Added resistance in short waves: a ray theory approach

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1. Introduction

Added resistance in waves it's a well-known effect which has been studied by numerous scientists over the last decades. The traditional calculation tools are able to predict with good accuracy the peak of the added resistance RAO; the peak usually occurs for $\lambda \cong L$. When instead the incident wave lengths are shorter than the ship length, the diffraction effects dominate the unsteady wave field and the accuracy of the numerical results decreases. In this case alternative methods are preferred but the literature material is limited; the most followed methodologies are suited only for specific hull forms and speeds.
Another possible approach is ray theory, which has been successfully applied to the problem of wave front tracing in many different fields like atmosphere physics or electromagnetism [1].
The present work proposes a numerical application of ray theory for the evaluation of added resistance in short waves. The theory has been presented by Hermans [3] and afterwards revised by Kalske [4]; here some additional features are presented by the authors. Few significant comparisons with model tests outline a good agreement of current results with experimental data.

2. Unsteady potential

A ship advancing in regular waves with constant speed U is considered. The origin of the Cartesian coordinates system is located at midship on the undisturbed free surface with the x-axis positive in forward direction and the z-axis positive in upward direction.
The typical assumptions are made of inviscid and incompressible fluid and irrotational flow. The flow is then characterized by the velocity potential Φ, which satisfies the Laplace equation $\Delta\Phi = 0$ in the fluid domain. Because of the short wave lengths the ship motions are neglected and the boundary condition on the ship hull is expressed by $\partial\Phi/\partial n = 0$ on S_H.
The linearized free surface condition was obtained by Sakamoto and Baba [2], who assume U as small parameter and $\Phi(x,y,z,t) = \phi_r(x,y,z) + \phi_0(x,y,z) + \phi(x,y,z,t)$ where ϕ_r is the double body potential, ϕ_0 is the steady potential and ϕ is the unsteady potential. With a non-conformal coordinates transformation $x' = x, y' = y, z' = z - \zeta_r$ the free surface condition for the unsteady potential is given as:

$$\frac{1}{g}\left(\frac{\partial}{\partial t} + u_r\frac{\partial}{\partial x'} + v_r\frac{\partial}{\partial y'}\right)^2 \phi + \frac{\partial\phi}{\partial z'} = 0 \quad \text{on} \quad z' = 0 \quad \text{and} \quad \zeta = -\frac{1}{g}\left(\phi_t + \mathbf{u}_r \cdot \nabla\phi\right)$$

The vector $\mathbf{u}_r$ represents the double body flow velocities computed at the undisturbed free surface and when computing the previous equation its derivatives must be neglected.
From now on the transformed coordinates (x', y', z') are considered and the primes are omitted.

3. Ray Expansion

We seek for the unsteady potential an asymptotic solution for large k of the form:

$$\phi(x,y,z,t) = a(x,y,z,k)e^{ikS(x,y,z)+i\omega t},$$

with $k = \omega^2 / g$, $\omega = \omega_0 - k_0 U \cos\beta$ and β is the incident waves direction according to the sea-keeping convention.

The functions $S(\mathbf{x})$ and $a(\mathbf{x},k)$ are the phase and amplitude functions, with $a = a_0 + \sum_{j=1}^{n} \frac{a_j}{(ik)^j} + O((ik)^{-n})$.

Only the first term of the series is considered, this means that at caustics the solution will become multivalued thus it will not be possible to evaluate the complete unsteady wave field with this expansion. Substituting ϕ in the Laplace equation and in the free surface boundary condition two equations are found one for $S(\mathbf{x})$ and one for $a_0(\mathbf{x})$,:

$$\nabla S \cdot \nabla S = \left(1+\mathbf{v}\cdot\nabla S\right)^4 \qquad a_0 MS + 2\nabla a_0 \cdot \left[\nabla S - 2\mathbf{v}\left(1+\mathbf{v}\cdot\nabla S\right)^3\right] = 0$$

$$\text{with} \quad MS = \Delta S - \frac{\nabla S \cdot \nabla\left(\nabla S\cdot\nabla S\right)}{\nabla S\cdot\nabla S} - 2\left|\nabla S\right| \mathbf{v}\cdot\nabla\left(\mathbf{v}\cdot\nabla S\right).$$

