Second order hydroelastic response of the vertical circular cylinder to monochromatic water waves

Malenica Š. & De Hauteclocque G.

BUREAU VERITAS - Research Department, Paris, France
Email: sime.malenica@bureauverias.com

Introduction

The problem of second order hydroelastic response of the floating bodies receives more and more attention nowadays in the context of the so called springing response of some large ships such as ultra large container ships or ultra large ore carriers. The full scale measurements which are performed on these ships clearly shows important vibrations around the first structural natural modes. Even if these ships are nowadays very large (L_{pp} up to 400m) the first natural frequencies are still relatively high (≈ 2.5 rad/s) so that the excitation forces will be dominated by the non linear effects. Since the initial work of Molin [6] on second order diffraction for rigid bodies, lot of work has been done in the past on the devlopment of the efficient numerical methods for second order diffraction in both monochromatic and bichromatic waves [1, 2, 3, 7] and we can say that this problem is properly solved today for rigid body case. Recently Shao & Faltinsen [8] proposed the second order theory for flexible structure. Due to the lack of reference analytical results it is very difficult to properly validate the proposed numerical approach. The purpose of this paper is to provide the semi analytical solution for simplified configuration which could be used for validation of numerical codes. At the same time the numerical developments using the BEM code Hydrostar are also undertaken and the results will be compared.

Mathematical model

The basic configuration is shown in Figure 1. Elastic vertical column of radius a and length L is fixed at the sea bottom in the water of depth D. The problem is formulated in frequency domain and an

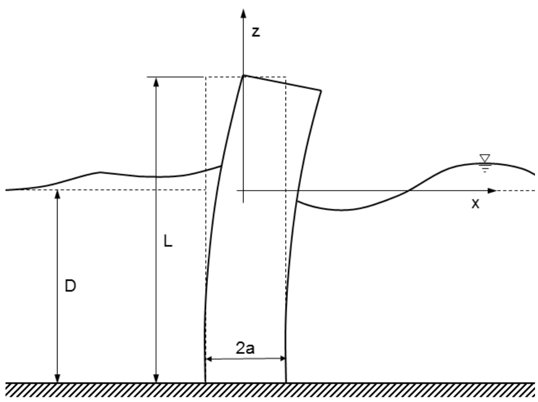

Figure 1: Basic configuration.

incomming monochromatic wave is defined, up to second order, by the following velocity potential:

$$\Phi_I(\boldsymbol{x},t) = \varepsilon\phi_I^{(1)}(\boldsymbol{x},t) + \varepsilon^2\phi_I^{(2)}(\boldsymbol{x},t) = \Re\{\varphi_I^{(1)}(\boldsymbol{x})e^{-i\omega t}\} + \Re\{\varphi_I^{(2)}(\boldsymbol{x})e^{-2i\omega t}\} \tag{1}$$

where:

$$\varphi_I^{(1)} = -\frac{igA}{\omega}\frac{\cosh k_0(z+D)}{\cosh k_0 D}e^{ik_0x} \quad , \quad \varphi_I^{(2)} = -\frac{3i\omega\nu A^2}{2\sinh^2 k_0 D}\frac{\cosh 2k_0(z+D)}{4\nu\sinh^2 k_0 D}e^{ik_0x} \tag{2}$$

Due to the action of the incident wave, the column will vibrate and these vibratory motions are described by the following vector field:

$$\boldsymbol{H}(\boldsymbol{x},t) = \sum_{i=1}^{N} \boldsymbol{\xi}_i(t)\boldsymbol{h}_i(\boldsymbol{x}) = \sum_{i=1}^{N} \left\{ \varepsilon\boldsymbol{\xi}_i^{(1)}(t) + \varepsilon^2\boldsymbol{\xi}_i^{(2)}(t) \right\} \boldsymbol{h}_i(\boldsymbol{x}) \tag{3}$$

where $\boldsymbol{h}_i(\boldsymbol{x},t)$ is the i-th modal shape function and $\boldsymbol{\xi}_i$ is its amplitude.
The interaction in between the incident wave and the column results in the perturbation velocity potential Φ_B which can be formally written as:

$$\Phi_B(\boldsymbol{x},t) = \varepsilon\phi_B^{(1)}(\boldsymbol{x},t) + \varepsilon^2\phi_B^{(2)}(\boldsymbol{x},t) = \Re\{\varphi_B^{(1)}(\boldsymbol{x})e^{-i\omega t}\} + \Re\{\varphi_B^{(2)}(\boldsymbol{x})e^{-2i\omega t}\} \tag{4}$$

