A SURFACE-PIERCING BODY MOVING ALONG THE FREE SURFACE

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ABSTRACT
We consider the two-dimensional steady free surface flow past a body of arbitrary shape floating on water of infinite depth. The fluid is assumed to be inviscid, incompressible and the flow is irrotational; surface tension of the free surface is neglected. Our concern is with the periodic waves generated downstream and the splash jet separating from the body. The problem is relevant to the bow and stern flows when a ship is moving at a constant speed on the free surface. An advanced hodograph method is employed to derive an analytical expression for the complex potential of the flow. The problem is then reduced to a system of two integro-differential equations in terms of the velocity modulus on the free surface and the slope of the body surface. The Brillouin –Villat criterion is used to determine the location of the point of flow separation from the body while the equivalent condition of ventilation is applied to determine the location of the splash jet separation arising at the fore part of the body. Results showing the effect of gravity on the flow detachment and the waves downstream are presented over a wide range of Froude numbers.

1. INTRODUCTION

Two-dimensional free surface flows past a fixed floating obstacle is a challenging analytical and numerical problem, which is relevant to the generation of bow and stern flows in ship hydrodynamics.

The study of bow/stern flows started to receive much attention in the late 1960s [1 – 3], aimed to reduce drag of ships and find bow geometry providing a waveless free surface. In order to formulate a mathematical problem, an assumption about the flow topology should be done. Several configurations have been proposed for bow flows [3]: a) the free surface approaches smoothly the bow; b) a splash appears near the bow; c) a jet rises along the bow and the returning is neglected.

Dagan and Tulin [3] used asymptotic methods to consider type a) solutions for small Froude numbers and type c) for large Froude numbers. Dias and Vanden-Broeck [4] studied a specific solution of type b) with presence of a stagnation point at the splash jet by solving complete nonlinear problem in the whole flow domain. Vanden-Broeck and Tuck [5] showed that type a) solutions are not possible for flat bows in water of infinite depth. In the water of finite depth Vanden-Broeck [6] found a range of supercritical Froude numbers for which type a) solutions are possible.

However, there are a limited number of studies considering both bow and stern flow-regions within the framework of one problem. The study of the complete problem may be important for ships whose length scale is comparable with the typical wave length generated downstream. In the present study we choose two kinds of body geometries which are the flat plate and the circular cylinder. The first one gives the possibility to study splash jet separation for bow flows and the second one requires the complete solution of the problem including both for the bow and stern.

The flow topology adopted in the present study corresponds to the case b) proposed by Dagan and Tulin [3] and very close to that by Dias and Vanden-Broeck [4] considering the splash jet appearing near the bow.

Our solution method follows that proposed by Joukovskii (1890) for steady jet flows of an ideal fluid, the key step being the analytical construction of two governing functions in a parameter plane: the complex velocity and the derivative of the complex potential with respect to the parameter variable defined in an auxiliary parameter region. In reference [7] the method has been extended to study unsteady and gravity flows with the free surface. It gave a new way to derive the expression for the complex velocity accounting the variation of the velocity magnitude along free surfaces and the variation of the velocity angle along the wetted part of the body.

For a given body shape, we derive an integro-differential equation in the velocity angle along the body and integral equation in the velocity magnitude along the free surface. These integral equations must be solved numerically to obtain the solution.

2. THEORETICAL ANALYSIS

Figure 1(a) shows the flow configuration for the bow/stern flow model. The fluid is inviscid and incompressible and the flow is steady and irrotational. Far upstream, the flow is uniform with constant velocity U and the free surface is flat and parallel to the x-axis. The origin of the x-y coordinate system is placed at the point of separation O on the body. The acceleration due to gravity g is in the negative y direction. The distance of the lowest of the point of body to the undisturbed free surface it h. The disturbance of the body to the flow will generate waves downstream.