The differential operators are two dimensional and are evaluated in the z = 0 plane; $\mathbf{v} = (u,v) = \mathbf{u}_r \tau / U$ and $\tau = \omega U / g$. The amplitude function equation is the transport equation. It can be obtained by using the wave action equation instead of the linearized free surface boundary condition; in that case the expression for MS changes as: $MSA = MS + 2\left|\nabla S\right|\left(1-\mathbf{v}\cdot\nabla S\right)\nabla\cdot\mathbf{v}.$

The phase function equation is an eikonal equation, an equation of the form $F\left(\mathbf{x},\nabla S\left(\mathbf{x}\right)\right) = 0$, and can be solved using the method of characteristics.

Introducing $\mathbf{p} = \left(p,q\right) = \nabla S$, the eikonal equation is written as: $F = \left(1+\mathbf{v}\cdot\mathbf{p}\right)^4 - \mathbf{p}\cdot\mathbf{p} = 0$. The PDE is then transformed into a system of five ODE with parameter σ , which represents a coordinate along the rays; the rays are defined in the Cartesian space by $\left(dx/d\sigma, dy/d\sigma\right)$.

From the Charpit-Lagrange equations the following system is obtained:

$$\frac{dx}{d\sigma} = F_p = 4\left(1+\mathbf{v}\cdot\mathbf{p}\right)^3 u - 2p \qquad \frac{dy}{d\sigma} = F_q = 4\left(1+\mathbf{v}\cdot\mathbf{p}\right)^3 v - 2q$$

$$\frac{dp}{d\sigma} = -\left(F_x + pF_S\right) = -4\left(1+\mathbf{v}\cdot\mathbf{p}\right)^3\left(\mathbf{v}_x\cdot\mathbf{p}\right) \qquad \frac{dq}{d\sigma} = -\left(F_y + qF_S\right) = -4\left(1+\mathbf{v}\cdot\mathbf{p}\right)^3\left(\mathbf{v}_y\cdot\mathbf{p}\right)$$

$$\frac{dS}{d\sigma} = \left(pF_p + qF_q\right) = -4\left(1+\mathbf{v}\cdot\mathbf{p}\right)^4 + 2\mathbf{p}\cdot\mathbf{p} \ .$$

The amplitude function is also solved along the rays and the transport equation is expressed as:

$$\frac{da_0}{d\sigma} = a_0 MS \quad \longrightarrow \quad a_0(\sigma) = a_0(\sigma_0) e^{\int_{\sigma_0}^{\sigma} MS \, \mathrm{d}\sigma} .$$

Because *MS* contains the second spatial derivatives of *S*, three additional equations, see [4], are introduced to evaluate S_{xx}, S_{xy}, S_{yy} along the rays:

$$\frac{dp_x}{d\sigma} = -12\left(1+\mathbf{v}\cdot\mathbf{p}\right)^2\left(\mathbf{v}\cdot\mathbf{p}\right)_x^2 + 2\mathbf{p}_x\cdot\mathbf{p}_x - 4\left(1+\mathbf{v}\cdot\mathbf{p}\right)^3\left(\mathbf{v}_{xx}\cdot\mathbf{p} + 2\mathbf{v}_x\cdot\mathbf{p}_x\right)$$

$$\frac{dp_y}{d\sigma} = -12\left(1+\mathbf{v}\cdot\mathbf{p}\right)^2\left(\mathbf{v}\cdot\mathbf{p}\right)_x\left(\mathbf{v}\cdot\mathbf{p}\right)_y + 2\mathbf{p}_x\cdot\mathbf{p}_y - 4\left(1+\mathbf{v}\cdot\mathbf{p}\right)^3\left(\mathbf{v}_{xy}\cdot\mathbf{p} + \mathbf{v}_x\cdot\mathbf{p}_y + \mathbf{v}_y\cdot\mathbf{p}_x\right)$$

$$\frac{dq_y}{d\sigma} = -12\left(1+\mathbf{v}\cdot\mathbf{p}\right)^2\left(\mathbf{v}\cdot\mathbf{p}\right)_y^2 + 2\mathbf{p}_y\cdot\mathbf{p}_y - 4\left(1+\mathbf{v}\cdot\mathbf{p}\right)^3\left(\mathbf{v}_{yy}\cdot\mathbf{p} + 2\mathbf{v}_y\cdot\mathbf{p}_y\right) \ .$$

