Abstract submitted to the 27th IWWWFB, Copenhagen, Denmark, 22-25 April 2012

Dissipation in the gap resonance between two bodies

L. Lu[1] and **X.B. Chen**[2]

[1]Centre for Deepwater Engineering, Dalian University of Technology, 116024 Dalian, China

[2]Research Department, BV, 92571 Neuilly-Sur-Seine, France

xiao-bo.chen@bureauveritas.com

The energy dissipation in the fluid domain is evaluated by using CFD method when a resonance of wave elevations occurs in the gap between two bodies. The introduction of dissipation in the classical potential method permits to predict the wave elevation close to model tests. The estimation of associated dissipation coefficient is performed by the equivalence of dissipative force and friction forces on gap walls, and by the equivalence of energy dissipation rate and the power consumed on the gap free surface by the dissipation force. It is shown that the friction on gap walls represents only 1/6 of total dissipation, and that the introduction of full dissipation in the domain around the gap gives correct predictions.

1 Introduction

Water wave-induced gap resonance can be observed as the field operations of multiple floating structures in ocean are arranged side by side with small separations. Much efforts have been devoted to this problem including the model tests, theoretical analysis and numerical investigations. This work concerns with the numerical modeling of the resonant phenomenon and the physical aspect of energy dissipation. The former aims to develop dissipative numerical model in the framework of potential flow theory, while the latter is achieved based on the CFD results of viscous fluid model.

The early numerical examinations are mainly based on the classical potential flow model. These previous potential flow results have demonstrated clearly the substantial over-predictions of the resonant wave height in the narrow gap and wave forces on the floating bodies although the classical potential flow theory is applied to the accurate prediction of the resonant frequency and is able to produce reasonable fluid oscillation and wave forces outside the resonant frequency region.

The straightforward method for the investigation of gap resonance problem might be using the viscous fluid model by solving the Navier-Stokes equations (Lu et al., 2011). In this manner, the energy dissipation, including the viscous effect, vortices motion and even turbulence, can be considered together. In addition, the coupling of potential flow model (in the far field) and CFD method (for near-solid region) seems also to be promising (Kristiansen and Faltinsen, 2011). However, the viscous fluid model is much time-consuming. It is desirable if the energy dissipation resulted from fluid viscosity, rotational flow and even vortex shedding can be considered or modeled partially in the context of potential flow theory with high computational efficiency. The energy dissipation is believed to play the first role in limiting the resonant wave height in reality. The typical attempts to introduce the energy dissipation into potential flow model include the methods of using the rigid lid (Huijsmans er al. 2001) and flexible mat associated with linear damping term (Newman, 2004). The introduction of linear dissipative term in free surface boundary condition (Chen, 2004) was based on the notion of fairly perfect fluid in which a fictitious dissipative force is introduced in the fluid. This approach was then followed by several recent work (Pauw et al., 2007; Bunnik et al., 2009).

The available numerical comparisons (Lu et al. 2011) show that the potential flow model with artificial dissipation term may produce satisfying predictions on the resonant wave height. However, the necessary dissipation coefficient has to be calibrated by using the experimental data or CFD results. Therefore, the remained question is that how to determine the dissipation coefficient under the situations that the experimental data or CFD results are not available. The recent progress focusing on this issue will be presented in this work based on the oscillating boundary layer theory. The dissipation rate and its space distribution will be examined in this work based on the CFD results.

2 Potential flow model with dissipation forces

The conventional potential flow model is based on the assumptions that the fluid flow is incompressible, inviscid and irrotational. The mass conservation in two-dimensional space can be expressed by the Laplace equation of velocity potential $\Phi(x, y, t)$ as $\Phi_{xx} + \Phi_{yy} = 0$. For the harmonic wave motion, the potential is defined as $\Phi(x, y, t) = \Re\{\phi(x, y)e^{-i\omega t}\}$ and the complex potential $\phi(x, y)$ can be written as the sum of the incident potential ϕ_I and the scattering potential ϕ_S, that is,

$$\phi(x, y) = \phi_I(x, y) + \phi_S(x, y) \quad \text{with} \quad \phi_I(x, y) = -\frac{igA}{\omega}\frac{\cosh(z+h)}{\cosh kh}e^{ikx} \tag{1}$$

in which, g is the gravitational acceleration, A is the incident wave amplitude, $\omega = 2\pi/T$ is the wave angular frequency and T is the wave period, $k = 2\pi/L$ is the wave number and L is the wave length and h is the water depth.

