Modelling of the oblique impact of an elongated body by a 2D+t approach

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1 Introduction

We consider the oblique impact on deep water of an elongated body at high horizontal velocity. In the global coordinate system (x, y, z), the problem is assumed to be symmetric with respect to the plane x-z, see figure 1(a). The orientation of the body is defined by the pitch angle $\alpha(t)$ and the position of the centre of the body in the $x - z$ plane, position in surge and heave, is ($X(t)$,$Z(t)$). As the body is assumed elongated, it is relevant to use a 2D+t approach, which makes it possible to simplify the three-dimensional problem into a sequence of two-dimensional (2D) problems in a succession of vertical planes. In contrast to studies on the steady planing of prismatic hulls where the 2D+t approach leads to the 2D impact of a wedge, for more general curved shapes such as an ellipsoid, the 2D+t approach leads to the water entry and exit of a fictitious 2D body, the shape of which is changing in time. The originality of our approach consists in using the 2D+t approach together with the Modified Logvinovich Model (MLM), see Korobkin (2004). This model is based on linearised potential flow theory similar to the Wagner approach, but includes nonlinearities in the pressure. The MLM has already been shown to be an attractive model for the prediction of hydrodynamic loads during vertical water impacts at constant speed, see Tassin et al. (2011), and is therefore expected to predict accurately the longitudinal force distribution in a 2D+t approach. In this work, the MLM is adapted to the water-entry problem at variable speed. Also, a method due to von Karman (1929), consistent with the MLM, is proposed to describe the water exit stage. The combined MLM/von Karman method has been used to study the water entry and exit of a wedge, showing very good agreement with the fully nonlinear numerical results of Piro and Maki (2011). The prediction of the longitudinal force distribution during the oblique impact of an elongated ellipsoid is also presented, showing the presence of significant suction forces which act vertically downward in certain positions along the x-axis.

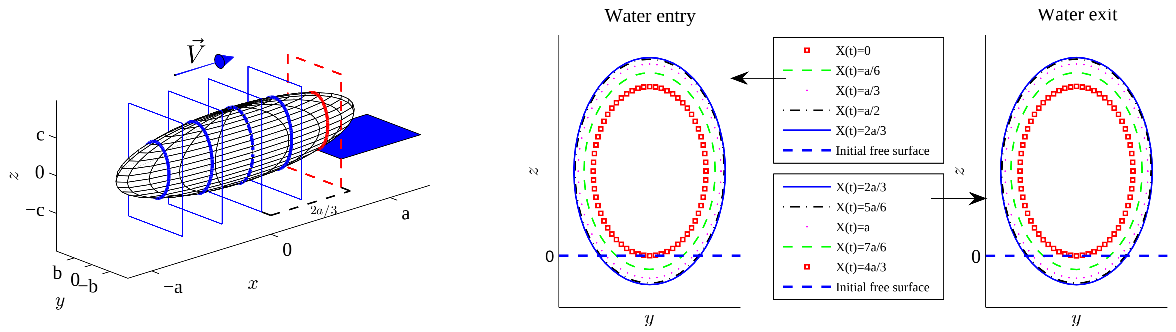
2 Mathematical formulation

2.1 2D+t approach

Within the 2D+t approach, the water impact problem is studied from the point of view of several vertical control planes fixed in the global coordinate system, distributed along the x-axis at locations $x = x_c$ and parallel to the y-z plane. Figure 1(a) shows the intersection between several control planes and an ellipsoid partially immersed, moving horizontally in the x-direction. In figure 1(a), the first front cross-section in contact with the initial water level $z = 0$ (horizontal plane in blue) is located at $x = 2a/3$ in the vertical plane depicted by a red dashed line (colour online). Figure 1(b) displays the shape of the cross-section in the plane $x_c = 2a/3$ for several positions of the centre of the body. It can be seen that from $X(t) = 0$ to $2a/3$ (left hand side), the cross-section enters the water as the body passes through the control planes, and on the same time the cross-section is expanding. However, once $X(t) > 2a/3$ (right hand side), the cross-section is going up and contracting. From the 2D point of view, this phenomenon is similar to water entry and exit, see Piro and Maki (2011). For a body whose shape is changing in time, the entry stage and the exit stage are defined as the periods during which the wetted area is expanding and contracting, respectively. In the next sections, it is assumed that the shape of the body can be described by $z = Zb(x, y, t)$ where t is time.

