The bounce of a blunt body from a water surface at high horizontal speed

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Introduction

We consider the two-dimensional blunt body impact into water with large horizontal speed, where the vertical motion of the body is free. For large horizontal speed of the body, the hydrodynamic loads can be high enough for the body to bounce out of the water without deep penetration of the body through the water surface. This phenomenon was utilised in Wallis's bouncing bomb (see Johnson, 1998). On the other hand, it can be a hazard for planing high-speed boats (see Faltinsen, 2005) and for the safe landing of aircraft on the water surface. Difficulties arise in the modelling and the computation of such impact processes, not only with identifying the location of the spray root zone but also the position of the water separation from the body surface. To understand the behaviour of the separation point during the oblique impact, simplified models of the phenomenon are required. Although we are aware of the roles of viscosity and surface tension in separation processes, we aim to model separation as inviscid (Sun and Faltinsen, 2007) which is suitable for bodies of large dimensions such as an aircraft fuselage. For oblique impact of a two-dimensional blunt body on the water surface, jets initially occur at the rear and front of the wetted area (see Fig. 1(a)); later the rear jet disappears and the fluid separates from the rear of the body surface (see Fig. 1(b)). We show that before separation starts the hydrodynamic pressure under the body falls below atmospheric pressure. In the absence of any well-established criterion, we consider three possible criteria of inviscid separation and show that the choice changes the body motion significantly.

Mathematical formulation

Initially the inviscid, incompressible fluid is at rest and occupies the lower half plane $y' < 0$. The blunt body initially touches the free surface tangentially at a single point which is taken as the origin of the Cartesian coordinate system $x'Oy'$. Then the body starts to penetrate the liquid with initial vertical velocity component V and constant horizontal velocity component U, where $\varepsilon = V/U$ is small. The shape of the body surface near its lowest point is approximated as parabolic $y' = (x' - Ut')^2/L + h'(t')$, where $L/2$ is the radius of curvature of the body surface at its lowest point and $h'(t')$ is the vertical displacement of the body at time t'. The body displacement is negative when the body moves downwards. The flow is assumed irrotational. The acceleration due to gravity is denoted by g. We assume that the Froude number $U/\sqrt{\varepsilon g L}$ is large, so gravity can be neglected in the hydrodynamic model. Surface tension and the presence of air are not taken into account. The atmospheric pressure is set to zero. We take εL as the horizontal lengthscale of the problem and the ratio $\varepsilon L/U$ as the timescale. The vertical displacement of the body is scaled by $\varepsilon^2 L$. The fluid velocity potential, pressures and the energies are scaled by $\varepsilon^2 LU$, $\varepsilon \varrho_F U^2$ and $\varepsilon^4 \varrho_F U^2 L^2$, respectively, where ϱ_F is the fluid density. We drop the primes for nondimensional variables. The body position at time t

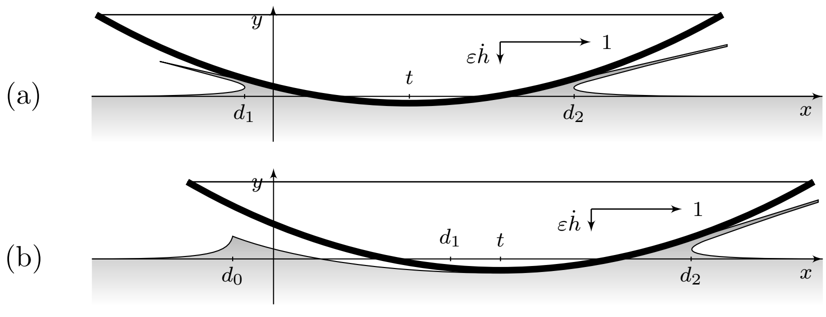

Figure 1: Blunt body impact onto deep water (a) $0 < t < t_0$, when a spray jet emerges at $x = d_1$ and (b) $t > t_0$, when the fluid separates smoothly from the body at $x = d_1$. Note that a jet is thrown forwards from the turnover point $x = d_2$ throughout. The body has unit horizontal velocity component.

