple sum for 41 buoys, a factor 2.35 for 11 buoys. For larger array spacings, the CW continues to be periodic but is generally smaller . The average improvement factor is close to 41 for 41 buoys, close to 11 for 11 buoys, as one might naively expect. With a wide spectrum sea the result will be some average over adjoining values of kd. In 1976 Budal [6] computed the energy balance in the far field for a line of 10 buoys and obtained a graph similar to figure 1. McIver [7] found essentially the same for a line of 5 buoys.

Two parallel lines of buoys, both parallel to the wave crests, have been recommended. The same calculation gives a poor result if the line spacing is one or two wavelengths. Optimum line spacing is 0.25 λ which gives figure 2 with an improvement factor 205 for 82 buoys, a ratio of 2.50 per buoy. Comparing with figure 1, it seems that the same number of buoys in a single line is slightly better.

The result for a linear array oriented parallel to the wave direction is given in figure 3, again for lines of 11 and 41 buoys. In this case there are pronounced minima when the buoy spacing is some multiple of $\lambda/2$, but on the whole the CW increases as the buoy spacing is increased. However the best results are disappointing: the improvement factors over the same number of independent buoys is only 0.7 for 11 buoys and 0.55 for 41 buoys.

Circular arrays

The polar diagram for a circular disc-shaped array can be obtained in the same way, (but in this case neglecting the hydrodynamic modifications of the incident wave [4]). The improvement ratio for a ring of 6 buoys with another at the centre, was caculated as a function of ring diameter. For very small diameters the ratio is 1, so the capture width is the same as that of a single buoy. The ratio rises to a maximum of 7.2 when the radius is 1.64 λ, hardly better than 7 separate buoys. A disc is a collection of rings, so will be no better. It seems that the compact arrays studied in [4] will have a broader resonance because of the close interactions but the peak response will hardly justify the extra expense.

A paradox

The array captures more power because on average the radiation to the far field is reduced by destructive interference. The radiation damping of each buoy will be correspondingly reduced. To capture more power each device must move more. Why should it do so when I have assumed that the incident wave at each device is unmodified? This is a mystery. Theorem (1) is valid only if the device motions have been optimised, which must imply less internal damping than for an isolated buoy. However less damping implies a narrower resonance; so in a wide-spectrum sea the gains predicted by figures 1 etc may prove illusionary.

Conclusions

1) A line of WECs perpendicular to the wave direction is the best shape for the array.
2) Spacing should be of order 3/4 times the wavelength, a bit smaller for a wide spectrum sea; one wavelength spacing is bad.
3) The total CW is then about a factor 2 more than for the individual elements deployed separately.
4) Two parallel broadside lines do not offer much improvement.

References

[1] F.J.M. Farley, (2004) *Optical theorem in atomic physics and wave power*, 19th International Workshop on Water Waves and Floating Bodies, Cortona, Italy (http://www.iwwwfb.org/)

[2] F.J.M. Farley, (2012) *Far field theory of wave power capture by oscillating systems*, Phil.Trans. A370, 278

[3] J.N. Newman, (1979) *The interaction of stationary vessels with regular waves*, Proc. 11th Symp. on Naval Hydrodynamics, London, UK, pp. 491 - 501

[4] C.C. Mei, (2012) *Hydrodynamic principles of wave power extraction*, Phil.Trans. A370, 208

[5] X. Garnaud and C.C. Mei, (2010) *Bragg scattering and wave-power extraction by an array of small buoys*, Proc. Roy. Soc. A466, 79

[6] K. Budal, (1977) *Theory for absorption of wave power by a system of interacting bodies*, Journ. Ship Research 21, 248; (and preprint 1976).

[7] P. McIver, (1995) *Arrays of wave energy devices*, 5th International Workshop on Water Waves and Floating Bodies, Oxford, UK (http://www.iwwwfb.org/)

Figure 1: *Capture width ratios for a line of buoys perpendicular to the wave direction, spacing d. For 11 buoys and 41 buoys. The total CW is compared to the CW for a single buoy*

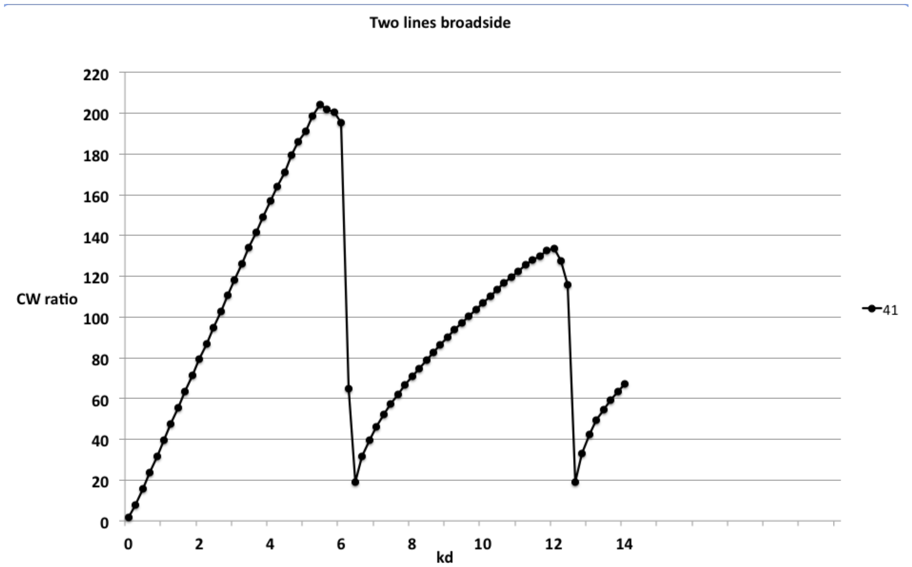

Figure 2: *Capture width ratio for two parallel lines of buoys perpendicular to the wave direction, buoy spacing d, line spacing* $\lambda/4$*. Each line has 41 buoys.*

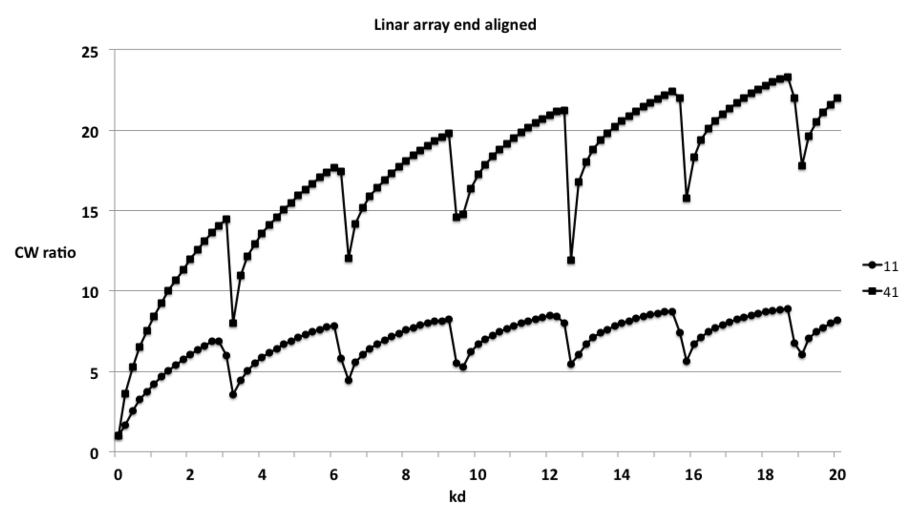

Figure 3: *Capture width ratios for a line of buoys parallel to the wave direction, spacing* d*. For 11 buoys and 41 buoys.*