The impact of a fractionally (viscoelastic) damped system onto the water free surface

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1 Introduction

Since the pioneering works of von Karmán [1] and Wagner [2], who were originally motivated by the landing of seaplanes, considerable effort has been made toward the comprehension and formulation of the physical phenomena which govern the impact of structures onto a liquid surface. It is well-known that the formal treatment of this problem is embedded in a great variety of difficulties which appear in both theoretical and experimental approaches. From a practical point of view, the water impact can be seen as the cornerstone for the analysis of important problems in ship dynamics and offshore engineering (see [3]). For this reason, interests on the subject are recursively renewed in accordance with the current necessities of technology and industry. From sixties to present, for instance, the hydroelastic behaviour of impacting marine structures has received a special attention when regarding design and safety (see [3], [4], [5]). Likewise, recent advances in modern engineering have revealed that viscoelastic materials play an important role on dissipation of vibration, active noise control and stability, particularly of fast boats and floating structures (see [6], [7]). In this sense, the present work aims at theoretically addressing the impact of a viscoelastic damped system onto a liquid surface. As analogously pointed out for the elastic case (see [5]), it is believed that the upcoming discussion might help with the understanding of important features of the structural water impact response when taking a viscoelastic damper. In this sense, fractional calculus is evoked to model viscoelastic effects (see [8], [9]). Additionally, physical insights from such coupled dynamics, which appears from the combination of the hydrodynamic impact force with the fractional-order dissipation term, are to be exposed. This article presents the preliminary results of an ongoing research which aims at bridging the water impact problem, as well as the following fluid-structure interaction, to the advances of fractional-order mechanics of the twentieth century. It was originally motivated by the current studies on the impact of an elastic system onto the water free surface (see [3], [4], [5]).

2 Fractional-order derivatives and viscoelastic theory

Fractional calculus deals with the uncustomary derivatives of non-integer order, which are named fractional derivatives (see [8]). In the treatment of mechanical systems, Caputo's definition is rather preferable for physical reasons, that is, if $0 < \alpha < 1$, the α- order derivative of $f = f(t)$, named $D^\alpha f$, is given as

$$D^\alpha f(t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{df(\tau)}{d\tau} \frac{d\tau}{(t-\tau)^\alpha} \tag{1}$$

where Γ is the gamma function.

Eq. 1 can be used to model the viscoelastic damping force F_v by means of the constitutive relation

$$F_v = cD^\alpha \delta \tag{2}$$

where δ is the displacement, and c and α are constants which depend on the viscoelastic material.

3 Mathematical formulation

Let us consider the vertical water entry of two rigid spheres of radius R that are connect by a viscoelastic material; that is, $0 < \alpha < 1$ applies on Eq. 2 (see Fig. 1(a)). Analogously to [4], [5], the equations of motion[1] follow from Newton's law:

$$m_1 \ddot{z}_1 + cD^\alpha \delta + F_h = 0 \tag{3}$$

$$m_2(\ddot{z}_1 - \ddot{\delta}) - cD^\alpha \delta = 0 \tag{4}$$

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[1] Note that the mass matrix renders non symmetric due to the adoption of the relative motion as generalized coordinate.

For the sake of simplicity of such preliminary analysis, the hydrodynamic impact force F_h, which acts on the sphere that touches water free surface, is written from the linearized Wagner's approach,

$$F_h(z_1, \dot{z}_1) = \frac{d}{dt}(M(z_1)\dot{z}_1), \quad M(z_1) = 4\sqrt{3}\rho(Rz_1)^{3/2} \tag{5}$$

where ρ is the density of the water and $M(z_1)$ is the added mass of the impacting body; see discussions on its proper definition in [10].

4 On energy distribution

As it has been addressed, the distribution of energy during water entry is an important point to be discussed. The initial kinetic energy of the solid system, that is, $T_s(0) = \frac{1}{2}(m_1 + m_2)v_0^2$, where v_0 is the velocity at the instant of impact, is not entirely transmitted to the bulk of liquid, once it is partially drained to spray jets and partially dissipated due to the viscoelastic damping. This implies the following energy balance $T_s(t) + T_b(t) + T_j(t) + \Im(t) = T_s(0)$, where $T_b(t) = \frac{1}{2}M(z_1(t))\dot{z_1}^2(t)$ is the kinetic energy of the bulk of liquid, $T_j(t) = \int_0^t \frac{1}{2}\frac{dM(z_1(\tau))}{d\tau}\dot{z_1}^2(\tau)d\tau$ is the kinetic energy that is drained to spray jets (see [10]), and $\Im(t) = \frac{1}{2}c(D^{\alpha/2}\delta(t))^2$ is the energy that is dissipated by the viscoelastic connection (see [8]).