is described in non-dimensional variables by the equation $y = \varepsilon\omega(x,t)$ where $\omega(x,t) = (x-t)^2 + h(t)$. The vertical motion of the body is governed by Newton's second law $\mu(\ddot{h}(t) + \kappa) = F(t)$, where $F(t)$ is the vertical component of the hydrodynamic force, $\mu = m\varrho_F^{-1}(\varepsilon L)^{-2}$ is the nondimensional mass, $\kappa = gLU^{-2}$ is the nondimensional gravity and m is the mass of the body. This is a second order differential equation with respect to the unknown function $h(t)$ subject to the initial conditions $h(0) = 0$ and $\dot{h}(0) = -1$. To calculate the hydrodynamic force, we formulate the problem for the velocity potential $\varphi(x,y,t)$ in the leading order as $\varepsilon \to 0$. In the leading order, the boundary conditions can be linearised and imposed on the initial position of the liquid boundary $y = 0$. The interval of the boundary, where $d_1 \leq x \leq d_2$, $y = 0$, corresponds to the wetted part of the moving body, which is in contact with the liquid. The rest of the boundary corresponds to the free surface, where the hydrodynamic pressure is zero. The hydrodynamic pressure $p(x,y,t)$ is given by the linearised Bernoulli's equation $p = -\varphi_t$. The free surface shape is $y = \varepsilon\eta(x,t)$, where $\eta(x,t)$ is given by the linearised kinematic boundary condition $\eta_t = \varphi_y(x,0,t)$. The velocity potential $\varphi(x,y,t)$ satisfies the following equations (see Reinhard et al. (2011)):

$$\nabla^2\varphi = 0 \qquad (y < 0) \tag{1}$$
$$\varphi_y = 2(t-x) + \dot{h}(t) \qquad (y = 0,\ d_1 < x < d_2) \tag{2}$$
$$\varphi_x = 0 \qquad (y = 0,\ x > d_2) \tag{3}$$
$$\varphi_x = \bar{\varphi}_x(x) \qquad (y = 0,\ x < d_1) \tag{4}$$
$$\varphi \to 0 \qquad (x^2 + y^2 \to \infty) \tag{5}$$

The point $x = d_2(t)$ models the forward overturning region, where a thin spray jet is formed (see Howison et al., 1991). Details of the flow in this overturning region are not considered in this study. Condition (3) implies that the speed of the forward contact point $x = d_2(t)$ is assumed positive. As to the rear contact point, $x = d_1(t)$, initially its speed is unbounded, $\dot{d}_1(t) \to -\infty$ as $t \to 0$. Therefore, there is a time interval $0 < t < t_0$, during which $\dot{d}_1(t) < 0$. This stage is referred to as the Wagner stage. During the Wagner stage the point $x = d_1(t)$ models the rear overturning region, where a spray jet is also formed. The time t_0 is such that $\dot{d}_1(t_0) = 0$. We introduce $d_0 = d_1(t_0)$ as the final position of the rear contact point at the end of the Wagner stage. In the present model, the next stage, $t > t_0$, of the impact is referred to as the separation stage. We expect that the fluid separates from the body surface in the rear of the body surface with the formation of a wake between $x = d_0$ and $x = d_1(t)$ (see Fig. 1(b)). The function $\bar{\varphi}_x(x)$ in (4) is zero during the Wagner stage, $0 < t < t_0$, and zero for $x < d_0$ during the separation stage. For $t > t_0$ the function $\bar{\varphi}_x(x)$ has to be determined as part of the solution for $d_0 < x < d_1$.

