The 27th International Workshop on Water Waves and Floating Bodies, Copenhagen, Denmark, April 22-25, 2012

Wave-energy Absorption Efficiency by a Rotating Pendulum-type Electric-power Generator Installed inside a Floating Body

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Abstract

A rotating pendulum-type wave-power generator is considered, and its performance in the wave-energy absorption is analyzed to understand the conditions for maximizing the absorption efficiency and the relation with reflected and transmitted waves on the free surface. In the present model, the electric-power generator is supposed to be set at the center of a smaller circular cylinder which rotates on the interior circular surface of a floating body without sliding. In the analysis, the linear wave-making theory is effectively used for describing hydrodynamic relations, which makes it possible to derive the conditions for maximizing the efficiency of wave-energy absorption for the present model. Numerical confirmation is also made using the boundary element method for computing hydrodynamic forces and reflected and transmitted waves.

1. Introduction

Recently much attention is focused again on the utilization of ocean energies. Since the amount of resources of wave energy is enormous, the wave-power generation will play an important role in the future strategy for the electric-power supply, provided that we can develop an efficient apparatus for the wave-energy absorption and transmission.

A large number of researches have been done so far on the performance of various kinds of wave-power generator in the world. In Japan, although a large amount of theoretical work had been made in the 1970s, most of the big projects on the system of wave-power generation were concerned with the use of air flow induced by oscillating water columns. Nowadays, many projects on the development of various types of apparatus for the wave-energy utilization are in progress mainly in Europe, including real-size experiments at actual seas. The contemporary trend in the development of wave-power generators is the use of floating bodies responding to the wave excitation, installed at an offshore site of deep water where large-amplitude waves exist.

In the present paper, a rotating pendulum-type wave-power generator is studied to make the efficiency in absorbing wave energy higher over a wide range of wave frequencies. In the proposed model, the interior surface of a floating body is of circular cylinder with larger radius, and a smaller circular cylinder is assumed to rotate on the interior circular surface without sliding. An electric-power generator is set at the center of the smaller circular cylinder. Both floating body and smaller circular cylinder inside rotate due to wave actions and cross-coupling effects, and then the smaller circular cylinder generates the electric power when the relative rotational motion becomes large. In this paper, coupled motion equations between the floating body and the smaller circular cylinder inside are established using the linear theory and solved analytically, providing the complex amplitude of wave-induced motions. Then the efficiency of wave-energy absorption by the smaller circular cylinder inside is computed and the conditions for maximizing the efficiency are studied. It is theoretically shown that, when those conditions are satisfied, the maximum efficiency in the wave-energy absorption becomes equal to $1/2$ and the anti-symmetric component of the wave on the free surface becomes perfectly zero (because the symmetric heave motion is not controlled in the present study). Numerical computations are also made to compute hydrodynamic forces appearing in the motion equations and to confirm correctness of the relations derived theoretically.

2. Theory

2.1 Outline of wave-power generator and assumption

The cross section of a rotating pendulum-type wave-power generator is shown in Fig. 1, where the problem is treated as two dimensional. We consider a floating body whose interior surface is of circular cylinder with radius R, and a smaller circular cylinder with radius r which rotates on the interior circular surface without sliding. By installing an electric generator inside the smaller cylinder with their centers set coincident, the electric generator can be actuated by the rotational motion of the cylinder. In order to evaluate the rotational motion of the electric generator, we need to consider coupled motion equations between the floating body and the smaller cylinder rotating inside.

