INCLINED IMPACT OF A SMOOTH BODY ON THIN LIQUID LAYER

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Inclined impact of a body is usually studied in connection with emergency landing of aircraft on the water. The body motions during the landing and the hydrodynamic loads acting on the body surface are of primary interest. The process of the inclined impact can be divided into two phases. During the first phase spray jets are formed along the periphery of the contact region. During the second phase the free surface of the liquid separates from the surface of the moving body in its rear part. The first phase is referred to as the impact phase and the second one as planing phase (Fig. 1 a,b).

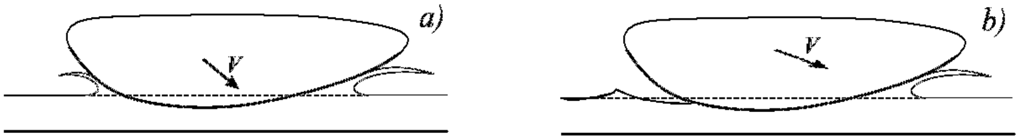

Fig. 1

Two-dimensional problem of the inclined impact of a rigid body with smooth surface onto the thin layer of an ideal incompressible fluid is considered. The problem is coupled: liquid flow, body motions with three degrees of freedom, hydrodynamic loads distributed along the contact region and the position of this contact region on the body surface should be determined simultaneously. The liquid flow during the impact phase is obtained by using the approach from [1]. This approach is based on the method of matched asymptotic expansions. The flow region is subdivided into several subdomains: the region beneath the entering body surface, the jet roots, the spray jets, and the outer regions. A complete solution is obtained by matching the solutions in these subdomains.

During the second phase a main attention is given to conditions at the separation point. The coupled problem is reduced to a system of integro-differential equations. The equations are solved numerically. Displacements and rotation of the body caused by the hydrodynamic loads during both phases are investigated.

Formulation of the problem

We use the global coordinate system (x, y) and the local coordinate system (ξ, η) moving with the body. The line $y = 0$ corresponds to the bottom of the liquid layer and $y = H$ to the initial position of the liquid free surface. Initially the liquid is at rest. Motions of the body are described by the coordinates of its centre of mass $x_0(t), y_0(t)$ and its angle of inclination $\alpha(t)$ (Fig. 2).

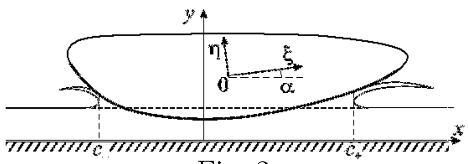

Fig. 2

The origin of the global coordinate system is on the bottom, with $x = 0$, $y = H$ corresponding to the point of the first contact between the body and the free surface. The lower part of the body surface in the local system is described by the equation $\eta = F(\xi)$, where $F(\xi)$ is a given smooth function. Initial value of the angle $\alpha(0)$ and values of the derivatives $\dot{x}_0(0), \dot{y}_0(0), \dot{\alpha}_0(0)$ are given. The values $x_0(0)$, $y_0(0)$ are calculated, as well as the local coordinate ξ_0 at which the first contact occurs. The local and global coordinates are related by the equations

$$x = x_0(t) + \xi \cos\alpha - \eta \sin\alpha, \qquad y = y_0(t) + \xi \sin\alpha + \eta \cos\alpha. \tag{1}$$

On the body surface, $\eta = F(\xi)$ and equations (1) provide x and y on the surface of the moving body as

$$x = X(\xi, t), \qquad y = Y(\xi, t). \tag{2}$$

The first equation in (2) can be inverted with respect to $\xi = \xi(x, t)$ and the second equation gives

$$y = y_b(x, t). \tag{3}$$

The body motions are governed by the equations

$$m\ddot{y}_0 = F_y(t) - mg, \qquad m\ddot{x}_0 = F_x(t), \qquad J\ddot{\alpha} = M(t), \tag{4}$$

where m is the mass of the body, J is the moment of inertia, $F_x(t)$ and $F_y(t)$ are the horizontal and vertical components of the hydrodynamic force acting on the body surface in the contact region, M is the moment of the hydrodynamic force, and g is the acceleration due to gravity. Dot stands for the time derivative.