Wagner stage of oblique impact

During the Wagner stage, $0 < t < t_0$, the positions of the contact points $d_1(t)$ and $d_2(t)$ are determined by Wagner's condition (Howison et al., 1991). The solution reads

$$d_1(t) = t - \sqrt{-2h(t)}\,, \qquad d_2(t) = t + \sqrt{-2h(t)}\,, \qquad F(t) = \tfrac{\pi}{2}\frac{\mathrm{d}^2}{\mathrm{d}t^2}\left(h^2(t)\right)\,, \tag{6}$$
$$h(t) = \tfrac{1}{\pi}\left(\mu - \sqrt{\mu^2 + \pi\mu(\kappa t^2 + 2t)}\right)\,, \tag{7}$$
$$p(x,0,t) = \frac{(x - d_1 - \dot{d}_1\sqrt{-2h})^2}{\sqrt{(d_2 - x)(x - d_1)}} - (\ddot{h} + 1)\sqrt{(d_2 - x)(x - d_1)}\,. \tag{8}$$

In dimensional form, the penetration depth h' and the contact point positions $c_1' = d_1' - Ut'$ and $c_2' = d_2' - Ut'$, in the frame of reference moving with the body, are independent of the horizontal velocity U for $0 < t' < t_0'$, whereas the pressure distribution depends on U. Note that the duration of the Wagner stage, t_0', decreases as U increases. In equation (8), the numerator of the first term approaches zero as $x \to d_1$ and $\dot{d}_1 \to 0$, and the second term starts to dominate. Hence the pressure starts to be negative inside the wetted area (see Fig. 2(a)). Cavities may occur, but are not taken into consideration, here. Below d_1 and $\ddot{h}$ are shown to be discontinuous at $t = t_0$. We superscript t_0 with a minus when we refer to values before the jump and with a plus after the jump.

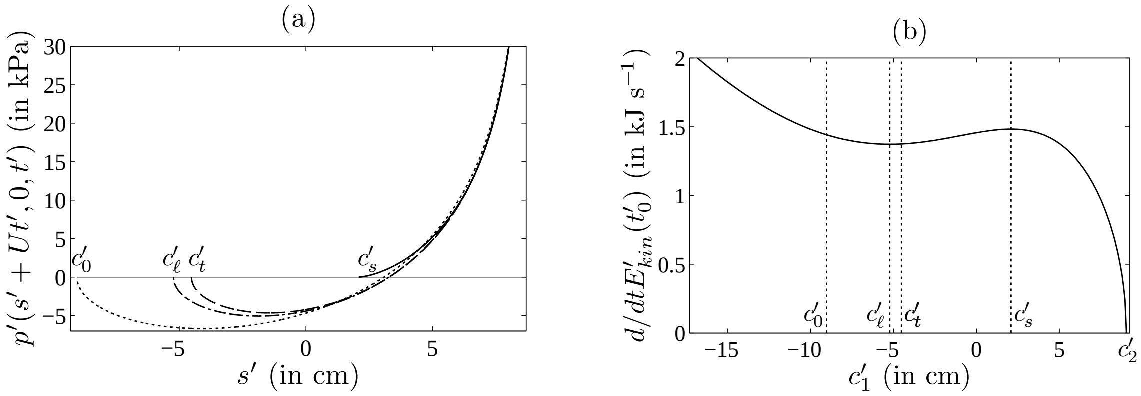

Figure 2: (a) pressure distribution at time $t = t_0'^{-}$ (dotted) as a function of the body-frame coordinate $s' = x' - Ut'$ with $c'_0 = d'_0 - Ut'$ and at $t = t_0'^{+}$ satisfying criterion 1 (solid), $c'_s = c'_1$, satisfying criterion 2 (dashed), $c'_t = c'_1$, satisfying criterion 3 (dashed-dotted), $c'_\ell = c'_1$, (b) time derivative of kinetic energy in the fluid at time $t' = t'_0$ and the position of the separation points at $t' = t_0'^{-}$ and $t' = t_0'^{+}$ (legend see (a)).

