Investigating interaction effects in an array of multi-mode wave energy converters

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1 Introduction

A number of wave energy device developers have now successfully tank-tested scale-model prototypes and several are attempting full-scale deployment at sea [1]. Many believe the superior survivability of simple, buoy-like designs make them the most economically viable solutions [2, 3]. Developers of so called 'point-absorbers' hope to install multiple devices in arrays, offering considerable savings in terms of moorings, grid connections and maintenance. It is recognised that the additional hydrodynamic interactions between devices, from scattered and radiated waves within the array, can significantly alter the surface elevation and enhance the interaction factor, q, defined as the ratio of power from the array to that from the same number of isolated devices [4-6]. In contrast to traditional offshore structures, like floating platforms [7], enhancements due to these interactions could have practical benefit in the effective design of wave energy converter (WEC) arrays [8]. However, these interactions depend on numerous system variables leading to a complex array transfer function, referred to here as the Configuration Response Amplitude Operator (CRAO). There exists a CRAO specific to each possible configuration, consisting of a set of q-factors which describe the output of the array, compared to isolated devices, as a function of incident wave frequency and direction. Research directly concerning WEC arrays has focussed primarily on optimal response; however, there has been limited success in designing optimal array configurations over a range of incident wave conditions. Some novel control methods have been suggested [9, 10] and this work considers combining multiple oscillatory modes as one possible method, differing from the majority of the literature which considers single mode oscillation only (usually heave).

Numerical modelling has become increasingly important in the assessment of a given concept before going to the expense of full scale deployment. There exist a number of complex numerical methods for solving the case by case diffraction and radiation problem for floating offshore structures like tension-leg platforms (TLPs). However, when designing WEC arrays, precise computation of the interactions between multiple floating bodies is widely considered too expensive to investigate the effects of various parameters efficiently [6, 11]. Therefore, there is still a need for simplified design tools which capture the essential hydrodynamic features without the need for computationally expensive simulations. As shown here, progress can be made towards such a tool using superposition of analytical solutions, linear wave theory and various simplifications and approximations [4, 6].

In this work, the direct matrix method of Siddorn and Eatock Taylor [8] is utilised to find the surface elevation around a single, floating, 'point-absorber' WEC when subject to incident waves of different frequencies and directions. These results are then used to extend the heuristic 'Parabolic Intersection' method of Child and Venugopal [4] to enable fast array designs that include the diffraction and radiation interactions arising from floating devices. Superposition of the free surface behaviour around an isolated device is used to estimate the CRAOs for simple staggered arrays and assess the potential for improved frequency response through combined oscillation in heave and pitch.

2 Numerical Method

2.1 The Model Definition

Each device consists of an axisymmetric, truncated, circular cylinder (radius a, draft $(h-c)$), floating vertically in water with constant depth h. The cylinders are secured to the seabed via a taut tether and the PTO is represented by a spring and damper system allowing oscillation in all six degrees of freedom. This is a model which represents a wide range of realistic devices while allowing the associated computations to be performed efficiently [4].

2.2 Harmonic Waves in Water

The algebraic method of Siddorn and Eatock Taylor [8] uses the simplifying assumptions of linear potential flow and offers an approach which does not require discretisation of the fluid volume (only truncation of some series). This leads to a fast analytical solution to the diffraction and radiation of gravity waves around floating cylinders. All time-dependent quantities are assumed time-harmonic with the same frequency as the incident wave, ω. Therefore, throughout the fluid there exists a time-dependent velocity potential which, written as a complex Fourier series in cylindrical coordinates originating from the intersection of the cylinder axis and the still water level, has the form:

$$\Phi = \Phi(r,\theta,z,t) = Re\{\varphi(r,\theta,z)e^{i\omega t}\} = Re\left\{e^{i\omega t}\sum\nolimits_{l=-\infty}^{\infty} e^{il\theta}\chi_l\,(r,z)\right\}. \qquad (1)$$

What follows is a well-documented boundary value problem [8, 11, 12], the time-independent, complex spatial potential $\varphi(r,\theta,z)$ must satisfy Laplace's equation everywhere within the fluid, subject to the linearized Neumann boundary conditions; on the seafloor, the free surface and at any point on the surface of the cylinder.

