Resonant scattering by an array of thin plates for wave energy extraction

E. Renzi*& F. Dias
UCD School of Mathematical Sciences
University College Dublin, Belfield Dublin 4, Ireland.
email: emiliano.renzi@ucd.ie & frederic.dias@ucd.ie

Introduction

This paper examines the wave scattering by an infinite array of thin plates in the open ocean, used for the purpose of wave energy extraction. Among the tasks necessary to describe the behaviour of such a system and to optimise its efficiency, the analysis of the scattering of the incident waves by the plates is of particular importance. Within the framework of a linear wave theory, the wave power P extracted by a single element of the array when pitching in incident waves depends on the square of the excitation torque F acting on the element when it is fixed in incoming waves, i.e. $P \propto F^2$ (Mei *et al.*, 2005). The model adopted here is based on the theoretical work of Renzi & Dias (2012), where a linear inviscid potential flow theory is devised for a single plate, either moving or fixed, in a channel. Due to the mirroring effect of the channel walls, those results can also be applied to describe the dynamics of an infinite array of plates in the open ocean.

In the following, the mathematical model of Renzi & Dias (2012) is summarised briefly. Then application of the model to the present case is discussed. The shape of the scattered waves at both sides of the plates is investigated. Asymptotic analysis of the wave field allows to obtain new expressions for the reflection and transmission coefficients. The effect of the difference in free-surface elevation at the sides of the plates in generating the excitation torque is then explained. Resonance is shown to occur at the cut-off frequencies of the system, for which transverse waves generated by the interaction of the incoming wave field with the array turn from propagating to trapped near the plates. Finally, parametric analysis highlights the dependence of the wave field and the excitation torque on the spacing between the plates. This analysis can be of interest either for the design of effective breakwaters in coastal areas (see for example Porter & Evans, 1996) and also for optimising power generating actions on a system of oscillating wave surge converters (Folley *et al.*, 2007).

Mathematical model

Consider an in-line array of identical rectangular plates hinged on a bottom foundation in an ocean of constant depth h (see for example Porter & Evans, 1996). The scattering of a train of monochromatic incident waves of amplitude A and frequency ω by such a system is investigated, assuming the plates are fixed in incoming waves. Let w and b be the width of each plate and the spatial period of the array respectively. Symmetry of the problem allows to reduce the geometry to that of a single plate within two waveguides placed at a mutual distance b (Renzi & Dias, 2012).

*Presenting author

A Cartesian coordinate system is assumed, with the x direction orthogonal to the plates, the y axis along the plate lineup and the z axis rising from the undisturbed water level $z = 0$, positive upwards. Within the framework of a linear inviscid potential-flow theory, the velocity potential Φ must satisfy the Laplace equation

$$\nabla^2\Phi(x, y, z, t) = 0 \tag{1}$$

in the fluid domain. The kinematic-dynamic boundary condition on the free surface reads

$$\frac{\partial^2\Phi}{\partial t^2} + g\frac{\partial\Phi}{\partial z} = 0, \quad z = 0, \tag{2}$$

where g is the acceleration due to gravity. Absence of normal flux at the bottom and through the waveguides requires

$$\frac{\partial\Phi}{\partial z} = 0, \quad z = -h; \quad \frac{\partial\Phi}{\partial y} = 0, \quad y = \pm b/2 \tag{3}$$

respectively. Finally, the no-flux boundary condition on the plate yields

$$\frac{\partial\Phi}{\partial x} = 0, \quad x = \pm 0, |y| < w/2, \tag{4}$$

where the thin-body approximation has been applied (Linton & McIver, 2001). Finally, the reflected and transmitted wave field respectively on the weather side and the lee side of the plate must be outgoing at large x. The set of governing equations (1)–(4) has been solved by Dalrymple & Martin (1990), Williams & Crull (1993), Porter & Evans (1996), Molin *et al.* (2005) and Renzi & Dias (2012) with different techniques. In the first three referenced works, the authors focus on the determination of the reflection and transmission coefficients, while Molin *et al.* (2005) analyse the free-surface elevation along the plate. Finally, Renzi & Dias (2012) investigate the resonant behaviour of the excitation torque in a channel. However, a complete theory treating all these aspects and their relative implications appears to be missing. Following the approach of Renzi & Dias (2012), application of the Green integral theorem to the fluid domain yields an integral equation for Φ, which is solved by decomposing the integrand into a singular part plus an analytic function. Careful treatment of the singularity ultimately allows to write the potential in a new fast-converging semi-analytical form:

$$\Phi(x, y, z, t) = \Phi^I(x, y, z, t) - \frac{1}{4\sqrt{2}}\Re\left\{igAbw\,kx\frac{\cosh k(z+h)}{\left(gh + (g/\omega)^2\sinh^2 kh\right)^{1/2}}\right.$$
$$\left.\times\sum_{p=0}^{N}\beta_p\sum_{m=-\infty}^{+\infty}\int_{-1}^{1}\left(1-u^2\right)^{1/2}U_p(u)\frac{H_1^{(1)}\left(k\sqrt{x^2+(y-\frac{1}{2}wu-bm)^2}\right)}{\sqrt{x^2+(y-\frac{1}{2}wu-bm)^2}}\,du\,e^{-i\omega t}\right\} \quad \left[m^2/s\right]. \tag{5}$$

In the latter expression, Φ^I is the well-known potential of a progressive plane wave (Mei *et al.*, 2005), k the wavenumber, U_p the Chebyshev polynomials of second kind and order $p \in \mathbb{N}$ and $H_1^{(1)}$ the Hankel function of first kind and first order. Finally, the β_p, $p = 0 \ldots N \in \mathbb{N}$, are the complex solutions of a system of linear equations which ensures that the boundary condition on the plate (4) is respected. This system is solved numerically with a collocation scheme, therefore the solution (5) is partly numerical. Once the velocity potential (5) is known, the free surface elevation $\zeta(x, y, t) = -\partial\Phi/\partial t$ and the excitation torque

$$F = -\rho\int_{S_B}\partial\Phi/\partial t\cdot(z+h-c)\,dA \quad [Nm]$$

are found accordingly, S_B being the plate surface and ρ the water density. By applying the method of stationary phase to evaluate the integral in (5) at large $|x|$, new asymptotic expressions of

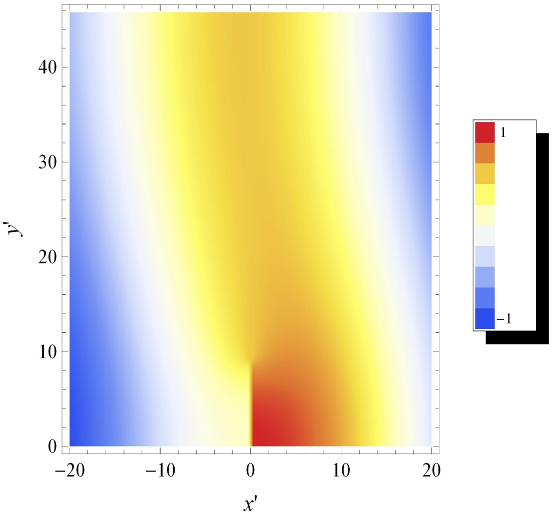

Figure 1: Density plot of the free-surface elevation around a plate. Incident waves come from the left. Half-plate is represented. Values are in metres.

the wave field can be obtained, from which new formulae for the zero-th order reflection and transmission coefficients are found, respectively

$$R_0 = -i\pi/(4\sqrt{2})w\beta_0\,\omega\cosh kh\left(gh + (g/\omega)^2\sinh^2 kh\right)^{-1/2} \tag{6}$$

and

$$T_0 = 1 + i\pi/(4\sqrt{2})w\beta_0\,\omega\cosh kh\left(gh + (g/\omega)^2\sinh^2 kh\right)^{-1/2}. \tag{7}$$

Results

Calculations have been performed for a reference array of plates of width $w = 18\,\mathrm{m}$ and spatial period $b = 91.6\,\mathrm{m}$ on a $h = 10.9\,\mathrm{m}$ deep ocean. The amplitude of the incoming wave is $A = 0.3\,\mathrm{m}$ and its period $T = 2\pi/\omega = 7\,\mathrm{s}$. Recall that this configuration is analogous to that of a single plate in a channel of width b. Figure 1 shows the density plot of the free-surface elevation on a region close to one of the plates. Incoming waves are partially obstructed by the plate and the free-surface elevation is maximum in front of it. Wave crests bend to overcome the obstacle, showing a three-dimensional refractive behaviour. In the meantime, the shading effect of the plate attenuates the wave amplitude at the lee side. Difference in free-surface elevation between the weather side and the lee side of the plate is ultimately responsible for the generation of the excitation torque F. The latter is plotted in figure 2 together with the modules of the reflection and transmission coefficients versus the ratio λ/b, varying the wavelength λ of the incident wave. Theoretical results compare satisfactorily with experimental data in wave tank. Note that spikes on the torque and the reflection and transmission coefficients occur at specific ratios $\lambda_m/b = 1/m, m \in \mathbb{N}$. The latter correspond to the cut-off wavelengths of the transverse modes generated by the interaction between the incoming wave field and the array (Chen, 1994; Renzi & Dias, 2012). Each of the transverse modes is a standing wave along y, while along x it turns from propagating to exponentially damped as soon as $\lambda \to \lambda_m$. The relevant wave energy is then trapped near the plates, inducing the spikes in the plots of figure 2. This is a particularly favourable situation for an array of wave energy converters, where the spacing b can be designed to tune the array to resonance, given the characteristics of the