5 Numerical simulations

A simple Euler's numerical scheme was found to be suitable to integrate the system of ordinary fractional differential equations, which is given by Eqs. (3) and (4), and so to furnish the time history of the physical quantities in in the energy balance (see Figs. 1(b) - 1(f)). In nondimensional form, Eqs. (3), (4) and the energy balance are respectively written as

$$\ddot{\bar{z}}_1 + \frac{\gamma}{1+\lambda(\bar{z}_1)}\bar{D}^\alpha\bar{\delta} + \frac{1}{\beta+\mu(\bar{z}_1)}\frac{d\mu(\bar{z}_1)}{d\bar{z}_1}\dot{\bar{z}}_1^{\,2} = 0 \tag{6}$$

$$(\ddot{\bar{z}}_1 - \ddot{\bar{\delta}}) - \kappa\gamma\bar{D}^\alpha\bar{\delta} = 0 \tag{7}$$

$$\phi_s + \phi_b + \phi_s + \psi = 1 \tag{8}$$

with the following nondimensional variables and parameters: $\mu(\bar{z}_1) = (3\sqrt{3}/\pi)\bar{z}_1^{\,3/2}$, $\lambda(\bar{z}_1) = \mu(\bar{z}_1)/\beta$, $\phi_s = T_s/T_s(0) = \frac{\kappa\dot{\bar{z}}_1^2 + (\dot{\bar{z}}_1 - \dot{\bar{\delta}})^2}{(\kappa+1)}$, $\phi_b = T_b/T_s(0) = \frac{3\kappa\sqrt{3}}{\pi\beta(\kappa+1)}\bar{z}_1^{\,3/2}\dot{\bar{z}}_1^{\,2}$,
$\phi_j = T_j/T_s(0) = \frac{9\kappa\sqrt{3}}{2\pi(1+\kappa)}\int_0^t \dot{\bar{z}}_1^3(\tau)\sqrt{\bar{z}_1(\tau)}d\tau$, $\psi = \Im/T_s(0) = \frac{\kappa\gamma}{\kappa+1}(\bar{D}^{\alpha/2}\bar{\delta})^2$

$\beta = \rho_1/\rho$, $\gamma = (cv_0^{\alpha-2})/(m_1R^{\alpha-2})$, $\kappa = m_1/m_2 = \rho_1/\rho_2$, $\bar{z}_1 = z_1/R$, $\bar{\delta} = \delta/R$, $\dot{\bar{z}}_1 = d\bar{z}_1/d\bar{t} = (dz_1/dt)/v_0$, $\ddot{\bar{z}}_1 = d^2\bar{z}_1/d\bar{t}^2 = R/v_0^2(d^2z_1/dt^2)$, $\ddot{\bar{\delta}} = d^2\bar{\delta}/d\bar{t}^2 = R/v_0^2(d^2\delta/dt^2)$, $\bar{D}^\alpha\bar{\delta} = R^{\alpha-1}/v_0^\alpha(D^\alpha\delta)$.

ρ_1 and ρ_2 are respectively the density of spheres 1 and 2.

6 Discussion

This paper presented some initial considerations concerning the inclusion of fractional-order effects within the study of the water impact problem. A preliminary parametric study was carried out by varying α, the fractional-order of the derivative, in the range of $[0, 1[$, keeping $\kappa = 1$, $\beta = 0.1$ and $\gamma = 10$ as fixed. Fig 1(b) shows the acceleration time history of the impacting sphere, penetrating the water surface, for six distinct values of α. It is interesting to notice that, despite the nonlinear nature of Eqs. (6) - (7), the acceleration peak exhibits a weak dependence on α, yet decreasing in intensity as α increases. This is clearer presented in Fig. 1(c), where the acceleration peak is plotted as function of α. However, the oscillatory character of the dynamic response, which occurs just after the acceleration peak, strongly depends on α. This is physically reasonable since the fractional-order dissipation can be seen as lying in the transition region from the purely elastic case, $\alpha = 0$ - strongest oscillatory character, to the purely viscous case, $\alpha = 1$ - non-oscillatory character. Likewise, Figs.

1(c) and 1(d) show that the peak of the relative acceleration between the connected bodies is weakly dependent on α. On the other hand, the amplitude of the oscillatory *relative* motion, which takes place after the acceleration peak, is greatly affected by the choice of α. In fact, the amplitude of the oscillatory relative motion for $\alpha = 0.1$ is approximately twice the amplitude of the oscillatory motion for $\alpha = 0.8$. This may play an important role for design and safety as well as for further analysis of the structural response of boat hulls and floating structures. Fig. 1(e) shows the ratio between the viscoleastic connecting force and the hydrodynamic impacting force. As it can be seen, the ratio is always much lower than unity, and its peak is attained much after the acceleration peak occurs. Finally, Fig. 1(f) presents the energy balance during the water impact of this fractionally damped system, taking $\alpha = 0.75$. Although taking into account a viscoelastic damper, energy is mostly drained through the spray jets. In fact, the energy contained in the bulk of the liquid and that dissipated due to the viscoelastic connection are much smaller. Moreover, the time history of the energy dissipated to spray jets exhibits, to a very good extent, the opposite behavior of the time history of kinetic energy of the solid system. Future investigations aim at addressing a more comprehensive parametric analysis, regarding κ, β, γ and α. Other nonlinear effects following from the coupling between the hydrodynamic impact force and the fractional-order dissipation term are to be studied. Possible generalizations to consider the water impact of a fractionally damped plate can also be taken into account.