2.3 Analysis of a single truncated cylinder

For the diffracted wave field at the surface, the complex velocity potential around an isolated, stationary, truncated cylinder subjected to purely progressive, plane waves is expressed, in general terms, as the summation of cylindrical waves in the coordinate system of the cylinder [8].

For the radiation problem of an oscillating truncated cylinder, in the absence of an incident wave, a complex velocity vector, $\underline{u}$, assigns the amplitude and phase of the velocity/angular velocity of all six degrees of freedom by assuming each mode is periodic with the same frequency, ω. The introduction of these oscillations alters the boundary conditions allowing a finite fluid velocity on the cylinder surface and adds an inhomogeneous term to the velocity potential [8, 11].

In both cases a 'direct-matrix' method can be used to find the coefficients of a Fourier-Bessel series, satisfying the specific boundary conditions, by truncating and simultaneously solving two, infinite systems of equations after a finite number of evanescent and core modes [8].

For the purpose of this analysis it has been assumed that the wave loading, and hence the power available to the device, depends directly on the local wave height. This makes it possible to analyse the effect of various system variables on the location of desirable device positions (where the local interaction factor, q_n, is enhanced) through consideration of the surface elevation only.

The surface elevation is found by integrating over the vertical component of the fluid velocity, at the surface, with respect to time:

$$\eta(r,\theta,z,t) = \int \left.\frac{\partial\Phi}{\partial z}\right|_{z=0} dt = Re\left\{\frac{e^{-i\frac{\pi}{2}}}{\omega}\left.\frac{\partial\Phi}{\partial z}\right|_{z=0}\right\}. \qquad (2)$$

2.4 Analysis of interference effects in a simple array

To design optimised array configurations in a fast and computationally efficient manner, a process similar to Child and Venugopal's 'parabolic intersection' (PI) method [4] has been developed. The PI method superposes wave interaction solutions, generated by isolated devices, to produce staggered array designs. Enhanced q-factors are achieved for a specific incident wave by positioning additional array members on the advantageous, down-wave intersections of parabolas resulting from constructive interference between the scattered and incident waves. Child and Venugopal [4] only optimise the array for a single incident wave frequency and direction and by ignoring the effect of radiated waves, the original PI design method approximates devices as fixed bodies. Furthermore, by constructing the array using only down-wave intersections, the original PI method neglects the effect devices have on array members up-wave of their position.

In this analysis we extend the PI method to floating bodies by considering the additional interference arising from radiated waves. To demonstrate the potential of multiple mode oscillation, this work considers devices able to oscillate freely in heave and pitch, but the concept is easily extended to encompass all degrees of freedom and different forms of mechanical damping. Finally, the effects of incident wave frequency and direction are considered to complete the design tool.

3 Results and Discussion

Figure 1 shows the interference between incident waves of a specific frequency (propagating from left to right) and each of the interacted waves around an isolated device. The parabolic patterns between incident and scattered waves, found by Child and Venugopal [4], are apparent, and it can be seen that similar patterns arise between incident waves and those radiated by heave and pitch. The discontinuity in Figure 1(c) marks a line, parallel to the incident wave crest, over which the anti-symmetric pitch wave has zero amplitude and a phase which reverses relative to the incident wave.

As an example of the array design process, for an equilaterally staggered array (with device spacing = 40*a*) the position of an additional device, up-wave and down-wave, has been marked. For this specific configuration, incident wave frequency and direction, the down-wave position neither benefits nor suffers from interference due to radiation, however, the equivalent position up-wave experiences constructive interference when heaving and deconstructive interference when pitching.