At each order of approximation the corresponding Boundary Value Problem (BVP) for different potentials needs to be formulated. This is done by introducing the perturbation series expansion in the original non linear BVP and using the Taylor series expansion in order to express the different quantities at their instantaneous position as a function of their position at rest. The corresponding BVP's are composed of the Laplace equation in the fluid domain, no flow condition at the sea bed and the following free surface and body boundary conditions:
$O(\varepsilon)$

$$-\nu\varphi^{(1)} + \frac{\partial\varphi^{(1)}}{\partial z} = 0 \tag{5}$$

$$\nabla\varphi^{(1)}\boldsymbol{n} = \boldsymbol{v}^{(1)}\boldsymbol{n} \tag{6}$$

$O(\varepsilon^2)$

$$-\nu\varphi^{(2)} + \frac{\partial\varphi^{(2)}}{\partial z} = \frac{i\omega}{g}[\nabla\varphi^{(1)}\nabla\varphi^{(1)} - \frac{1}{2}(\frac{\partial^2\varphi^{(1)}}{\partial z^2} - \nu\frac{\partial\varphi^{(1)}}{\partial z})] \tag{7}$$

$$\nabla\varphi^{(2)}\boldsymbol{n} = [\boldsymbol{v}^{(2)} - (\boldsymbol{H}\nabla)\nabla\varphi^{(1)}]\boldsymbol{n} + (\boldsymbol{v}^{(1)} - \nabla\varphi^{(1)})\boldsymbol{n}^{(1)} \tag{8}$$

where $\varphi^{(1)}$ and $\varphi^{(2)}$ includes both the incident and perturbation parts, $\boldsymbol{n}$ is the local normal vector at rest, $\boldsymbol{n}^{(1)}$ is its first order correction, $\boldsymbol{v}^{(1)}$ and $\boldsymbol{v}^{(2)}$ are the first and second order local body velocities respectively.
In order to solve for the dynamic motion equation of the body, the perturbation potential φ_B is further decomposed into two parts: the first one the diffracted part φ_D which is independent of the body motions/deformations and the second one, the radiated part φ_{Rj} which depends on the body motions.
Once the different potentials calculated the pressure is calculated from Bernoulli's equation:

$$p = -\varrho[gz + \frac{\partial\Phi}{\partial t} + \frac{1}{2}(\nabla\Phi)^2] = -\varrho\{gz + \varepsilon\frac{\partial\phi^{(1)}}{\partial t} + \varepsilon^2[\frac{\partial\phi^{(2)}}{\partial t} + \frac{1}{2}(\nabla\phi^{(1)})^2 + \boldsymbol{H}\nabla\frac{\partial\phi^{(1)}}{\partial t}]\} \tag{9}$$

Finally the pressure is integrated over the wetted part of the body and the generalized modal forces are obtained:

$$\boldsymbol{F} = \int\int_{\tilde{S}_b} p\boldsymbol{H}\tilde{\boldsymbol{n}}dS \tag{10}$$

The above equation is written at the instantaneous position of the body and special attention should be given to the proper separation of different terms in order to write the final motion equations at first and second orders. These motion equations can be formally written in the following form:

$$\{-\omega^2([\boldsymbol{M}] + [\boldsymbol{A}]) - i\omega[\boldsymbol{B}] + [\mathbf{C}]\}\{\boldsymbol{\xi}^{(1)}\} = \{\boldsymbol{F}_E^{(1)}\} \tag{11}$$

$$\{-4\omega^2([\boldsymbol{M}] + [\boldsymbol{A}]) - 2i\omega[\boldsymbol{B}] + [\mathbf{C}]\}\{\boldsymbol{\xi}^{(2)}\} = \{\boldsymbol{F}_E^{(2)}\} \tag{12}$$

where $[\boldsymbol{M}]$ is the modal mass matrix, $[\boldsymbol{A}]$ is the associated added mass matrix, $[\boldsymbol{B}]$ is the damping matrix, $[\mathbf{C}]$ is the stiffnes matrix (including both hydrostatic and structural parts) and $\{\boldsymbol{F}_E^{(2)}\}$ and $\{\boldsymbol{F}_E^{(2)}\}$ are the first and second order excitation forces. The most complex part of the excitation forces is the part associated with the second order diffraction potential:

$$F^{(22)} = 2i\omega\varrho\int\int_{S_b} \varphi_D^{(2)}\boldsymbol{h}_i\boldsymbol{n}dS \tag{13}$$

Here below we concentrate on the detailed evaluation of this part in the context of the semi-analytical and numerical methods.