Separation stage

For $t > t_0$ we obtain the wake functions $\bar{\varphi}_x(x)$ in (4) and $\Phi_x(x, 0, x)$ (where Φ is the displacement potential $\Phi(x, y, t) = \int_0^t \varphi_x(x, y, \tau)\,\mathrm{d}\tau$) by using Kutta's condition at the separation point $|\nabla\varphi(d_1, 0, t)| < \infty$ together with the condition of continuous separation of the free surface $\eta(d_1, t) = \omega(d_1, t)$ (see Reinhard et al., 2011). These two conditions guarantee the fluid free surface meets the body's surface smoothly. A further condition is needed to determine the horizontal motion of the separation point $x = d_1(t)$ for $t > t_0$. We introduce three different separation criteria:

Separation criterion 1: Since Kutta's condition holds for $t > t_0$, the pressure has the form

$$p(x, 0, t) = -B(t)\dot{d}_2\sqrt{\tfrac{x-d_1}{d_2-x}} - (2 + \ddot{h})\sqrt{(x - d_1)(d_2 - x)} \tag{9}$$

with a square root behaviour at $x = d_1$ where

$$B(t) = \tfrac{1}{\pi(d_2-d_1)} \int_{d_0}^{d_1} \sqrt{\tfrac{d_1-\xi}{d_2-\xi}}\,\bar{\varphi}_x(\xi)\,\mathrm{d}\xi + \tfrac{1}{4}(2\dot{h} + 4t - d_1 - 3d_2) \tag{10}$$

contains an integral over the wake function $\bar{\varphi}_x(x)$. The first separation criterion assumes that, close to the separation point, the hydrodynamic pressure does not fall below atmospheric pressure, which means $p_x(d_1, 0, t) = 0$. Hence,

$$(d_2 - d_1)(2 + \ddot{h}) + \dot{d}_2 B(t) = 0\,. \tag{11}$$

If the separation is further downstream, the pressure would be negative close to the separation point, and if the separation is further upstream the wake free surface would intersect the body. This condition is equivalent to the Brillouin-Villat condition, which assumes a finite curvature of the free surface at the separation point (Crighton, 1985). It implies that $\omega(x, t) - \eta(x, t) \sim C(d_1 - x)^{5/2}$ for $x \to d_1$.

Separation criterion 2: In high-Reynolds-number cavity flow past a body experiments show that the free surface separates close to the shoulder of the body in a steady fluid flow (see Batchelor, 1967). Hence it is reasonable to model the separation (for a body moving in a fluid initially at rest) as that point on the body surface whose tangent is parallel to the body velocity $(1, \varepsilon\dot{h})$. We obtain

$$d_1 = t + \tfrac{\dot{h}}{2} \tag{12}$$

as our second separation criterion.

Separation criterion 3: We choose the position of the separation point such that the rate of increase of fluid kinetic energy, $\frac{\mathrm{d}}{\mathrm{d}t}E_{\mathrm{kin}}(t) = \int p(x, 0, t)\omega_t(x, t)\,\mathrm{d}x$, is minimised at each time $t > t_0$ when