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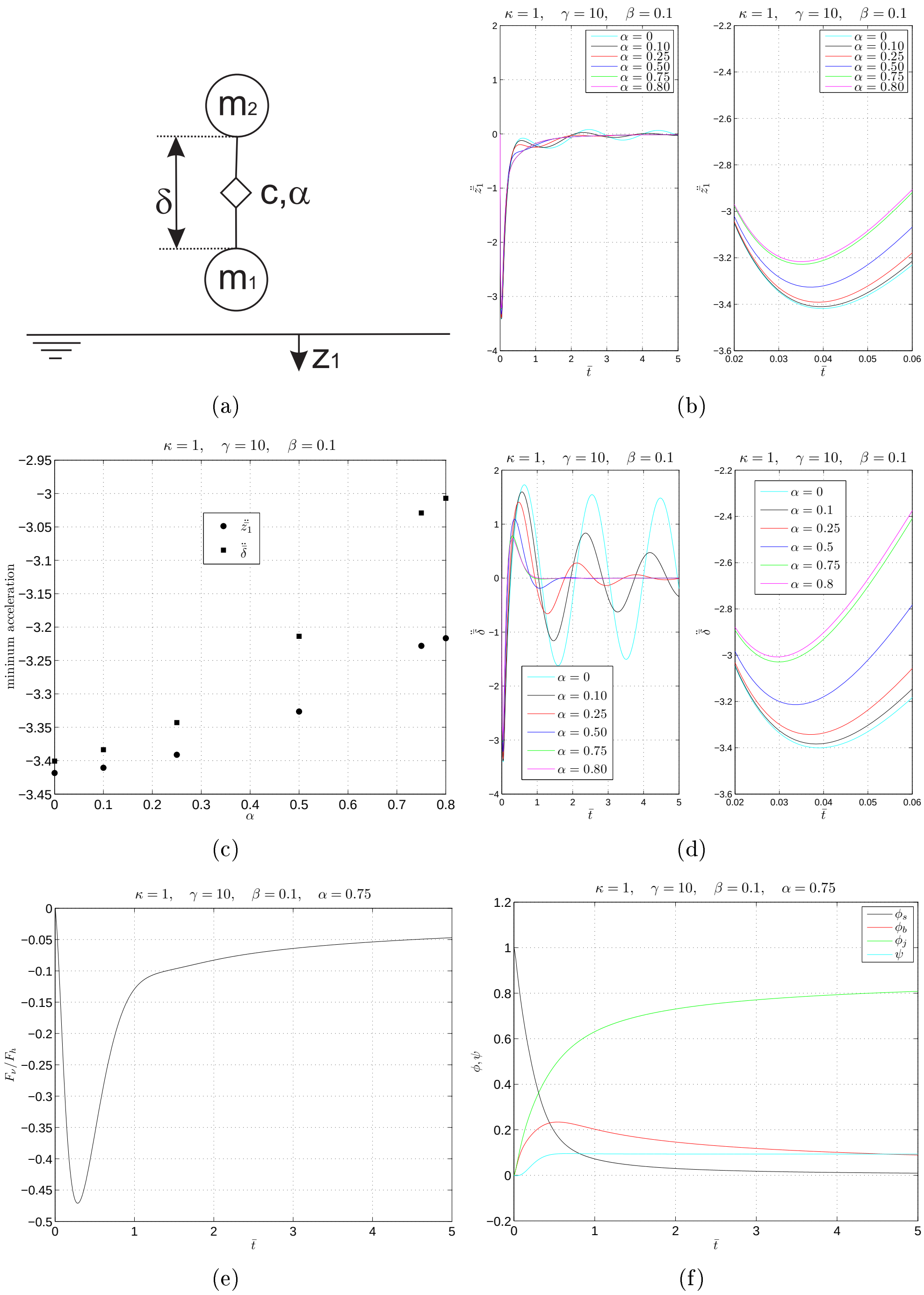

Figure 1: 1(a) - Schematic representation; 1(b) - Nondimensional acceleration of the impacting sphere vs time; 1(c) - Peak of nondimensional acceleration (impact and relative) vs α; 1(d) - Nondimensional relative acceleration vs time; 1(e) - Ratio between viscoelastic and impact force versus time; 1(f) - Energy balance vs time.