Semi analytical solution

The corresponding BVP for $\varphi_D^{(2)}$ can be written in the form:

$$\left.\begin{array}{lr}\Delta\varphi_D^{(2)} = 0 & \text{in the fluid} \\ -4\nu\varphi_D^{(2)} + \dfrac{\partial\varphi_D^{(2)}}{\partial z} = Q_D(x,y,0) & z = 0 \\ \dfrac{\partial\varphi_D^{(2)}}{\partial n} = 0 & S_b \\ \dfrac{\partial\varphi_D^{(2)}}{\partial z} = 0 & \text{on } z = -D\end{array}\right\} \tag{14}$$

This BVP is solved using the eigenfunction expansions method combined with the integral equation technique as explained in [5]. The final solution at the surface of the cylinder can be expressed in the following form:

$$\varphi_D(r,\theta,z) = \sum_{m=0}^{\infty} \epsilon_m \varphi_{Dm}(r,z)\cos m\theta \tag{15}$$

where ϵ_m is equal to 1 for $m = 0$ and 2 for $m > 0$, and φ_{Dm} is:

$$\varphi_{Dm}(a,z) = f_0(z)A_{m0} + \sum_{n=1}^{\infty} f_n(z)A_{mn} \tag{16}$$

with:

$$f_0(z) = \frac{\cosh k_0(z+D)}{\cosh k_0 D} \quad , \quad f_n(z) = \frac{\cosh k_n(z+D)}{\cosh k_n D} \quad , \quad 4\nu = k_0 \tanh k_0 D = -k_n \tan k_n D \tag{17}$$

and:

$$A_{m0} = -\frac{2C_0 \int_a^{\infty} H_m(k_0\rho)Q_{Dm}(\rho)\rho d\rho}{k_0 a H_m'(k_0 a)} \quad , \quad A_{mn} = -\frac{2C_n \int_a^{\infty} K_m(k_n\rho)Q_{Dm}(\rho)\rho d\rho}{k_n a K_m'(k_n a)} \tag{18}$$

Where H_m denotes the Hankel functions and K_m the modified Bessel functions and C_0 and C_n are the integration constants.
Here we are considering the cylinder vibratory response in bending only, so that the mode shape can be written in the following form:

$$\boldsymbol{h}_i = h_{ix}(z)\boldsymbol{i} + 0\boldsymbol{j} + 0\boldsymbol{k} \tag{19}$$

Knowing that $n_x = \cos\theta$ on the cylinder surface, the second order force $F^{(22)}$ becomes:

$$F_i^{(22)} = 4i\omega\varrho\pi\{A_{10}\int_{-D}^{0} f_0(z)h_{ix}(z)dz + \sum_{n=1}^{\infty} A_{1n}\int_{-D}^{0} f_n(z)h_{ix}(z)dz\} \tag{20}$$

Numerical solution

The numerical model is based on the well know Boundary Integral Equation (BIE) technique in the context of the BV numerical code Hydrostar which has already been used successfully for evaluation of the second order springing excitation for TLP platforms. Within this method the second order potential is obtained from the following BIE:

$$2\pi\varphi_D(\boldsymbol{x}) + \int\int_{S_b} \varphi_D(\boldsymbol{\xi})\frac{\partial G(\boldsymbol{\xi};\boldsymbol{\xi})}{\partial n_\xi} = \int\int_{S_F} Q_D(\boldsymbol{\xi})G(\boldsymbol{x};\boldsymbol{\xi})dS \tag{21}$$

where $G(\boldsymbol{x};\boldsymbol{\xi})$ is the Green function and subscript $_\xi$ means that the derivative should be taken with respect to the variable $\boldsymbol{\xi}$.
This BIE is solved by discretising the mean wetted surface of the body S_b into a certain number of panels on which the constant distribution of the potential is assumed. One of the main difficulties in solving the above equation is the evaluation ofthe integral over the free surface S_F. Indeed this integral is highly