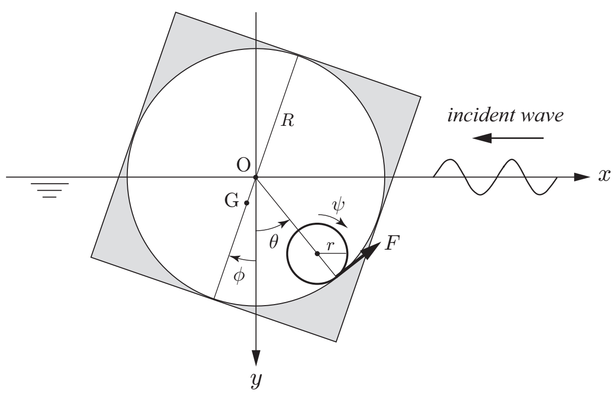

Fig. 1 Coordinate system and notations

As shown in Fig. 1, the origin of the coordinate system is taken at the center of the floating body and on the undisturbed free surface. The roll angle of the floating body is denoted as ϕ (which is positive in the clock-wise direction), and the swing angle of the center of smaller circular cylinder measured from a vertical line is denoted as θ (which is defined positive in the counter clock-wise direction). Denoting the friction force from the interior circular surface to the smaller circular cylinder as F, the same friction force in magnitude but opposite in sign works from the smaller circular cylinder to the floating body on the interior circular surface. By introducing this internal friction force, we can consider the roll motion of the floating body and the rotational motion of the smaller circular cylinder separately.

Since the floating body is assumed symmetric in the weather and lee sides, the heave motion can be analyzed separately in the linear theory. For brevity in the analysis, the sway motion is not considered, but fundamental features to be elucidated in this paper may not largely be changed by the effect of sway.

2.2 Coupled motion equations

The equation of roll motion of the floating body about the origin can be expressed as

$$I_0\ddot{\phi} = -A_{44}\ddot{\phi} - B_{44}\dot{\phi} - Mg\,\overline{GM}\sin\phi + M_4 + FR \tag{1}$$

where I_0 denotes the moment of inertia; A_{44} the added moment of inertia resulting from ambient fluid; B_{44} the damping coefficient due to wave making (the viscous damping should be included in reality, but that component is neglected in the analysis below); M the mass of the floating body; g the acceleration due to gravity; $\overline{GM}$ the metacentric height; M_4 the wave-exciting moment by an incident wave.

The translational motion equation in the tangential direction and the rotational motion equation of the smaller circular cylinder are expressed as follows:

$$m(R-r)\ddot{\theta} = -mg\sin\theta + F \tag{2}$$

$$I_C\ddot{\psi} = -N\dot{\psi} - Fr \tag{3}$$

where m and I_C denote the mass and moment of inertia, respectively, of the smaller circular cylinder; N the damping coefficient due to resistance of an electric-power generator; ψ the rotation angle of the smaller cylinder shown in Fig. 1. The condition of no sliding between the smaller circular cylinder and the interior circular surface of floating body can be expressed as

$$(R-r)\dot{\theta} - r\dot{\psi} = -R\dot{\phi} \tag{4}$$

This equation states that the velocity of the smaller circular cylinder at a contact point, $(R-r)\dot{\theta} - r\dot{\psi}$, and the velocity on the interior circular surface of the floating body, $-R\dot{\phi}$, must be equal.

Eliminating F and ψ from (1) – (4), we can obtain the following coupled equations:

$$I_A\ddot{\phi} + \Big(B_{44} + N\frac{R^2}{r^2}\Big)\dot{\phi} + Mg\,\overline{GM}\,\phi + \frac{R(R-r)}{r^2}\Big\{I_C\ddot{\theta} + N\dot{\theta}\Big\} = M_4 \tag{5}$$

$$I_C\ddot{\phi} + N\dot{\phi} + \frac{R-r}{R}\Big\{I\ddot{\theta} + N\dot{\theta} + mg\frac{r^2}{R-r}\theta\Big\} = 0 \tag{6}$$

where

$$I_A \equiv I_0 + A_{44} + I_C\frac{R^2}{r^2}\,, \quad I \equiv I_C + mr^2 \tag{7}$$

and we have assumed that ϕ and θ are of small quantities to linearize the problem.

We consider the case that the wave-exciting moment M_4 is time harmonic due to incident wave with circular frequency ω and amplitude ζ_a, and the resulting responses, ϕ and θ, are also time harmonic with the same circular frequency. Thus we write them as follows:

$$M_4 = \mathrm{Re}\big[E_4 e^{i\omega t}\big],\ \phi = \mathrm{Re}\big[\varPhi e^{i\omega t}\big],\ \theta = \mathrm{Re}\big[\varTheta e^{i\omega t}\big] \tag{8}$$

where E_4, $\varPhi$, and $\varTheta$ denote the complex amplitudes of corresponding quantities.