The shape of the body is given by the angle $ß$ as a function of the spatial coordinate s along the body, which starts at the separation point O. At the stagnation point A, the streamline splits into two streamlines, which go along the upper and lower sides of the body. The first streamline forms the splash jet which truncated at such distance from the body where it does not influences the main flow. The second streamline starting at the stagnation point *A* detaches at point *O* of the body and generates the wave downstream. The Brillouin –Villat criterion has been applied to determine the location of point *O*. The equivalent condition of ventilation has been applied to determine the location of point *B* at which the pressure becomes equal to pressure on the free surface.

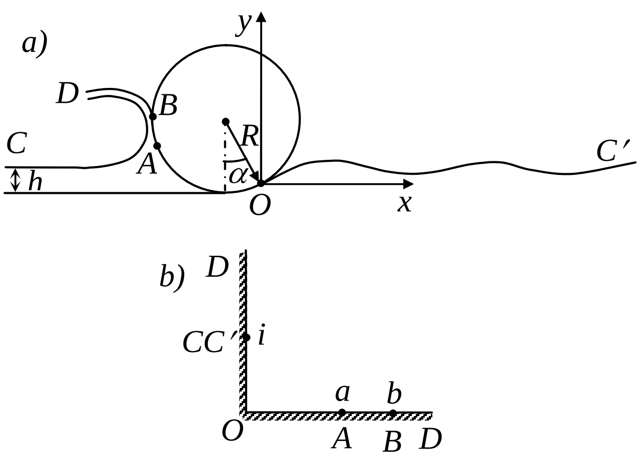

Figure 1. Sketch of the surface-piercing circular cylinder moving along the free surface: (*a*) the physical plane; (*b*) the parameter plane.

The Bernoulli equation can be written for reference point at upstream infinity

$$\rho\frac{V^2}{2}+\rho gY+p=\rho\frac{U^2}{2}+p_a\,, \tag{1}$$

where ρ is the liquid density; p_a is the atmospheric pressure;

Using the radius of the cylinder *R* as the characteristic dimension and the inflow velocity *U* as the characteristic velocity, Eq. (1) takes the form

$$v^2=1-c_p-\frac{2y}{Fn^2}\,, \tag{2}$$

where $F=\frac{U}{(gR)^{1/2}}$, $c_p=\frac{p-p_a}{0.5\rho U^2}$, and $v = V/U$ and $y = Y/R$ are the dimensionless velocity and coordinate, respectively. Eq. (2) also determines the velocity magnitude along the free surface where $c_p=0$.

Joukovskii proposed to map a parameter plane ς onto the planes of two functions, which are the complex potential $W=\phi+i\psi$ and the function $\omega=-\ln\left(dW/dz\right)$. If $W(\varsigma)$ and $\omega(\varsigma)$ are known functions of the parameter variable ς, the velocity field and the function mapping the parameter plane onto physical plane can be determined as follows:

$$\frac{dW}{dz}=\exp[-\omega(\varsigma)]\,,\quad z(\varsigma)=z_0+\int_0^{\varsigma}\frac{dW/d\varsigma}{dW/dz}d\varsigma\,. \tag{3}$$

We choose the first quadrant of the ζ -plane, where the complex variable $\varsigma=\xi+i\eta$ (see figure 1b), to correspond to the physical plane. A conformal mapping allows us to fix three points *O*, *C* and *D* as shown in figure 1b, then $\varsigma=a$ and $\varsigma=i$ are the images of points *A* and *C* in the physical plane, which should be determined from additional conditions. The interval $0<\eta<1$ of the imaginary axis corresponds to the free boundary OC' (the region of the stern flow), the interval $1<\eta<\infty$ corresponds to the free surface CD (the region of the bow flow). Interval $0<\xi<b$ of the real axis corresponds to the wetted part *OAB* of the body and $b<\xi<\infty$ corresponds to the upper side of the splash jet BD.