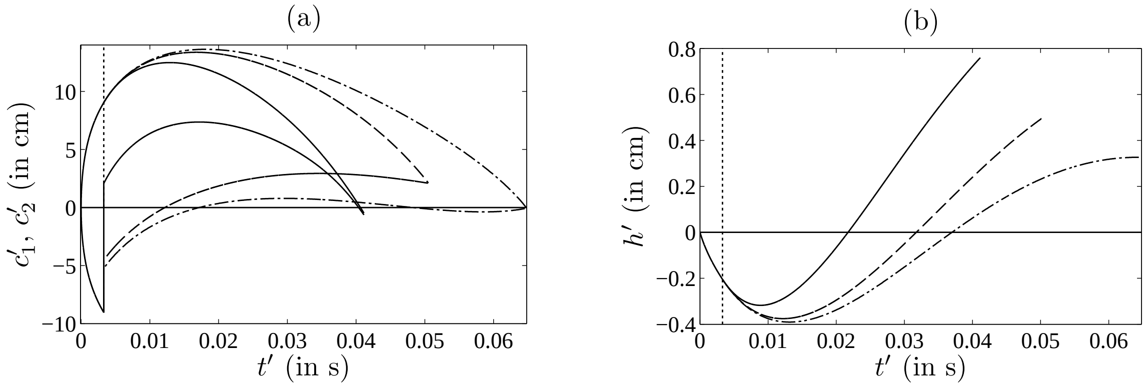

Figure 3: (a) The contact point c'_1 and c'_2 in dependence of time for the cases when the separation criterion (11) (solid), criterion (12) (dashed) and criterion (13) (dashed-dotted) are satisfied, (b) the penetration depth h' for all three separation criteria (legend see (a)).

compared with all possible separation positions. We have found that $\frac{\mathrm{d}}{\mathrm{d}t}E_{\mathrm{kin}}$ has a *local maximum* when d_1 satisfies criterion 1 (equation (11)) and has a *local minimum* when d_1 satisfies

$$\tfrac{\pi}{8}(d_2 - d_1)^3 + \mu(4t - 3d_1 - d_2 + 2\dot{h}) = 0\,, \tag{13}$$

see Fig. 2(b). We cannot choose the separation point upstream of the point $x = d_1$ satisfying (11), so we conclude equation (13) determines a *global minimum* of $\frac{\mathrm{d}}{\mathrm{d}t}E_{\mathrm{kin}}$.

Solution for the case $t > t_0$ and numerical results

For all three separation criteria the point $x = d_1$ jumps at $t = t_0$ such that $d_1(t_0^+) > d_0$. Since the pressure distribution changes abruptly at $t = t_0$ (see Fig. 2(a)), $\ddot{h}$ also jumps such that $\ddot{h}(t_0^+) > \ddot{h}(t_0^-)$. The other functions h, $\dot{h}$, d_2, $\varphi_x(x, 0, t)$ and $\Phi_x(x, 0, t)$ are continuous at $t = t_0$. We use a numerical scheme similar to that in Reinhard et al. (2011) to compute d_1, d_2, h, $\varphi_x(x, 0, t)$, $\Phi_x(x, 0, t)$ for $t > t_0$. The functions d_2 and h are calculated by applying a modified Euler's method for the ODE system provided by Newton's second law and Wagner's condition at $x = d_2$. Separation criteria 2 and 3 can be incorporated in the ODE system. In contrast, when using criterion 1, d_1 depends on $\ddot{h}$ and $\dot{d}_2$, so we update d_1 afterwards in each time step. Finally $\bar{\varphi}_x(x)$ and $\Phi_x(x, 0, x)$ are obtained from square-root singular Volterra integral equations, which follow from Kutta's condition and the continuous separation condition, by a time-marching scheme.

Numerical results are presented in Fig. 3 for a body with curvature $L/2 = 1$m, mass $m = 10$kg, horizontal velocity $U = 10\mathrm{m\,s^{-1}}$, initial vertical velocity $V = 1\mathrm{m\,s^{-1}}$ and with $g = 9.81\mathrm{m\,s^{-2}}$. In Fig. 3(a) the evolution of the contact points c'_1 and c'_2 during the Wagner stage with two jets is illustrated for $0 < t' < t'_0 = 3.3$ms. When $t' = t'_0$ the rear contact point c'_1 jumps beyond the lowest point of the body in the case when separation criterion 1 is applied. In this case the hydrodynamic loads are highest compared with separation criteria 2 and 3, and the body is quickly forced out of the fluid (Fig 3(b)). Separation criterion 3 provides the smallest jump of d_1 at $t = t_0$ and delays the exit of the body from the water at most. In this way we can model the complete bounce, from initial contact to final separation of the body from the fluid.

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