An experimental study of near-cloaking

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In the past years there has been much research work on "cloaking", whereby some object is made "invisible". From electromagnetics the topic has been extended to other fields such as acoustics, structural mechanics and, more recently, hydrodynamics. In the water wave context, "invisibility" means that the diffracted wave field is nil in the far-field at all azimuthal angles. This can usually be achieved only at one wave frequency. At the 26th IWWWFB Porter (2011) presented an application to the case of a vertical cylinder, rendered invisible by a local modification of the bathymetry in an otherwise constant depth ocean. This case was further investigated by Newman (2012).

In this paper we report a similar study, where a vertical dihedral, at the end of the ECM wavetank, is attempted to be rendered invisible.

Experimental set-up

Our wave flume is about 15 m long and 65 cm wide. In the reported experiments the waterdepth was set at 40 cm. The beach at the end of the tank was removed and a rigid vertical plate was installed, from wall to wall, at an angle of 60 degrees, thereby achieving a dihedral. In this configuration a first series of regular wave tests was run, with wave number k mainly in the range π/b through $2\,\pi/b$ (b being the tank width), meaning the reflected wave system, in the far-field, consists of two modes: the inline mode and the first sloshing (plus progressive) mode. The two components were separated by an array of 5 wave gauges over the width of the tank, set at different inline positions (the same experimental case being run as many times as different positions were used).

In a second stage an "invisibility carpet", consisting in 18 vertical poles, with trapezoidal cross-sections, was set in front of the dihedral. The same regular wave tests were run, and the reflected inline and sloshing modes were separated from the wave gauge measurements. Successful "invisibility" implies that the sloshing modes vanish and only the inline reflected mode remains.

Numerical determination of the reflected wave system

The problem was formulated within the frame of linearized potential flow theory, and solved numerically with the COMSOL Multiphysics software. In the dihedral alone case (without the "invisibility carpet") a semi-analytical method, described below, was also used to validate COMSOL's results.

Due to the wall-sided geometry the linearized velocity potential writes

$$\Phi(x,y,z,t) = \Re\left\{-\mathrm{i}\,\frac{A_I\,g}{\omega}\,\frac{\cosh k(z+h)}{\cosh kh}\,\varphi(x,y)\,\mathrm{e}^{-\mathrm{i}\,\omega t}\right\} \tag{1}$$

where A_I is the amplitude of the incoming waves, h the waterdepth, ω the frequency and k the wave number. The reduced potential φ satisfies the Helmholtz equation $\Delta\varphi + k^2\,\varphi = 0$ in the fluid domain, no-flow conditions at the solid walls and appropriate ingoing and outgoing conditions at $x \to \infty$.

Dihedral alone

Figure 1 shows the geometry at the end of the tank. It consists in two overlapping rectangular sub-domains:
– the angular sector $0 \le R \le 2\,d$; $0 \le \theta \le \pi/3$ (inside the green contour in figure 1).
– the semi-infinite strip $d \le x < \infty$; $0 \le y \le b$ with $d = b\,\sqrt{3}/3$ (inside the red contour).

Within the first sub-domain the reduced potential φ takes the general form:

$$\varphi_1(R,\theta) = \sum_{m=0}^{\infty} A_m\,\frac{J_{3m}(kR)}{J_{3m}(2kd)}\,\cos 3m\theta \tag{2}$$

with J_{3m} the Bessel function of the first kind.

Within the second sub-domain it can be written as:

$$\varphi_2(x,y) = \mathrm{e}^{-\mathrm{i}\,kx} + \sum_{n=0}^{\infty} B_n\,\cos\lambda_n y\,\mathrm{e}^{-\alpha_n\,(x-d)} \tag{3}$$