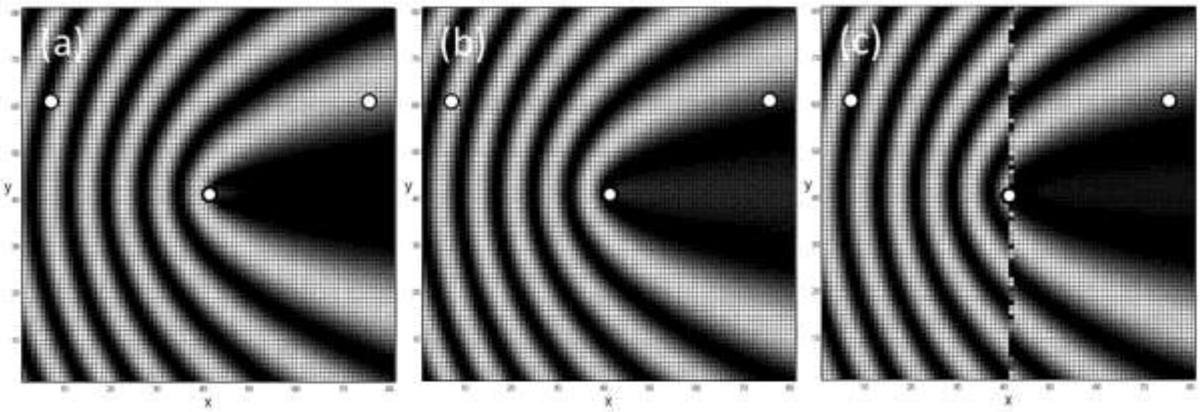

Figure 1: Phase relations between interacted waves ((a) scattered, (b) heave, (c) pitch) and incident waves for an isolated body at the free surface. White indicates completely in phase and black indicates completely out of phase. Device radius a = 1m, draught d =1m, water depth h = 8m, wave frequency, ω = 1.981rad s^{-1}, equivalent to 5m buoy in water 40m deep and wave period of 5.9s [4]. x and y units are metres/a.

Figure 2 shows the phase relations between pairs of interacted waves, isolated from the incident waves. This gives an insight into optimal device locations and the array design concept by showing where interference effects, from Figure 1, can be combined to further enhance the output of additional devices. Down-wave; heave and pitch waves are in phase with each other (2(c)) and almost in phase with the scattered waves (particularly directly down wave) (2(a) and (b)). This suggests that devices oscillating in heave and pitch simultaneously could generate particularly advantageous positions down wave. However, up wave; pitch and heave are out of phase (2(c)) and neither combines with the scattered waves (2(a) and (b)), highlighting an important difference in effects up- and down-wave.

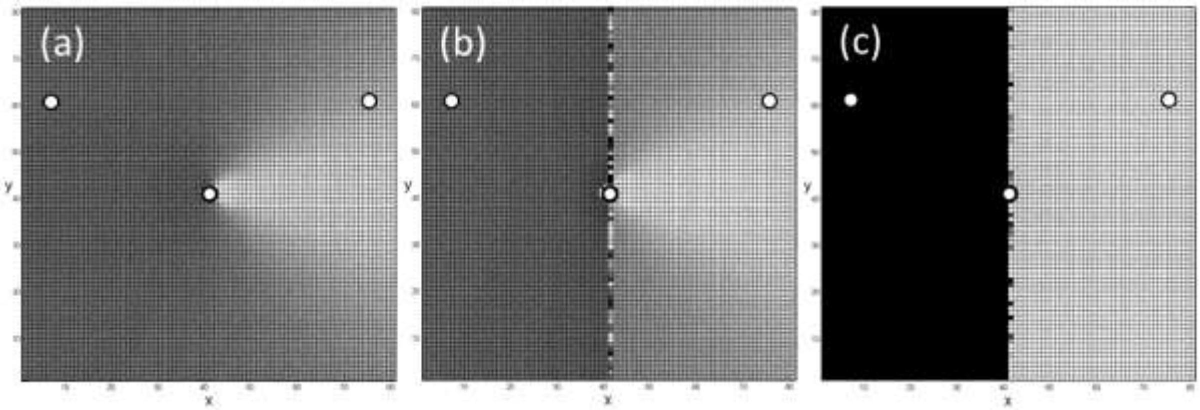

Figure 2: Phase relations between pairs of interacted waves ((a) scattered + heave, (b) scattered + pitch, (c) heave + pitch), using the same parameters as Figure 1. White = in phase, black = out of phase.