Coupled solution of the problem

The interval $c_-(t) < x < c_+(t)$ corresponds to the contact region between the body surface and the liquid. In the model of thin liquid layer, the hydrodynamic pressure $p(x,t)$ and horizontal component of the flow $u(x,t)$ are independent of the vertical coordinate y, and satisfy the following equations 1

$$\frac{\partial}{\partial x}(u(x,t)y_b(x,t)) + \frac{\partial y_b}{\partial t} = 0, \qquad u_t + uu_x = -\frac{1}{\rho}p_x \tag{5}$$

beneath the body surface, $c_- < x < c_+$. The right-hand sides in equations (4) are given by

$$F_y(t) = \int_{c_-}^{c_+} p(x,t)\,dx + \int_{c_-}^{c_+} \rho g(H - y_b(x,t))\,dx, \tag{6}$$

$$F_x(t) = \int_{c_-}^{c_+} p(x,t)y_{bx}(x,t)\,dx + \int_{c_-}^{c_+} \rho g(H - y_b(x,t))y_{bx}\,dx, \tag{7}$$

$$M(t) = \int_{c_-}^{c_+} p(x,t)[(x-x_0)+(y-y_0)y_{bx}]\,dx + \int_{c_-}^{c_+} \rho g(H - y_b(x,t))[(x-x_0)+(y-y_0)y_{bx}]\,dx, \tag{8}$$

where the buoyancy force is included.

Differentiating equations (1)-(3) in time, we find

$$\frac{\partial y_b}{\partial t} = \dot y_0 - \dot x_0 y_{bx} + \dot\alpha[(x-x_0)+(y-y_0)y_{bx}],$$

which makes it possible to integrate the first equation in (5) as

$$u(x,t)y_b(x,t) = \dot x_0 y_b - \dot y_0 x - \frac{\dot\alpha}{2}[(x-x_0)^2 + (y_b-y_0)^2] + C(t), \tag{9}$$

where $C(t)$ is a constant of integration. The function $C(t)$ has to be determine as part of the solution.

The time derivative $\dot u(x,t)$ which is required to calculate the pressure distribution $p(x,t)$ is presented here in the form

$$\dot u(x,t) = \ddot x_0 - \ddot y_0\,\frac{x}{y_b(x,t)} - \ddot\alpha\,\frac{(x-x_0)^2+(y_b-y_0)^2}{2\,y_b(x,t)} + \dot C\,\frac{1}{y_b(x,t)} + \tilde u(x,\mathbf{z},\dot{\mathbf{z}},C), \tag{10}$$

where the vector-function $\mathbf{z}(t) = (x_0, y_0, \alpha)$, and $\tilde u$ is independent of $\ddot{\mathbf{z}}$.

Substituting (5)-(10) in equations (4), we obtain the following system of three linear equations with respect to the second derivatives $\ddot x$, $\ddot y$, $\ddot\alpha$ and $\dot C$

$$A_{j1}\ddot x_0 + A_{j2}\ddot y_0 + A_{j3}\ddot\alpha + A_{j4}\dot C = f_j(\mathbf{z},\dot{\mathbf{z}},C) \qquad (j=1,2,3). \tag{11}$$

The coefficients A_{ji} and the right-hand side functions f_j are convenient to calculate by using a parametric representation of the body shape $\xi = \xi(\gamma)$, $\eta = \eta(\gamma)$, where γ is a parameter. The values of the parameter which correspond to $x = c_-(t)$ and $x = c_+(t)$ are denoted by $\gamma_-(t)$ and $\gamma_-(t)$. In particular, equations (1) give $c_+(t) = x_0(t) + \xi(\gamma_+)\cos\alpha - \eta(\gamma_+)\sin\alpha$, and $x = x(\gamma,t)$, $y = y(\gamma,t)$. The elements of the equation (11) with $j = 1$ are only shown here as

$$A_{11} = \frac{1}{2}(c_+^2 - c_-^2), \qquad A_{12} = -\int_{\gamma_-}^{\gamma_+} \frac{x^2\,x_\gamma(\gamma,t)\,d\gamma}{y(\gamma,t)} - \frac{m}{\rho},$$

$$A_{13} = -\frac{1}{2}\int_{\gamma_-}^{\gamma_+}(\xi^2+\eta^2)\frac{x x_\gamma(\gamma,t)\,d\gamma}{y(\gamma,t)}, \qquad A_{14} = \int_{\gamma_-}^{\gamma_+}\frac{x x_\gamma(\gamma,t)\,d\gamma}{y(\gamma,t)}, \tag{12}$$