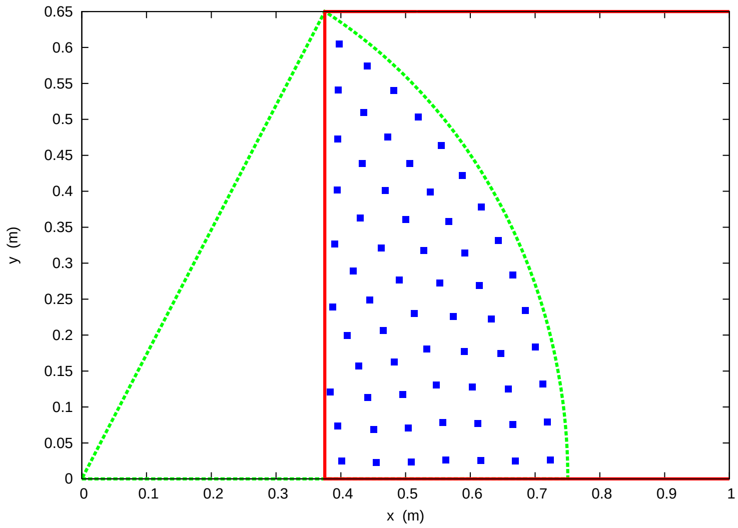

Figure 1: Sub-domains 1 and 2 and control points.

where $\lambda_n = n\,\pi/b$ and

$$\alpha_n = -\mathrm{i}\ \sqrt{k^2 - \lambda_n^2} \quad \text{for} \ \ n \leq N_2 \qquad\qquad \alpha_n = \sqrt{\lambda_n^2 - k^2} \quad \text{for} \ \ n > N_2 \tag{4}$$

N_2 being the largest integer n such that $\lambda_n < k$.

The unknown coefficients A_m and B_n can be determined by enforcing that the two expressions coincide in the common region. To this end the series are truncated at orders M and N, N_{pt} (with $N_{\mathrm{pt}} \gg M + N + 2$) control points (shown as blue square symbols) are distributed over the common region, and the following quantity

$$E = \sum_{i=1}^{N_{\mathrm{pt}}} \Big| \varphi_1(x_i, y_i) - \varphi_2(x_i, y_i) \Big|^2 \tag{5}$$

is minimized.

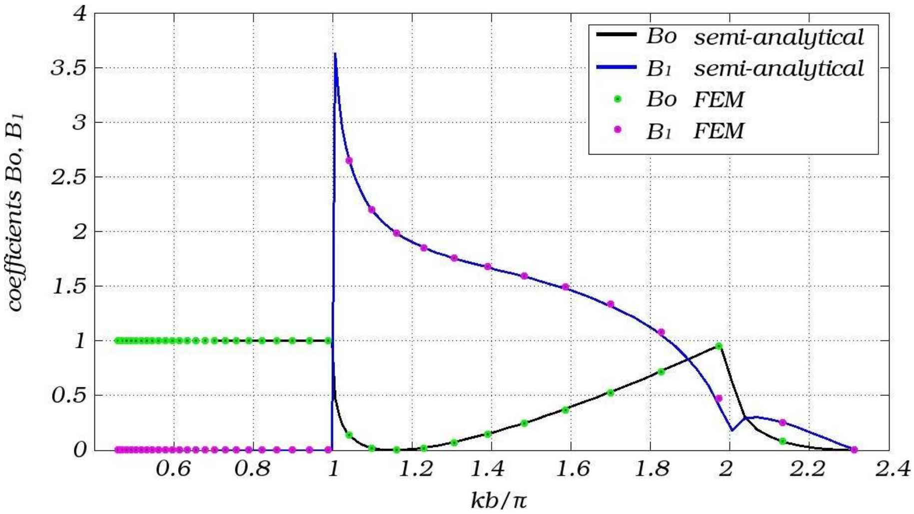

Figure 2: Coefficients $|B_0|$ and $|B_1|$ vs kb/π.

Figure 2 shows the moduli of the non-dimensional amplitudes B_0, B_1 of the propagating modes for $0.5 \leq k\,b/\pi \leq 2.3$, as obtained semi-analytically with this method and as obtained numerically with the Finite Element Method of COMSOL. The agreement is excellent.

Invisibility carpet

The trapezoidal end part of the wave-flume is covered with vertical poles of prismatic cross-sections, as shown in figure 4. The void fraction is close to 50 %, and the "carpet" extends 1.5 m from the end point of the tank. These choices are somewhat arbitrary. Keeping constant the void fraction and extension of the carpet, the number of inclusions in the inline and transverse directions are varied in COMSOL computations.