In order to model the flow resistance in the narrow gap within the framework of potential flow theory, an artificial dissipation force is introduced (Chen, 2004) so that the boundary condition on the free surface is augmented by an additional term :

$$-\omega^2\phi_S + g\partial\phi_S/\partial z - i\epsilon\omega\phi_S = 0 \tag{2}$$

where ϵ is the artificial dissipation coefficient. Accordingly, the first order wave elevation reads

$$\eta = (i\omega - \epsilon)\phi/g \tag{3}$$

The boundary condition (2) for the scattering potential ϕ_s is applied on S_F shown on Fig.1 with $\epsilon = 0$ and $\epsilon \neq 0$ at the free surface in gap S_G. $\partial\phi_S/\partial n = -\partial\phi_I/\partial n$ along the body hull S_B and at the sea bottom S_D (in fact, $\partial\phi_S/\partial z = 0$). At the left and right surfaces (S_L, S_R), we have $\partial\phi_S/\partial x = ik\phi_s$ on S_L and $\partial\phi_S/\partial x = -ik\phi_S$ on S_R, respectively.

The above governing equations together with the boundary conditions are solved using a boundary element method. The previous numerical investigations (Lu et al., 2011) have shown that the potential flow model with artificial dissipation force is able to predict satisfactory results for resonant wave heights if an appropriate dissipation coefficient is used. The dissipation coefficient can be obtained by the comparisons with available experimental data or the results of viscous fluid model. It was also confirmed that the dissipation coefficient is not sensitive to the changes of geometric characteristics, including the gap width, draft, breadth ratio of bodies and floating body number, regarding the evaluations of not only the resonant condition but also the resonant wave height and wave forces.

3 Dissipation coefficient associated with friction forces

Considering that the fluid oscillation in the narrow gap might be thought of as the oscillation boundary layer flow, it implies that the available boundary layer theory might be used to approximate the drag force on the wet surface of floating bodies. In this way, at least the viscous contribution can be considered partially in the potential flow model by using the concept of dissipation force.

Assuming that the difference between the wave amplitudes obtained by the potential flow models with and that without the dissipation force, following (3), is given by

$$\Delta\eta = -\epsilon\phi/g \tag{4}$$

which represents an additional pressure p and force F_{gap} on the gap free surface :

$$\Delta p = \rho g \Delta\eta = -\rho\epsilon\phi \quad \text{and} \quad F_{\text{gap}} = \int_B^{B+B_G} dx\,\Delta p \tag{5}$$

This additional pressure can be interpreted as an external force on the free surface to resist the large amplitude wave oscillation in the narrow gap. Now we assume that the physical resistance of the oscillating fluid in the narrow gap is only resulted from the shear stress on the both sides of the narrow gap. In other words, only

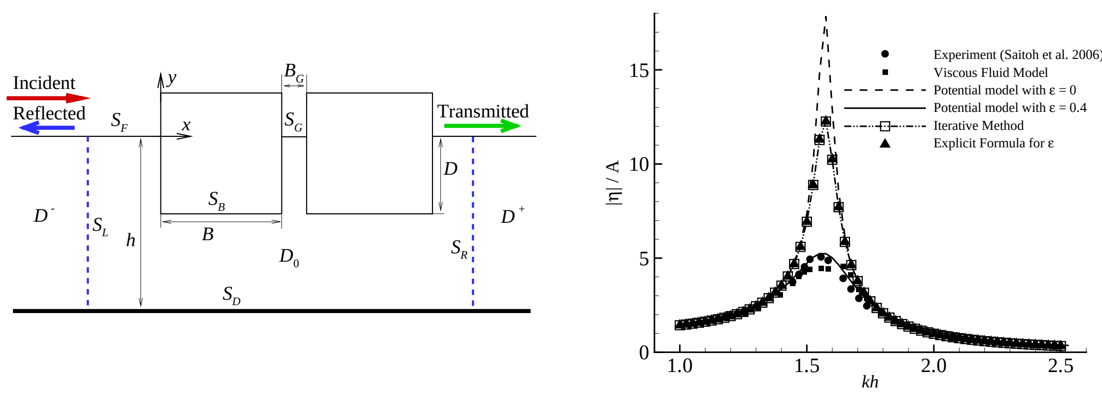

Figure 1: Definitions (left) and Comparison of wave elevation in the gap (right)

the frictional force is considered. According to the oscillating boundary layer theory, the shear stress on the solid wall can be evaluated by

$$\tau = -(1/2)\rho C_f |\phi_z|\phi_z \tag{6}$$

where C_f is the frictional coefficient and ϕ_z is the vertical velocity of fluid particle along the gap walls. For the laminar oscillating boundary (Jensen et al., 1989), the frictional coefficient reads :