$$f_1 = \frac{mg}{\rho} - \int_{\gamma_-}^{\gamma_+} U(\gamma,t)x x_\gamma(\gamma,t)\,d\gamma + \frac{c_-p_- - c_+p_+}{\rho} + g\int_{\gamma_-}^{\gamma_+} y x_\gamma(\gamma,t)\,d\gamma - gH(c_+ - c_-),$$

where $U(\gamma,t) = \tilde u(x(\gamma,t),\mathbf{z},\dot{\mathbf{z}},C) + uu_x$, $p_\pm = p(c_\pm(t),t)$ are the pressure at the periphery of the contact region. The unknown functions in equations (11) are x_0, y_0, α, $C(t)$, $\gamma_+(t)$, $\gamma_-(t)$, $p_+(t)$ and $p_-(t)$. We need five more equations to arrive at the complete system.

One equation follows from the second equation in (5). By integrating this equation over the contact region, we find

$$A_{41}\ddot x_0 + A_{42}\ddot y_0 + A_{43}\ddot\alpha + A_{44}\dot C = f_4(\mathbf{z},\dot{\mathbf{z}},C), \tag{13}$$

where

$$A_{41} = c_+ - c_-, \qquad A_{42} = -\int_{\gamma_-}^{\gamma_+} \frac{x x_\gamma(\gamma,t)\, d\gamma}{y(\gamma,t)}, \qquad A_{43} = -\frac{1}{2}\int_{\gamma_-}^{\gamma_+} (\xi^2+\eta^2)\frac{x_\gamma(\gamma,t)\, d\gamma}{y(\gamma,t)},$$

$$A_{44} = \int_{\gamma_-}^{\gamma_+} \frac{x_\gamma(\gamma,t)\, d\gamma}{y(\gamma,t)}, \qquad f_4 = \frac{p_- - p_+}{\rho} - \int_{\gamma_-}^{\gamma_+} U(\gamma,t) x_\gamma(\gamma,t)\, d\gamma. \qquad (14)$$

To calculate all elements in (11) and (13), we need to evaluate 13 integrals over the contact region at each time instant. The four equations (11) and (13) can be resolved with respect to the derivatives $\ddot{x}_0$, $\ddot{y}_0$, $\ddot{\alpha}$, $\dot{C}(t)$ which can be integrated in time if $c_+(t)$, $c_-(t)$, $p_+(t)$ and $p_-(t)$ are known. These equations are valid during both the first and second phase of the inclined impact. Equations for $c_\pm(t)$ and $p_\pm(t)$ follow from the matching conditions at the periphery of the contact region, which are different for the first phase and for the second one.

We assume that there is always jet flow region at $x = c_+(t)$, which separates the region beneath the body and the outer region of the liquid in front of the moving body, which is at rest. The matching conditions at $x = c_+(t)$ (see [1] and [2]) are

$$p_+ = \frac{\rho u^2(c_+,t)}{2\left(\sqrt{y_b(c_+,t)/H} - 1\right)}, \qquad \frac{dc_+}{dt} = \frac{u(c_+,t)}{2\left(1 - \sqrt{H/y_b(c_+,t)}\right)}. \qquad (15)$$

During the first phase a jet is formed also at the rear point $x = c_-(t)$, where the conditions similar to (15) must be satisfied

$$p_- = \frac{\rho u^2(c_-,t)}{2\left(\sqrt{y_b(c_-,t)/H} - 1\right)}, \qquad \frac{dc_-}{dt} = \frac{u(c_-,t)}{2\left(1 - \sqrt{H/y_b(c_-,t)}\right)}. \qquad (16)$$

The system (11), (13), (15), (16) of eight equations is integrated in time up to a time instant t_* at which $\dot{c}_- = 0$. The value of the parameter γ at the first contact point, γ_0, is evaluated from the equation $(dy_b/d\gamma)(\gamma_0, 0) = 0$ which reads

$$\xi'(\gamma_0)\, \sin\alpha_0 + \eta'(\gamma_0)\, \cos\alpha_0 = 0, \qquad (17)$$

where α_0 is the initial angle of the body inclination. Equations $y_b(\gamma_0, 0) = H$, $x(\gamma_0, 0) = 0$ provide

$$y_0(0) = H - \xi(\gamma_0)\, \sin\alpha_0 - \eta(\gamma_0)\, \cos\alpha_0, \qquad x_0(0) = \eta(\gamma_0)\, \sin\alpha_0 - \xi(\gamma_0)\, \cos\alpha_0. \qquad (18)$$