An efficiency function is defined from the quantity

$$F = \sum_n \int_0^b \left| \varphi(x_n, y) - \overline{\varphi(x_n, y)} \right|^2 \, \mathrm{d}y \tag{6}$$

where a number of reference abscissas x_n are taken, from the edge of the carpet toward the wavemaker. When there is no carpet F takes the value F_0. The efficiency is then defined as F/F_0. An efficiency equal to zero means that the dihedral has been made "invisible".

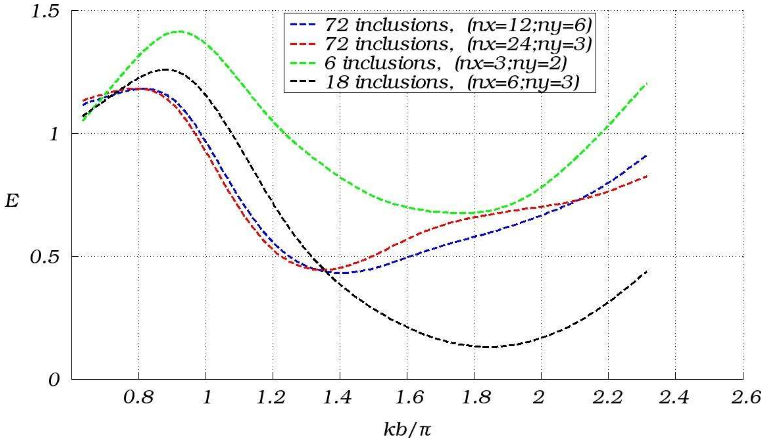

Figure 3: Efficiency function F/F_0, for different numbers of inclusions.

Figure 3 shows the obtained efficiencies for 4 different arrangements, and for $k\,b/\pi$ in the range 0.8 to 2.2. It is somewhat puzzling that the most efficient carpet, in the range $1.4 \leq k\,b/\pi < 2.2$, is the carpet with the intermediate number of inclusions (18). This is the carpet that was modeled experimentally. A photograph is shown in figure 4.

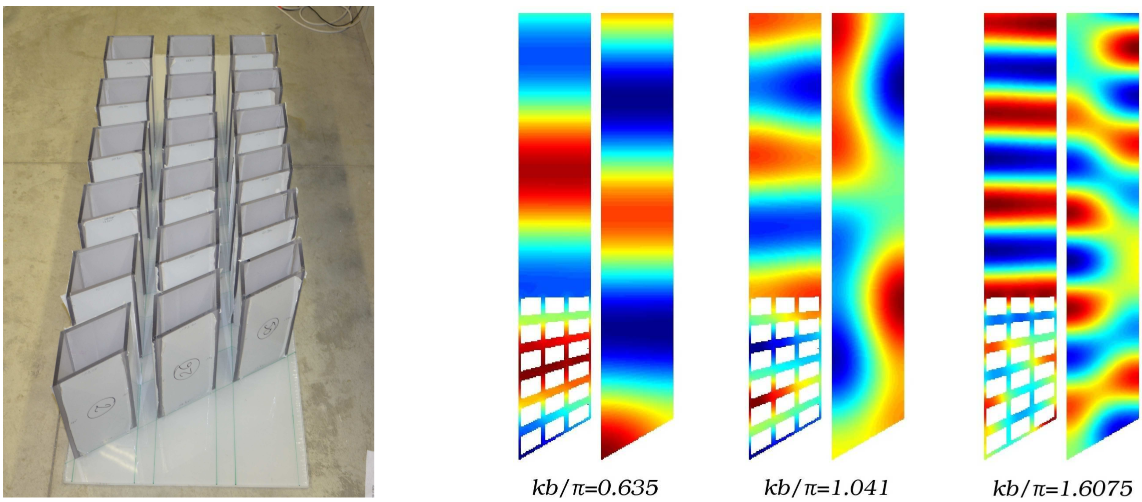

Figure 4: Photograph of the invisibility carpet (left). Calculated wave patterns in the end part of the tank for 3 kb values (right).