The function dW/dz satisfies the following boundary conditions

$$\left|\frac{dW}{dz}\right|_{\varsigma=i\eta}=v(\eta)\,,\quad 0<\eta<\infty\,, \tag{4}$$

$$\arg\left(\left.\frac{dW}{dz}\right|_{\varsigma=\xi}\right)=\gamma(\xi)=\begin{cases}-\pi-\beta, & 0<\xi<a,\\ -\beta, & a<\xi<\infty,\end{cases} \tag{5}$$

where *ß* is the slope of the wetted part of the body and the upper side of the splash jet. By using the integral proposed in [7], the final expression for the complex velocity takes the form

$$\frac{dW}{dz}=v_0\left(\frac{\zeta-a}{\zeta+a}\right)\exp\left[\begin{array}{l}-\frac{1}{\pi}\int_0^{\infty}\frac{d\beta}{d\xi}\ln\left(\frac{\xi+\zeta}{\xi-\zeta}\right)d\xi-\\ \frac{i}{\pi}\int_0^{\infty}\frac{d\ln v}{d\eta'}\ln\left(\frac{i\eta'-\zeta}{i\eta'+\zeta}\right)d\eta'-i\beta_0\end{array}\right], \tag{6}$$

where $v_0=v(0)$ and $\beta_0=\beta(0)$ are the velocity magnitude and direction at point O, respectively.

For steady free-boundary flows, the stream function ψ takes a constant value along the body and free boundaries, and therefore the region boundary in the *W*-plane forms a polygonal region. According to Chaplygin's singular point method [8], to determine the function $W=W(\varsigma)$, it is sufficient to analyse all singular points where the mapping is not conformal. The function $W=W(\varsigma)$ has singularities at points *O* ($\zeta = 0$), *A* ($\zeta = a$), *C* ($\zeta = i$), which correspond to the corner points of the region boundary in the ζ -plane and the

W - plane. The analysis of the behaviour of the function $\arg(W)$ at each corner point makes it possible to determine the order of the singularities in the expression $W = W(\varsigma)$, whose differentiation yields

$$\frac{dW}{d\zeta} = K\frac{\zeta(\zeta^2 - a^2)}{(\zeta^2+1)^2}, \qquad (7)$$

where K is a real scale factor.

Dividing Eq. (7) by Eq. (6), we obtain the derivative of the mapping function

$$\frac{dz}{d\zeta} = \frac{K}{v_0}\frac{\zeta(\zeta+a)^2}{(\zeta^2+1)^2}\exp\left[\begin{array}{l}\dfrac{1}{\pi}\displaystyle\int_0^\infty \frac{d\beta}{d\xi'}\ln\left(\frac{\xi'+\zeta}{\xi'-\zeta}\right)d\xi' + \\ \dfrac{i}{\pi}\displaystyle\int_0^\infty \frac{d\ln v}{d\eta'}\ln\left(\frac{\varsigma - i\eta'}{\varsigma + i\eta'}\right)d\eta' + i\beta_0\end{array}\right], \qquad (8)$$

whose integration along the imaginary axis in the parameter region provides the free boundaries OC' and CD in the z-plane. The parameters a and K and the functions $v(\eta)$ and $\beta(\xi)$ are determined by solving integro-differential equations derived from boundary conditions and physical considerations.

The region of the flow corresponding to splash jet maps to $|\zeta| \to \infty$ in the parameter plane as seen from figure 1. Then the asymptotic expression for the derivative of the flow potential in the region of the splash jet is obtained from Eq. (7), $dW/d\zeta \sim K/\zeta$. By integrating Eq. (7) and using the last asymptotic expression it is possible to express the parameter variable ζ as function of the potential W and then determine the complex velocity using Eq.(6). This technique overcomes the logarithmic singularity at infinity in the parameter plane and makes it possible to predict relatively long thin jets.