Initial asymptotics of the functions $\gamma_\pm(t)$ are

$$\gamma_\pm(t) \sim \gamma_0 \pm \sqrt{6\left(x_0(0)\dot{\alpha}(0) - \dot{y}_0(0)\right)/y_{\gamma\gamma}(\gamma_0, 0)}, \qquad y_{\gamma\gamma}(\gamma_0, 0) = \xi''(\gamma_0)\sin\alpha_0 + \eta''(\gamma_0)\, \cos\alpha_0, \qquad (19)$$

and the initial value of the function $C(t)$ is

$$C(0) = -H\dot{x}_0(0) + \frac{1}{2}\,\dot{\alpha}(0)\,[x_0^2(0) + (H - y_0(0))^2]. \qquad (20)$$

Initial values of the functions $p_\pm(t)$ are

$$p_\pm(t)(0) = 3\rho\left(x_0(0)\dot{\alpha}(0) - \dot{y}_0(0)\right)^2 \frac{x_\gamma^2(\gamma_0, 0)}{H\, y_{\gamma\gamma}(\gamma_0, 0)}. \qquad (21)$$

Calculations show that the pressure in the contact region is positive at the early stage. Then the pressure decays and becomes negative inside the contact region, but still positive near the contact points $x = c_\pm(t)$. The region of negative pressures expand toward the rear point $x = c_-(t)$ and reach this point at $t = t_*$. At the end of the first phase we have $\dot{c}_-(t_*) = 0$, $u(c_-(t_*), t_*) = 0$ and $p(c_-(t_*), t_*) = 0$. Another method describing the body impact during the first phase was employed in [3]. In this method it was observed that the integral of the velocity $u(x,t)$ along the contact region is zero during the first phase. This integral made it possible to integrate equation (13) once in time.

In the present model, the liquid is allowed to separate instantly from the body surface. The position of the separation point is determined by the following two conditions

$$p(c_-(t), t) = 0, \qquad p_x(c_-(t), t) = 0. \qquad (22)$$

Equations (11), (13) with $p_- = 0$ and equations (15) are still valid but equations (16) should be replaced by the condition $p_x(c_-(t), t) = 0$. This condition together with the second equation in (5) provides

$$u_t + u u_x = 0 \qquad \text{at} \qquad x = c_-(t). \qquad (23)$$

By using (10) and the definition of the function $U(\gamma, t)$, we obtain

$$\ddot{x}_0 - \ddot{y}_0\, \frac{x(\gamma_-,t)}{y(\gamma_-,t)} - \ddot{\alpha}\, \frac{\xi^2(\gamma_-) + \eta^2(\gamma_-)}{2y(\gamma_-,t)} + \dot{C}\, \frac{1}{y(\gamma_-,t)} + U(\gamma_-, t) = 0. \qquad (24)$$

Equation (24) has the same form as equations (11) and (13). It is suggested to solve the linear system (11), (13) of four equations with respect to $\ddot{x}_0$, $\ddot{y}_0$, $\ddot{\alpha}$, $\dot{C}(t)$ and substitute the results in (24). In this way, we derive a non-linear equation for $\gamma_-(t)$, which is numerically solved at each time instant.

Numerical results

The ordinary differential equations of the model were integrated in time by the second-order predictor-corrector method. During the first phase we solve the system of ordinary differential equations

$$\frac{d\mathbf{W}}{dt} = \mathbf{G}(\mathbf{W}, t) \qquad \text{where} \qquad \mathbf{W} = (x_0, y_0, \alpha, C, \dot{x}_0, \dot{y}_0, \dot{\alpha}, \gamma_+, \gamma_-), \tag{25}$$

which follows from (11), (13), (15), (16). At time t_n the solution $\mathbf{W}_n$ is known. Then we calculate the value $\mathbf{W}_{n+1}$ at the time step $t_n + \Delta t$ as

$$\tilde{\mathbf{W}} = \mathbf{W}_n + \Delta t \mathbf{G}(\mathbf{W}_n, t_n), \qquad \mathbf{W}_{n+1} = \mathbf{W}_n + \Delta t \frac{\mathbf{G}(\mathbf{W}_n, t_n) + G(\tilde{\mathbf{W}}, t_n)}{2}. \tag{26}$$

During the second phase with separation point the equation (25) is replaced as follows

$$\frac{d\mathbf{W}^*}{dt} = \mathbf{G}^*(\mathbf{W}^*, \gamma_-, t) \qquad \text{where} \qquad \mathbf{W}^* = (x_0, y_0, \alpha, C, \dot{x}_0, \dot{y}_0, \dot{\alpha}, \gamma_+), \tag{27}$$

and $\gamma_-(t)$ is calculated as a solution of equation $Q(\mathbf{W}^*, \gamma_-) = 0$ which follows from (24) and should be solved together with the system (27).