4. Initial Conditions

Far ahead of the ship $S\left(\mathbf{x}\right) = S_0$ on $\Gamma : ax + by = c$, with $c \gg L_{PP}$. The line Γ is parameterized by r . The conditions for *p* and *q* are given by the eikonal equation together with the chain-rule: $S' = px' + qy'$, which yields $bp = aq$. The coefficients (a,b) are related to the incident wave angle by: $\tan(\frac{\pi}{2} - \beta) = -\frac{a}{b}$.

Three different cases are separated depending on the wave angle:

- $\mu = 0^\circ \vee \mu = 180^\circ \longrightarrow b = 0$ $\qquad \begin{bmatrix} x_0 = const & y_0 = r & S_0 = S_0 \\ p_0 = \dfrac{-2u - 1 + \sqrt{1+4u}}{2u^2} & q_0 = 0 & \end{bmatrix}$
- $\mu = 90^\circ \longrightarrow a = 0$ $\qquad \begin{bmatrix} x_0 = r & y_0 = const & S_0 = S_0 \\ p_0 = 0 & q_0 = \dfrac{-2v - 1 + \sqrt{1+4v}}{2v^2} & \end{bmatrix}$
- $a \neq 0 \wedge b \neq 0$

$$\begin{bmatrix} x_0 = r & y_0 = -\dfrac{a}{b} r + \dfrac{c}{b} & S_0 = S_0 \\ p_0 = \dfrac{-2\left(u+kv\right) - \left(1+k^2\right) + \sqrt{\left(1+k^2\right)^2 + 4\left(1+k^2\right)\left(u+kv\right)}}{2\left(u+kv\right)^2} & q_0 = kp_0 & \dfrac{b}{a} = k \end{bmatrix}$$

The initial conditions for p_x, p_y, q_y are found through numerical differentiation of $\dfrac{\partial p}{\partial r}$ and $\dfrac{\partial q}{\partial r}$, which is straightforward since the rays are parallel straight lines in the far field.

The initial condition $a_0(\sigma_0)$ for the transport equation is given by the expression of the unsteady wave amplitude, which after substituting the ray expansion yields:

$$\zeta_a = \frac{\omega a_0}{g}\left(1 + \mathbf{v} \cdot \nabla S\right).$$

The boundary condition $\partial\phi/\partial n = 0$ on the ship hull must be satisfied by the superimposition of the incident and reflected rays. Again after substituting the ray expansion, the boundary condition is expressed as:

$$\left(ika_0^i \mathbf{n} \cdot \nabla S^i + \mathbf{n} \cdot \nabla a_0^i\right) e^{ikS^i + i\omega t} + \left(ika_0^r \mathbf{n} \cdot \nabla S^r + \mathbf{n} \cdot \nabla a_0^r\right) e^{ikS^r + i\omega t} = 0,$$

from which it follows:

$$S^i = S^r, \quad \nabla S^i \cdot \mathbf{t} = \nabla S^r \cdot \mathbf{t}, \quad a_0^r = a_0^i .$$

Assuming the incident rays are known these conditions together with the eikonal equation are sufficient to compute the reflected rays.