2.2 Water entry stage

The MLM is used to study the water entry stage in each cross-section. Within the MLM, the position of the contact point between the free surface and the body at the section $x = x_c$ is $y = c(t)$. The function $c(t)$ is given by the so-called Wagner condition. However, in contrast to the classical Wagner theory, the hydrodynamic pressure $P(x, y, t)$ on the body is calculated by the nonlinear Bernoulli



(a) Sketch of the vertical cross-sections

(b) Evolution of the body shape in the fixed control plane $x_c = 2a/3$ for several positions of the centre $X(t)$ in the x-direction

Figure 1: 2D+t approach for an ellipsoid moving horizontally in the x-direction (a, b and c are the semi-axes of the ellipsoid in the x, y and z-directions, respectively)

equation, which can be reformulated as follow by using the kinematic boundary condition on the body:

$$P(x,y,t) = -\rho\left(\phi_t - Zb_t\frac{\phi_x Zb_x + \phi_y Zb_y}{1+|\nabla Zb|^2} - \frac{Zb_x Zb_y \phi_x \phi_y}{1+|\nabla Zb|^2} + \frac{1}{2}\frac{(1+{Zb_y}^2)\phi_x^2 + (1+{Zb_x}^2)\phi_y^2 - Zb_t^2}{1+|\nabla Zb|^2}\right), \tag{1}$$

where ρ is the fluid density and $\phi(x,y,t)$ is the velocity potential on the body. Note that, in a strictly 2D approach Zb_x and ϕ_x are equal to 0 and the results presented in section 3 are based on this assumption. In addition, $\phi(x,y,t)$ is approximated by a Taylor expansion of the velocity potential of Wagner ($\varphi(x,y,z,t)$):

$$\phi(x,y,t) \approx \varphi(x,y,0,t) + Zb(x,y,t)\cdot Zb_t(x,y,t). \tag{2}$$

One can see from equations (1) and (2) that it is necessary to evaluate $\varphi_t(x,y,0,t)$ and $\varphi_y(x,y,0,t)$ to calculate the pressure. Within the 2D+t approximation, $\varphi_t(x_c,y,z,t)$ and $\varphi_y(x_c,y,z,t)$ satisfy Laplace's equation and are solutions of two 2D boundary value problems, which can be written as:

$$\varphi_t(x_c,y,0^-,t) = \frac{-1}{\pi\sqrt{c(t)^2-y^2}}\int_{-c(t)}^{c(t)}\frac{\theta_t(x_c,y,t)\sqrt{c(t)^2-\tau^2}}{(\tau-y)}\mathrm{d}\tau - \frac{C_0}{\sqrt{c(t)^2-y^2}} \quad \text{for } |y|<c(t), \tag{3a}$$

$$\varphi_y(x_c,y,0^-,t) = \frac{1}{\pi\sqrt{c(t)^2-y^2}}\int_{-c(t)}^{c(t)}\frac{Zb_t(x_c,\tau,t)\sqrt{c(t)^2-\tau^2}}{\tau-y}\mathrm{d}\tau \quad \text{for } |y|<c(t), \tag{3b}$$

where $\theta_t(x_c,y,t) = -\int_0^y Zb_{tt}(x_c,\tau,t)\mathrm{d}\tau$ is the time derivative of the stream function on the body and C_0 is a constant to be determined. Note that C_0 affects the asymptotic behaviour of φ_t close to $y = c(t)$. By substituting φ_t and φ_y into equation (1), one can see that the pressure can be expressed as $P = P_v + P_a$, with P_v depending on Zb_t only and P_a depending on Zb_{tt} only. At constant speed, Korobkin (2004) obtains the force by integrating the pressure from $y = -c^*$ to c^*, c^* being the closest point to $c(t)$ where $P(x_c,c^*,t) = 0$. In the present case, the pressure can be negative all over the wetted surface. It is therefore suggested to treat the singularity aspect of P_v by defining c^* as the closest point to $c(t)$ where $P_v(x_c,c^*,t) = 0$ and integrating P_v from $y = -c^*$ to c^*. In order to be consistent with the method used for the exit stage, P_a is integrated from $y = -c(t)$ to $c(t)$.