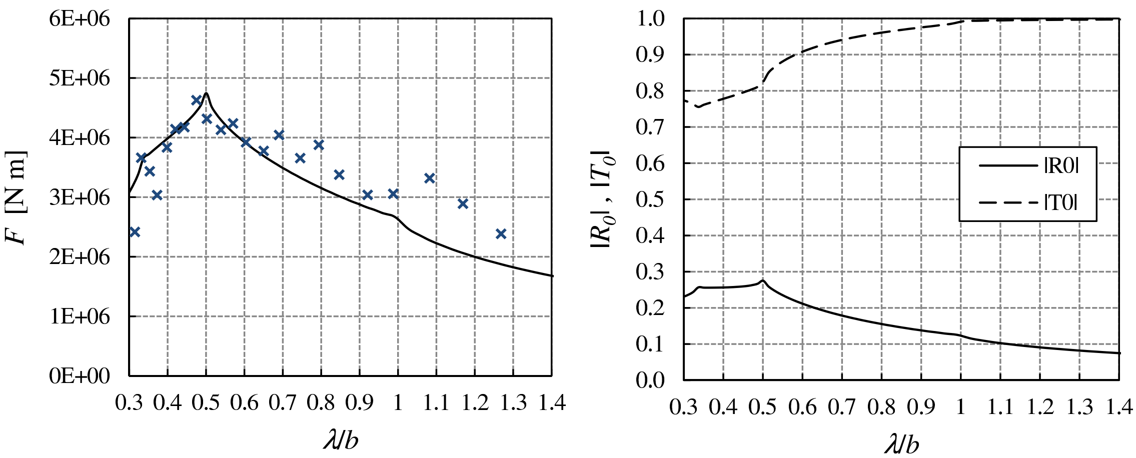

Figure 2: Left panel: torque acting on a single flap versus wavelength-to-width ratio. The solid line refers to the analytical results, while the markers refer to experimental data on a flap-type device (Folley *et al.*, 2007) for a similar geometry. Right panel: reflection and transmission coefficients versus wavelength-to-width ratio for the same system.

incoming wave field. A strong point of the theoretical model is that it easily allows the investigation of the parametric behaviour of the system's main quantities with respect to the spacing b of the plates. Parametric analysis is not included here for lack of space, but will be presented at the Workshop.

References

Chen, X. 1994. On the side wall effects upon bodies of arbitrary geometry in wave tanks. *Appl. Ocean Res.*, 16, 337–345.

Dalrymple, R.A. & Martin, P.A. 1990. Wave diffraction through offshore breakwaters. *J. Waterway, Port, Coastal and Oc. Eng.*, 116, 727–741.

Folley, M., Whittaker, T. & van't Hoff, J. 2007. The design of small seabed-mounted bottom-hinged wave energy converters. In *7th European Wave and Tidal Energy Conference*, Porto, Portugal.

Linton, C.M. & McIver, P. 2001. Mathematical techniques for wave/structure interactions. Chapman & Hall/CRC, USA.

Mei, C.C., Stiassnie, M.& Yue, D. K.-P. 2005. Theory and applications of ocean surface waves. World Scientific, USA.

Molin, B., Remy, F., Kimmoun O. & Jamois, E. 2005. The role of tertiary wave interactions in wave-body problems. *J. Fluid Mech.*, 528, 323–354.

Porter, R. & Evans, D.V. 1996. Wave scattering by periodic arrays of breakwaters. *Wave motion*, 23, 95–120.

Renzi, E. & Dias, F. 2012. Resonant behaviour of the Oscillating Wave Surge Converter in a channel. *J Fluid Mech*, under consideration for publication, 2012.

Williams, A.N. & Crull, W.W. 1993. Wave diffraction by thin-screen breakwaters. *J. Waterway, Port, Coastal and Oc. Eng.*, 119, 606–617.