The free-surface elevations for flow past the circular cylinder at Froude number $F = 1$ and three submergences are shown in figure 2. The wave steepness increases with the submergence while the wavelength somewhat decreases. For case (c) in figure 2 the steepness reaches its maximal value for which the free surface forms an angle of 120^0. For larger submergence, the iteration procedure diverges, which may correspond to wave breaking on the wave crest.

The free surface profiles near the cylinder are shown in figure 3. It is seen that the location of the stagnation point shown as an open circle is very close to the free surface and the flow rate ψ_0 through the splash jet is quite small. Such thin splash jets in real cases are destroyed due to interaction with air, and they return to the free surface as a spray. The similar thin splash jets and the location of the stagnation point near the free surface were predicted by Yeung [9] using a numerical method. The separation of main flow occurs at the upstream part of the cylinder so that the angle α is negative. With increase of submergence the magnitude of the angle α also increases.

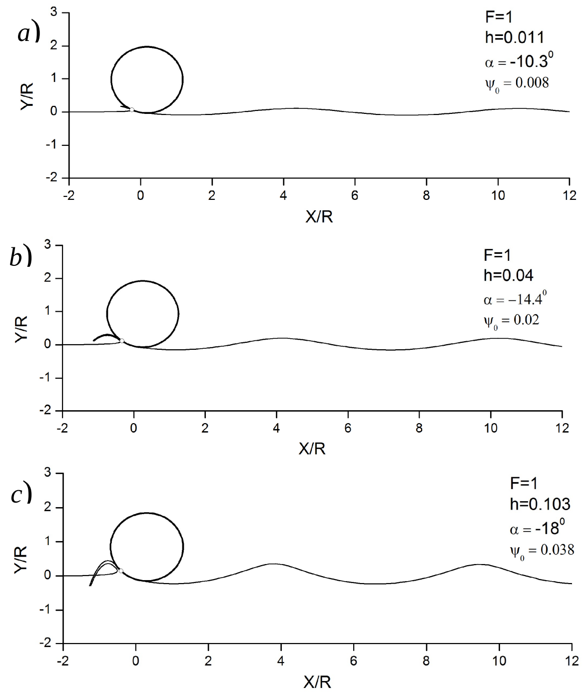

Figure 2. The free surface profile at Froude number $F = 1$ for depth of submergence (a) $h/R = 0.036$, (b) $h/R = 0.087$, (c) $h/R = 0.164$.

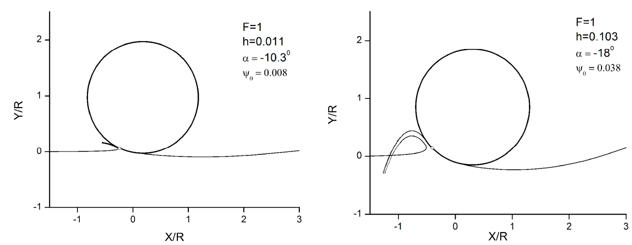

Figure 3. The free surface profile near the circular cylinder for cases (a) and (c) shown in figure 2.

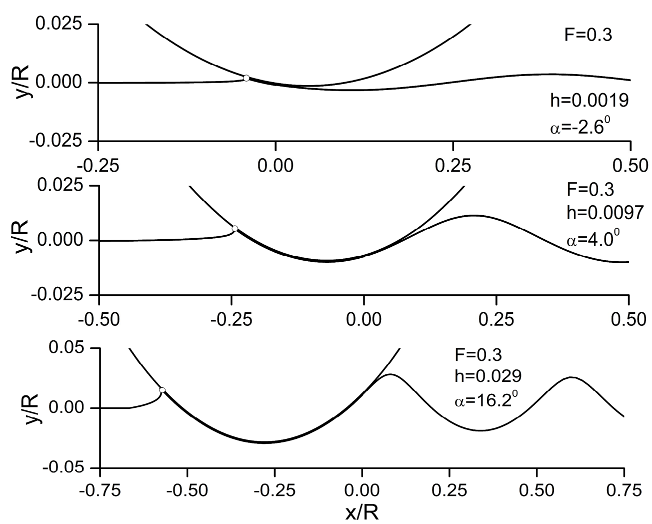

Figure 4. The free surface profile at Froude number $F = 0.3$ for three depths of submergence.