Calculations were performed for the elliptic cylinder $\xi^2/a^2 + \eta^2/b^2 = 1$ with semi-axis $a = 0.5$m and $b = 0.125$m. The mass of the cylinder was varied from 150 to 1500 kg. Results presented below are for the water with density $\rho = 1000$ kg/m^3 and depth $h = 0.05$m, The angle $\alpha(0)$ of the cylinder inclination at the impact instant was varied from 0^o up to 15^o. The time step Δt was chosen as 10^{-4} s.

Figures 3 present results of calculations for three different sets of impact parameters. Fig. 3a is for heavy cylinder with $m = 1500$kg. Initial velocity components of the body are $\dot{x}(0) = 10$ m/s and $\dot{y}(0) = 3$ m/s, inclination angle $\alpha(0) = 6^o$. In this case body finally touches the bottom. It is worth to notice that the angle $\alpha(t)$ initially decreases but then increases before the cylinder touches the bottom.

Figures 3b,c are for light bodies made of wood with $m = 150$kg. Initial velocity components of the body are $\dot{x}(0) = 10$ m/s and $\dot{y}(0) = 1$ m/s, inclination angle $\alpha(0) = 6^o$ for the case b and $\alpha(0) = 12^o$ for case c. In both cases the penetration depth is small. In the case c the body lifts the liquid above its equilibrium level. The calculations stop when equation (24) has no solution. At this time instant the cylinder is above the initial liquid level.

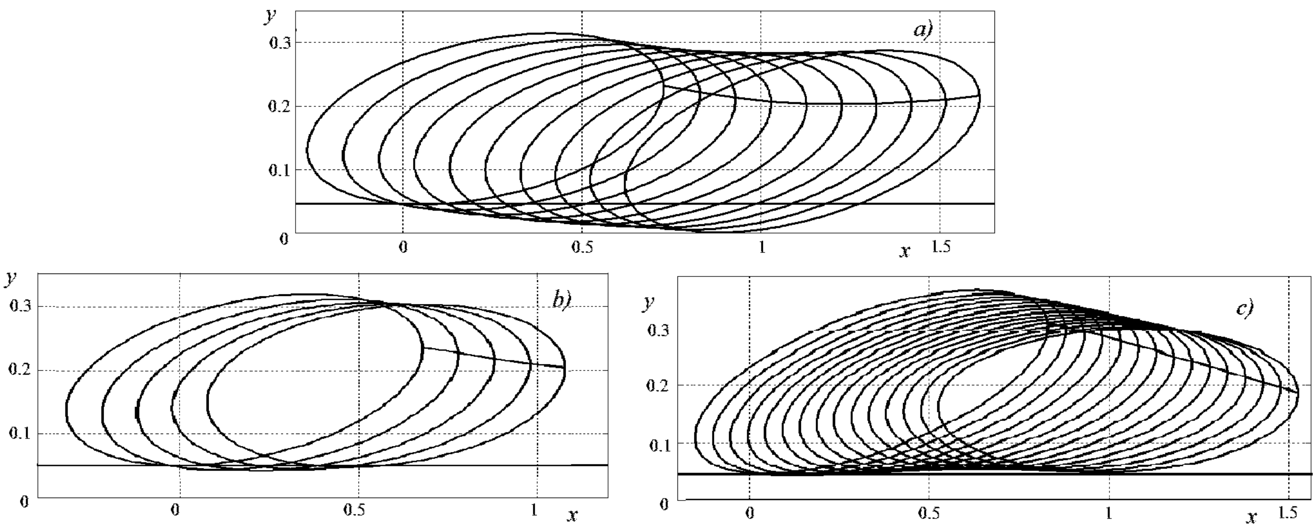

Fig. 3

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