oscillatory and, strictly speaking, the integration should be performed up to infinity. In practice, close to the body a numerical integration is performed and far from the body asymptotic expressions are used for different quantities and the integration is performed semi analytically. Once the potential calculated on each panel it is integrated numerically over the wetted body surface and the second order forces $F_i^{(22)}$ are obtained. Let us also mention that the second order forces can also be evaluated using the indirect approach where the use of an assisting radiation potential is made in order to avoid the direct calculation of the potential. This method is known as Haskind method and was commonly applied in the past for rigid body. Here we show that it is also applicable to flexible body provided the assisting radiation potential accounts for correct body boundary condition.

Few preliminary results

First preliminary results are presented below for rigid body mode shape. In Figure 2 we show the comparison of second order surge ($\boldsymbol{h} = \cos\theta \boldsymbol{i}$) diffraction force obtained by 3 different methods: semi analytical, direct numerical and indirect numerical. The cylinder radius is $a = 20m$ and water depth is $D = 3a$. More detailed results will be presented at the Workshop.

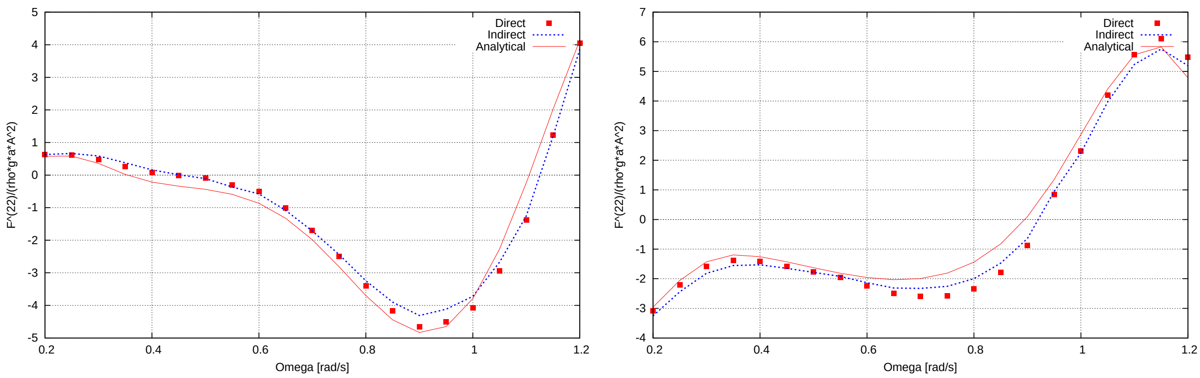

Figure 2: Real (left) and imaginary (right) parts of the second order diffraction force.

References

[1] Chau F.P. 1989. : "The second order velocity potential for diffraction of waves by fixed offshore structures.", PhD Dissertation, University College London.

[2] Chen X.B., Molin B. & Petitjean F. 1991. : "Faster evaluation of resonant exciting loads on tension leg platforms.", 7th Symp. on Offshore Engg., Brasil Offshore 91, Rio de Janeiro, Brasil.

[3] Kim M.H. & Yue D. 1995. : "The complete second-order diffraction solution for an axisymmetric body. Part 1. Monochromatic incident waves.", Journal of Fluid Mechanics, vol. 200, pp.235-264.

[4] Malenica S. 1998. : "Hydroelastic coupling of beam structural model with 3D hydrodynamic model.", Int. Conf. on Hydroelasticity, Kyushu, Japan.

[5] Malenica S. & Molin B. 1995. : "Third harmonic wave diffraction by a vertical cylinder.", Journal of Fluid Mechanics, vol. 302, pp.203-229.

[6] Molin B. 1979. : "Second order diffraction loads upon three dimensional bodies.", Applied Ocean Research, Vol. 1., pp. 197-202.

[7] Newman J.N. & Lee C.H. 1992. : "Sensitivity of wave loads to the discretisation of bodies.", Int. Conf. on behavior of Off Shore structures, BOSS 92, London, UK.

[8] Shao Y.L. & faltinsen O.M. 2012. : "A numerical study of the second order wave excitation of ship springing with infinite water depth.", Journal of Engg. for Maritime Environment, Submitted for publication