2.3 Water exit stage

Very little work has been dedicated to the water exit problem and there is no well established theory to describe this phenomenon, which could be wrongly seen as the time reversal of the entry problem. It is in particular difficult to describe precisely the separation of the water from the body, and a wake region like in oblique impact at high horizontal speed should also be considered, see Reinhard et al.

(2011). Indeed, there is so far no condition to determine the contact point position in the exit stage which would be equivalent to the Wagner condition. Furthermore, no experimental data are available. In contrast to the Wagner approach, the prediction of the contact point position by the von Karman theory is only based on geometry, the contact point being the intersection between the body and the initial water level surface. It is therefore possible to use a similar approach for the water exit. However, in order to ensure the continuity of the contact point position between the entry and exit stages, the reference water level for the von Karman approach has to be equal to the free-surface altitude at the contact point when exit starts. This is given by the following condition:

$$Zb(x_c, c(t), t) = Zb(x_c, c(t_0), t_0), \tag{4}$$

where t_0 is the instant of transition between the entry and exit stage defined such that $\dot{c}(t_0) = 0$ and $\ddot{c}(t_0) < 0$. The calculation of the hydrodynamic pressure by the von Karman theory, based as it is on the linearised Bernoulli equation, shall be written as follows in order to be consistent with the MLM:

$$P(x, y, t) = -\rho(\varphi_t + Zb_{tt} \cdot Zb). \tag{5}$$

However, attention must be paid in the calculation of φ_t. Indeed, for 2D impacts, it seems to us that the different authors referring to the early work of (6) calculate φ with the boundary condition $\varphi(x_c, y, 0, t) = 0$ for $|y| > c(t)$, leading to $\varphi(x_c, y, 0, t) = \dot{Z}(t)\sqrt{c(t)^2 - y^2}$. Then, the velocity potential is derived, but only part of the terms are used for the pressure:

$$\varphi_t(x_c, y, 0, t) = \ddot{Z}(t)\sqrt{c(t)^2 - y^2} + \cancel{\dot{Z}(t)\dot{c}c/\sqrt{c(t)^2 - y^2}}. \tag{6}$$

We believe that the second term of the right hand side of equation (6), commonly referred to as the slamming term, is arbitrarily cancelled because it gives positive contribution to the pressure for both positive and negative $\dot{Z}$, but no clear justification for doing so is given. This issue can be explained by the fact that the free surface condition $\varphi(x_c, y, 0, t) = 0$ should be replaced by $\varphi_t(x_c, y, 0, t) = 0$ for $c(t) < |y| < c(t_0)$ in the exit stage. However, equation (3a) remains valid in the exit stage, as the boundary conditions on φ_t are unchanged, but a Kutta condition ($|\varphi_t(x_c, c(t), 0, t)| < \infty$) is added at the contact point. The Kutta condition is consistent with the observation that no jet is generated during the water exit stage and will lead to $C_0 = -\pi^{-1} \int_{-c(t)}^{c(t)} \theta_t(x_c, y, t)\sqrt{c(t)^2 - \tau^2}[\tau - c(t)]^{-1} \mathrm{d}\tau$. This approach finally leads to the same expression as Kaplan (1987).

3 Results

3.1 Vertical water entry and exit of a wedge

We consider the water entry and exit of a wedge whose shape is defined by $Zb(y, t) = y \cdot \tan\beta - Z(t)$, with $\beta = 10°$, $Z(t) = t^2 V_1/2 + tV_0$ [m], $V_0 = -4$ [m/s] and $B = 1 \cdot \cos\beta$ [m] is the width of the specimen in the y-direction. The instant t_0 at which the body ceases descending and starts to go up is defined in equation (4) and V_1 is such that $Z(t_0) = -\sin\beta/4$ [m]. Note that the chines remain dry during the impact ($c(t_0) < B/2$). Figure 2 depicts the evolution of the non-dimensional force $F^* = F/(0.5(V_0/2)^2 B/2)$ as a function of the non-dimensional time $t^* = t/t_0$, where $V_0/2$ is the time-averaged velocity during the entry stage. The results of the present method (MLM/vK) are compared to the numerical results of Piro and Maki (2011) obtained by a fully nonlinear CFD method based on the Arbitrary Eulerian-Lagrangian approach. It can be seen that during the entry stage ($t^* < 1$) the two approaches agree very well and that during the exit stage the proposed method is in rather good agreement with the numerical results. However, at the end of the water exit ($t^* > 2$), although the up-rise of the water level is partially taken into account by the proposed method, the results of Piro and Maki (2011) predict a longer exit stage which is not captured by our method.