Furthermore, real sea states consist of many waves of different frequencies and headings. Presenting these interference patterns for a single frequency and direction can be misleading. The regions of constructive interference and so the optimisation method of aligning them is heavily reliant on the incident wave characteristics. Figure 3 gives an insight into how the regions in Figures 1 and 2 vary with incident wave frequency by displaying the local interaction factor, q_n, for the down-wave, 3(a), and up-wave, 3(b), positions. The main observations are: Down-wave, due to the phase relations in Figure 2, radiation in pitch and heave generally act to increases the interference effects of the scattered waves but with a slightly reduced peak frequency. This enhances the constructive effects over a range of frequencies, but also compounds the deconstructive effects; Up-wave, the *q*-factor is much more sensitive to changes in frequency due to the behaviour of the parabolas in Figure 1. Due to the phase relations in Figure 2, heave now increases the frequency at which peaks in local *q*-factor occur, whereas pitch still decreases it. This suggests that for devices able to move selectively in heave and pitch, the frequency range over which enhanced *q*-factors occur could be improved.

It is expected that variations in incident wave heading will have similarly interesting effects, on the *q*-factor, as the interference patterns are effectively rotated around the device. The combination of frequency and directional results would then form the complete CRAO for this simple array design.

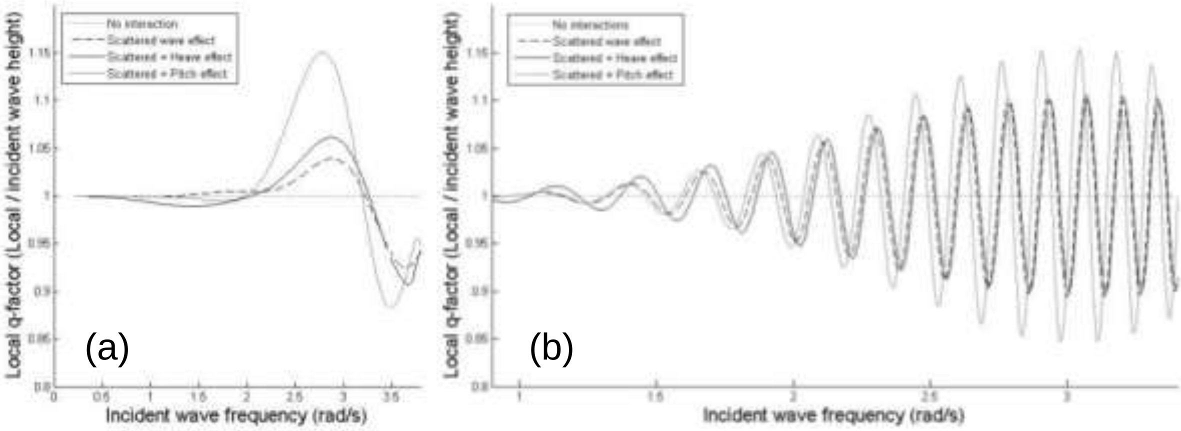

Figure 3: Frequency dependence of the *q*-factor, corresponding to the surface elevation enhancement, down-wave (a) and up-wave (b), for a fixed device (dashed), heaving device (solid black) and pitching device (solid grey). The velocity amplitude has be fixed, over the entire frequency range, to simulate a device tuned to a particular frequency ($u_{Heave} = -i\omega_0 A$, $u_{Pitch} = \omega_0^3 A/g$ where A is the incident wave amplitude and ω_0 = 1.981rad/s, is the tuning frequency), this helps to reduce unrealistically rapid oscillation at high frequencies.

4 Conclusions and Further Work

A rapid design concept for optimised arrays of floating WECs has been developed. In addition, multi-mode devices have been shown to have the potential to outperform single mode devices, over a wider frequency band. Further investigation into the directional dependence of these interactions, their effect on resonance effects and their implications on device survivability remains a task for the future along with comparisons with boundary element methods and experimental validation.

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