Figure 4 (right) shows calculated patterns of the free surface elevation, with and without the invisibility carpet, for 3 values of $k\,b/\pi$. The first value ($kb/\pi = 0.635$) is below the cut-off frequency, so the wave pattern is unidirectional in both cases.

Results for the non-dimensional amplitudes B_0 and B_1, from COMSOL computations, for the 18 inclusions arrangement, are shown in figure 5, and compared with the dihedral alone case. With the carpet the inline coefficient B_0 takes values very close to 1 all over the kb range, while the B_1 coefficient is strongly decreased.

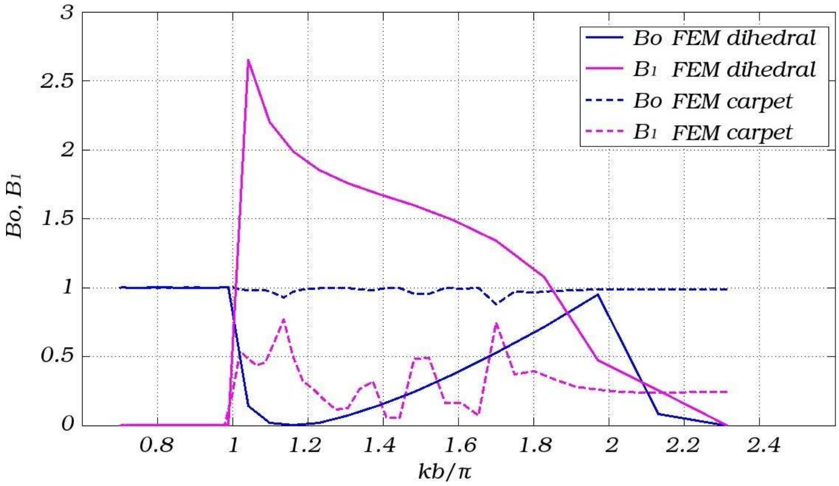

Figure 5: Results from COMSOL computations: B_0 and B_1 without (full lines) and with (dashed lines) the carpet.

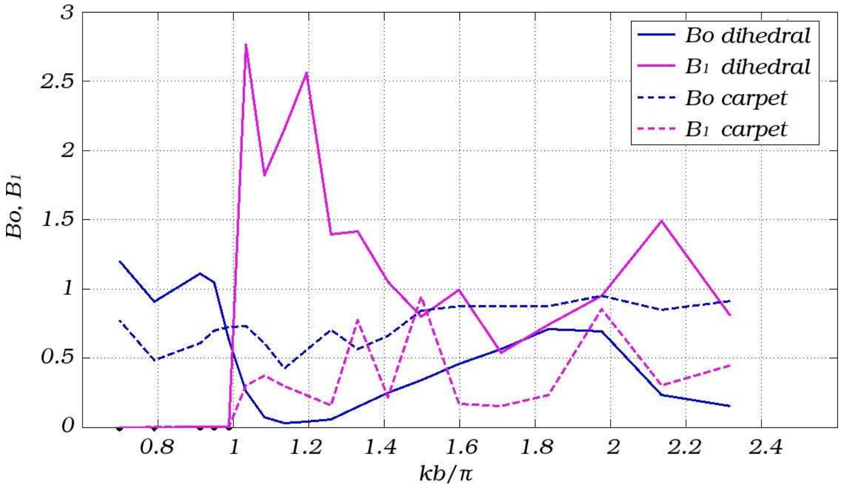

Figure 6: Results from experiments: B_0 and B_1 without (full lines) and with (dashed lines) the carpet.

Finally figure 6 shows the experimental B_0 and B_1 coefficients, as derived from the wave gauges measurements. The agreement with the results in figure 5 is fair in the dihedral alone case. In the carpet case it is only qualitative. In particular the experimental B_0 coefficient, in the low kb range, is much lower than 1, suggesting that appreciable energy dissipation takes place in the carpet, most likely through viscous effects. Another reason for discrepancies might be that at low kb values, the wave flume being only about 15 m long, with no active absorption mechanism, the steady state window is rather short.

Acknowledgement

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References

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Newman J.N. 2012 Scattering by a cylinder with variable bathymetry, in *Proc. 27th Int. Workshop Water Waves & Floating Bodies*, Copenhagen.