3.2 Oblique impact of an ellipsoid

We consider the oblique impact of an ellipsoid ($a = 10$ [m], $b = c = 1$ [m]) at constant speed with $\dot{X}(t) = 50$ [m/s], $\dot{Z}(t) = -1$ [m/s] and $\alpha(t) = 0$, the body touching the water at $t = 0$. The longitudinal distribution of the vertical force at different instants is depicted in figure 3, showing that the wetted area is expanding in x with time. The red point indicates the position of the last cross-section in entry stage. One can observe that important negative forces appear over the regions in entry stage which is consistent with the results on the water entry and exit of a wedge.

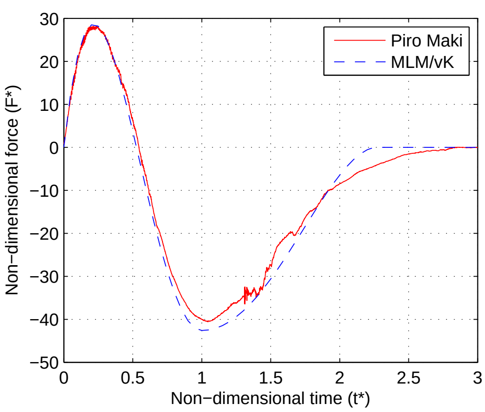


Figure 2: Non-dimensional force evolution during the water entry and exit of a wedge

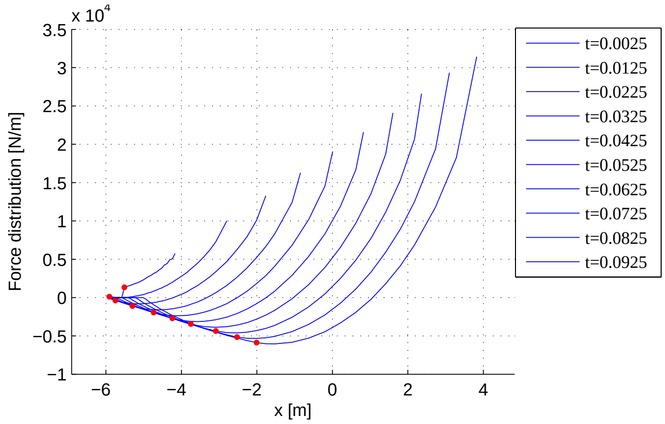


Figure 3: Longitudinal force distribution during the oblique impact of an ellipsoid

4 Conclusions

A method based on the 2D+t approach and a combined MLM/von Karman method has been proposed in order to predict the forces acting on elongated body during oblique impact at high horizontal speed. It has been shown that the method was able to predict suction forces. Predictions of motion for different conditions will be presented at the workshop. The proposed method can be enhanced in order to take into account the deformations of the structure by hydroelastic coupling. In the long term, complex flow phenomena such as aeration, ventilation and cavitation should be investigated. Impacts on a deformed free surface (waves) could also be studied. Finally, 3D effects may be investigated by using the 3D-MLM pressure equation (1) and the distribution of 2D velocity potential.

Acknowledgments

The authors thank Dominic J. Piro and Kevin J. Maki from the University of Michigan for kindly providing their results on the water entry and exit of a wedge published in Piro and Maki (2011). This work is part of the project SMAES - Smart Aircraft in Emergency Situations - project number 266172 - of the Seventh Framework Programme (FP7).

References

Kaplan, P. (1987) Analysis and prediction of flat bottom slamming impact of advanced marine vehicles in waves. *International Shipbuilding Progress* **34**, 391, pp. 44-53

Korobkin, A.A. (2004) Analytical models of water impact. *Euro. J. App. Math.* **15**, 821–838.

Piro, D.J., Maki, K.J. (2011) Hydroelastic wedge entry and exit. Fast 2011, USA, September 2011

Reinhard, M., Korobkin, A.A. and Cooker, M.J. (2011) Elastic plate impact onto water at high horizontal speed. IWWWFB 2011, Athens, Greece, 2011

Tassin et al. (2011) Hydrodynamic loads during water impact of three-dimensional solids: Modelling and experiments. *Journal of Fluids and Structures*, 2011, In press

von Karman, T. (1929) The impact of seaplane floats during landing. *NACA TN 321*