From comparison results presented in figures 2 and 4 it is seen that separation of the main flow occurs at the rear part of the cylinder when the wave length becomes smaller than the wetted part of the cylinder. At small Froude numbers the splash jet becomes too thin that is not shown in figure 4.

The presented solution makes it possible to study flow past a half-submerged flat plate moving along the free surface as a special case. We choose the length between the trailing edge of the plate and the stagnation point as a characteristic size *L*, because the total wetted length of the plate is unknown *a priory* and should be determined from the solution of the problem. Then the Froude number $F = U / \sqrt{gL}$.

In figure 5 are shown the effect of Froude number on the free surface profiles. The open circles show the location of the stagnation point while the closed circles show the point where the splash jet separates and free-falling under gravity. The interaction between the splash jet and the main incoming flow is not considered. Mathematically it means that the splash jet moves on the second sheet of the Riemann surface.

From figure 5 it is seen that the distance between the stagnation point and the point of flow separation increases with increase of the Froude number. For weightless fluid, $F=\infty$, the splash moves along the plate to infinity without separation. It should be noted that the ordinate of the trailing edge of the plate is located above the undisturbed free surface for higher Froude number.

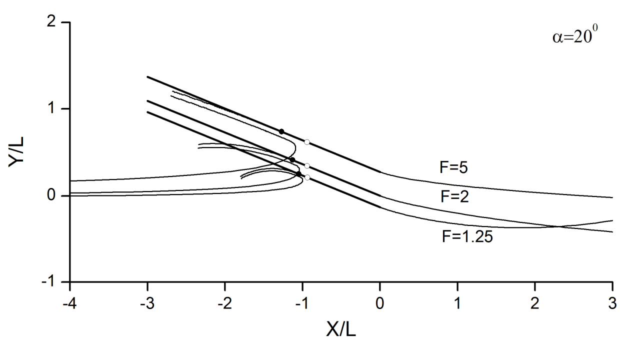

Figure 5. The free surface profile near the flat plate at the angle of attack a=20^0 for three Froude numbers.

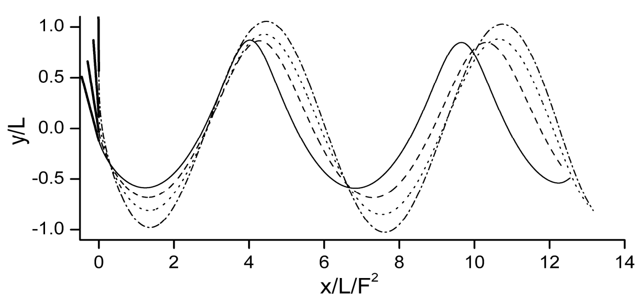

Figure 6. Free surface waves generated by the flat plate at the angle of attack a=30^0: F=1.55 (solid lines), F=2 (dashed lines), F=3 (dotted lines), F=10 (dash-dotted lines).

3. CONCLUSIONS

A complete nonlinear solution for a surface-piercing circular cylinder moving along a free surface is presented together with a special case of a plate. The method employed leads to the derivation of an analytical expression for the complex potential defined in the first quadrant of the parameter plane. The obtained solution includes both the bow and the stern flow regions.

The presented numerical results show how the free-surface shapes change as the submergence of the cylinder increases to its maximal value of a nonbreaking wave crest. It is shown that the wave profile downstream of the body corresponds to progressive waves in a fluid of infinite depth. The width of the splash jet in the bow region increases with both the Froude number and the submergence. The flow rate at the splash jet tends to zero as Froude number tends to zero.

The method presented allows one to study a more intricate geometry of the body, in particular a geometry that reflects more ship hulls.

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