$$C_f = 2/Re^{1/2} \quad \text{with the Reynolds number} \quad Re = a|\phi_z|/\nu \tag{7}$$

where a is the amplitude of oscillating fluid particle and ν the kinematic viscosity. The total viscous resistance is given by :

$$F_{\text{vis}} = \int_{-D}^{0} dy \, (\tau|_{x=B} + \tau|_{x=B+B_G}) \tag{8}$$

By making the equation between $F_{\text{gap}} = F_{\text{vis}}$, we have

$$2\epsilon \int_{B}^{B+B_G} dx \, \phi = C_f \int_{-D}^{0} dy \, (|\phi_z|\phi_z\,|_{x=B} + |\phi_z|\phi_z\,|_{x=B+B_G}) \tag{9}$$

Since the solutions ϕ and ϕ_z depends on the value of ϵ, an iterative algorithm can be applied to (9) to determine the dissipation coefficient. In general, only several iterations are necessary to make convergence.

In the narrow gap $a = |\eta|$, we assume the water goes up and down uniformly since the variation of ϕ and ϕ_z is negligible along the gap width. Introducing the identity $\phi_z = \phi\omega^2/g$ in (7), we have the friction coefficient :

$$C_f = 2\sqrt{\nu}g/(\omega^3\phi^2)^{1/2} \tag{10}$$

and (9) is reduced to :

$$2\epsilon B_G \phi = 2\sqrt{\nu}g/(\omega^3\phi^2)^{1/2} \cdot 2D|\phi|\phi\omega^4/g^2 \tag{11}$$

and we have an explicit formula of the dissipation coefficient :

$$\epsilon = 2\sqrt{\nu}\omega^{5/2}D/(gB_G) \tag{12}$$

which shows that the dissipation coefficient is dependent on the square root of kinematic viscosity and proportional to 2.5th power of wave frequency and linearly to the ratio D/B_G.

The numerical results of wave elevation in the gap η/A obtained by the iterative method and the explicit formula (12) are depicted on the right of Figure 1 together with the available experimental data, numerical results from the viscous fluid model and the potential model with $\epsilon = 0$ and $\epsilon = 0.4$.

On the right of Figure 1, it shows that the iterative method and the method using explicit dissipation coefficient produce almost identical wave elevation in the narrow gap throughout the frequency band considered here. This means that the assumption of uniform kinematics in the gap is valid. The predicted resonant wave elevation by using the dissipation coefficient to assimilate the friction force based on the oscillating boundary layer theory, is smaller than that predicted by the conventional potential flow model (dissipation free). The reduction of the wave height is about 30% percent of the wave height predicted by the method without dissipation but still much larger than the results from model tests or CFD computations. The over-prediction of the resonant wave height can be explained by the fact that the dissipation other than the friction in the gap and the dissipation in the fluid domain outside of the gap is important and not yet taken into account.

4 Energy dissipation evaluated by CFD computations

The viscous fluid model is based on the finite element solution of the two-dimensional Navier-Stokes equations for the incompressible Newtonian fluids

$$\frac{\partial u_i}{\partial x_i} = \begin{cases} 0 & \mathbf{x} \notin \Omega_s \\ q(\mathbf{x}, t) & \mathbf{x} \in \Omega_s \end{cases} \tag{13}$$

and

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = \frac{1}{\rho}\frac{\partial p}{\partial x_i} + \frac{2\mu}{\rho}\frac{\partial D_{ij}}{\partial x_j} + f_i + \frac{\mu}{3\rho}\frac{\partial q}{\partial x_i} \tag{14}$$

where u_i and f_i are the velocity and body forces components in the ith direction and $i = (1, 2) = (x, y)$ is used, p is pressure, ρ=1000kg/m^3 and μ=0.001 kg/(m·s) are the fluid density and dynamic viscosity, respectively, $q(x, t)$ is a source term defined in the restricted domain Ω_s used for internal wave maker method. Finally, $D_{ij} = (\partial u_i/\partial x_j + \partial u_j/\partial u_i)/2$ is the strain rate tensor. The details of the numerical implement and validation against experimental data for the gap resonance problem are referred to Lu et al. (2011).

The total rate of dissipation in viscous flows can be evaluated as given in Lamb (1932) :

$$\Theta = \int_\Omega d\Omega \, 2\mu D_{ij} D_{ij} \tag{15}$$

It is believed the energy dissipation mainly happens in the near region around the floating structure. For this purpose, three different zones in which the rate of energy dissipation is evaluated are defined as the gap zone Z1 limited by the gap walls and free surface, the middle zone Z2 extending Z1 to the sea bed and the full zone Z3 being Z2 plus the areas under body bottom.