5. Added Resistance and Numerical Solution

Added Resistance is computed by pressure integration over the ship waterline; in fact, since we are interested in short waves, the unsteady disturbance decays rapidly along the z-axis and its mean effect can be evaluated on the mean waterline:

$$R_{aw} = \overline{\int_{WL} \frac{1}{2} \rho g \zeta^2 \mathbf{n}_x \mathrm{d}l} - \overline{\iint_{S_0} \frac{1}{2} \rho \nabla_3 \phi \cdot \nabla_3 \phi \mathbf{n}_x \mathrm{d}s} = \frac{1}{4} k\rho \int_{WL} a_0^2 \left(\left|\nabla S^i\right| - \frac{\nabla S^i \cdot \nabla S^r}{\left|\nabla S^i\right|} \right) \mathbf{n}_x \mathrm{d}l .$$

In order to implement numerically this theory we should focus on the evaluation of the double body flow velocities and their derivatives, because while solving the ODE system these quantities should be known in several (*x*,*y*) locations of the mean free surface for each integration step.

In our algorithm the double body flow is calculated, up to the second spatial derivatives, by a panel method program using direct evaluation of the Green function coefficients in each collocation point of the free surface mesh. The third spatial derivatives of the double body potential are evaluated numerically and finally all the base flow quantities are obtained at any (x,y) location through numerical interpolation.
Since the free surface mesh is modeled around the ship waterline we are dealing with scattered data interpolation and derivatives evaluation; the method we chose is based on multiquadratics radial basis functions which offers a good control on the accuracy level and reasonable computational effort.
With this approach the most time consuming computations are done only once for each advance speed U.
The present study proved that ray theory can efficiently be implemented to evaluate added resistance in short waves and, as shown in the figure, the predicted values are in good agreement with the experiments results. Great care should be given to the treatment of the basis flow, in terms of evaluation, differentiation and interpolation. On the other hand the ray expansion shown here is not appropriate to represents the complete unsteady wave field and thus the current theory cannot be extended to include additional effects, i.e. wave breaking, unless the ray expansion is also modified.

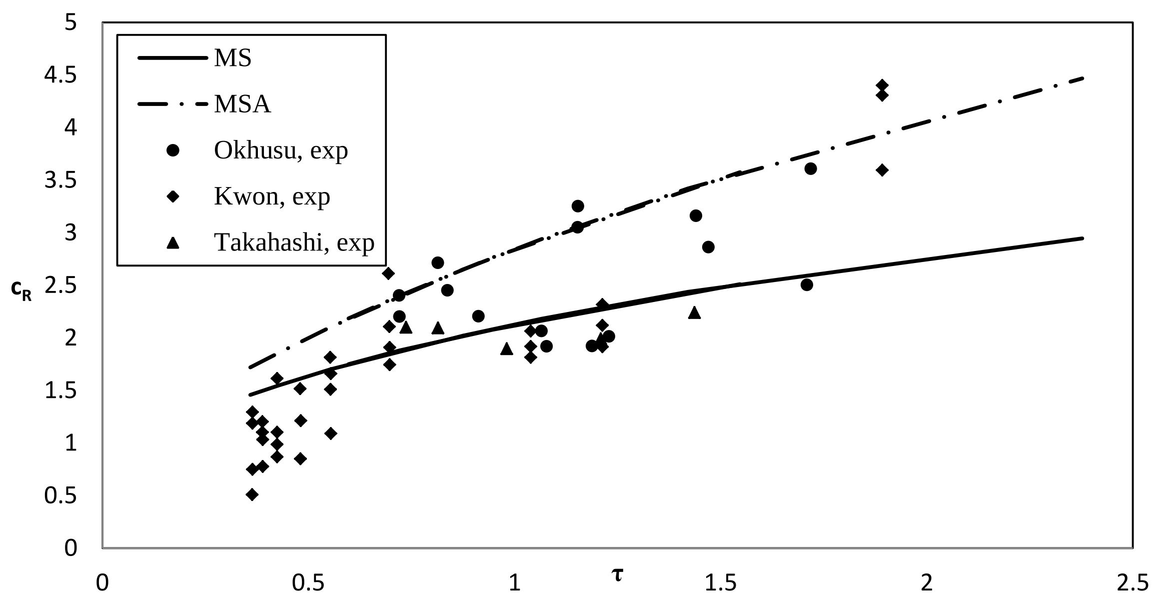

Figure 1: Added Resistance for the Series 60 cb 0.8 hull, 0.1 < Fn <0.2

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