The variation of energy dissipation rate defined by (15) with respect to the wavenumber kh of incoming waves is illustrated on Figure 2. On the left, the value of dissipation Θ is non-dimensionalized by dividing $\mu(\eta/A)^2/T$ while, on the right, the value of Θ is further divided by the area of relevant zone and depicted. It can be observed that the non-dimensional dissipation is more or less constant in the close interval around the resonant wavenumber $kh = 1.56$ and that the density of dissipation rate is the largest in Z1, and that the density in Z2 is larger than that in Z3. Furthermore, if we assimilate the dissipation by the power (average

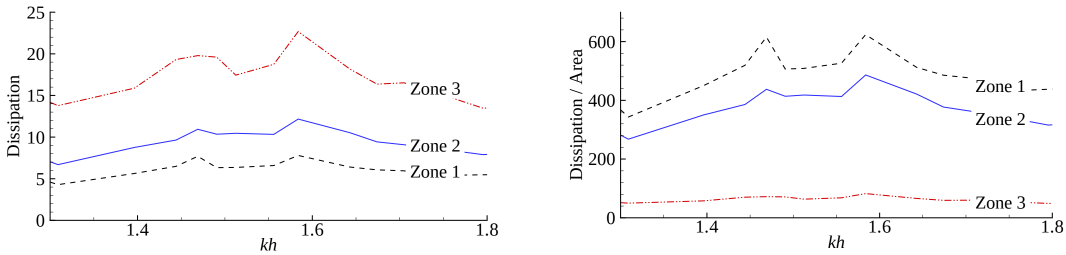

Figure 2: Energy dissipation rates (left) and dissipation density (right) in different zones

in one period) performed by the additional pressure (5) over the gap free surface :

$$\Theta|_{Z1} = \int_B^{B+B_G} dx \, \Delta p \phi_z \approx \epsilon \rho g |\eta|^2 B_G/2 \tag{16}$$

which gives $\epsilon \approx 0.4$ if we take $\Theta|_{Z1} \approx 17\mu(\eta/A)^2/T$. Indeed, as shown on Figure 1, the prediction with $\epsilon = 0.4$ is in good agreement with model tests and CFD computations.

Acknowledgement

This work was conducted when the first author worked at the Research Department of Bureau Veritas as a visiting scholar. The finical supports from NSFC with granted Nos. 50909016 and 50921001 are acknowledged.

References

[1] X.B. Chen, 2004 "Hydrodynamics in offshore and naval applications Part I". Keynote lecture on 6th ICHD'2004, Perth, Australia.

[2] T. Bunnik, W. Pauw, A. Voogt, 2009, "Hydrodynamic analysis for side-by-side offloading". Proc. 19th ISOPE, Osaka, Japon, 648653.

[3] R.H.M. Huijsmans, J.A. Pinkster, J.J. Wilde, 2001, "Diffraction and radiation of waves around side by side moored vessels". Proc 11th ISOPE, Cupertino, Italy, 406-412.

[4] B.L. Jensen, B.M. Sumer and J. Fredsoe, 1989, "Turbulent oscillatory boundary layers at high Reynolds numbers". J. Fluid Mech, 206, 265-297.

[5] T. Kristiansen, O.M. Faltinsen, 2010, "A two-dimensional numerical and experimental study of resonant coupled ship and piston-mode motion". App. Ocean Res, 32, 158-176.

[6] H. Lamb, 1932, "Hydrodynamics". Dover Publications, New York, USA.

[7] L. Lu, B. Teng, L. Cheng, L. Sun and X.B. Chen, 2011, "Modelling of multi-bodies in close proximity under water waves - Fluid resonance in narrow gaps". SCIENCE CHINA Physics, Mechanics & Astronomy, 54 (1): 16-25.

[8] J.N. Newman, 2004, "Progress in wave load computations on offshore structures". Proc 23th OMAE, Vancouver, Canada.

[9] W.H. Pauw WH, R. Huijsmans and A. Voogt, 2007, "Advances in the hydrodynamics of side-by-side moored vessels". Proc. 26th OMAE, New York, USA.

[10] T. Saitoh, G.P. Miao and H. Ishida, 2006, "Theoretical analysis on appearance condition of fluid resonance in a narrow gap between two modules of very large floating structure". Proc. 3rd Asia-Pacific Workshop Marine Hydro, Shanghai, China, 170-175.