With the energy conservation and the Haskind relation, the wave-damping coefficient B_{44} and the wave-exciting moment E_4 can be expressed in terms of the Kochin function in the radiation problem of roll motion H_4^+, in the form

$$\left.\begin{aligned} B_{44} &= \rho\omega R^4\,\big|H_4^+\big|^2 \equiv \rho\omega R^4 h \\ E_4 &= \rho g\zeta_a R^2 H_4^+ \end{aligned}\right\} \tag{9}$$

where ρ is the density of fluid, and radius R is used as a representative length for making quantities nondimensional and thus H_4^+ and $h\,(=|H_4^+|^2)$ are supposed to be nondimensional.

The damping coefficient of electric-power generator N is expressed in connection with B_{44}, in terms of a proportional coefficient β, in the form

$$N\frac{R^2}{r^2} \equiv \beta\,B_{44} = \rho\omega R^4\beta h \tag{10}$$

In order to facilitate subsequent transformation, the restoring and inertia terms are written as

$$\left.\begin{aligned} Mg\overline{GM} - \omega^2 I_A &\equiv \rho\omega^2 R^4 P^2 \\ \omega^2 I_C &\equiv \rho\omega^2 R^2 r^2 Q^2 \\ mg\frac{r^2}{R-r} - \omega^2 I &\equiv \rho\omega^2 R^2 r^2 S^2 \end{aligned}\right\} \tag{11}$$

where it should be noted that newly-introduced symbols, P^2, Q^2 and S^2, are also nondimensional.

Then, solutions for (5) and (6) can be analytically obtained and written in the form

$$\left.\begin{aligned} \frac{\varPhi}{K\zeta_a} &= \frac{H_4^+}{(KR)^2}\frac{(\,S^2 + i\beta h\,)}{\varDelta} \\ \frac{R-r}{R}\frac{\varTheta}{K\zeta_a} &= \frac{H_4^+}{(KR)^2}\frac{(\,Q^2 - i\beta h\,)}{\varDelta} \end{aligned}\right\} \tag{12}$$

where $K = \omega^2/g$ is the wavenumber and the denominator $\varDelta$ is given by

$$\varDelta = P^2S^2 - Q^4 - \beta h^2 + ih\big\{\beta(P^2 + S^2 + 2Q^2) + S^2\big\} \tag{13}$$

which is the determinant in the simultaneous equations to be obtained from (5) and (6).

In what follows, we will also use the following notations:

$$\left.\begin{aligned} &a \equiv P^2 + Q^2\,, \quad b \equiv S^2 + Q^2 \\ &c \equiv a + b = P^2 + S^2 + 2Q^2\,, \quad d \equiv S^2 \\ &e \equiv P^2 - Q^4/S^2\,, \quad q \equiv Q^2 \end{aligned}\right\} \tag{14}$$

Then the results of (12) and (13) can be written as

$$\varPhi + \frac{R-r}{R}\varTheta = K\zeta_a\,\frac{H_4^+}{(KR)^2}\,\frac{b}{\varDelta} \tag{15}$$

where

$$\varDelta = de - \beta h^2 + ih\big(\beta c + d\big) \tag{16}$$

2.3 Wave-energy absorption efficiency

The efficiency of wave-energy absorption is defined as the ratio between the power of regular incident wave per unit length P_W and the power of actuating the electric generator P_E. Here P_W is given by

$$P_W = \frac{1}{2}\rho g \zeta_a^2 \Big(\frac{g}{2\omega}\Big) = \frac{\rho g^2 \zeta_a^2}{4\omega} \tag{17}$$

and the power P_E can be calculated as the average of work over one period done by the damping force of electric generator, which is given by

$$P_E = \frac{1}{T}\int_0^T N\dot{\psi}^2\,dt = \frac{1}{T}\int_0^T N\Big\{\frac{R}{r}\dot{\phi} + \frac{R-r}{r}\dot{\theta}\Big\}^2 dt$$
$$= \frac{1}{2}N\frac{R^2}{r^2}\omega^2 \left|\Phi + \frac{R-r}{R}\Theta\right|^2 \tag{18}$$

Substituting (10) and (15) in the above and using (17), we can see that the absorption efficiency can be written in the form

$$\eta \equiv \frac{P_E}{P_W} = 2\beta h^2 \left|\frac{b}{\Delta}\right|^2 = \frac{2\beta h^2 b^2}{|\Delta|^2} \tag{19}$$

where
$$\left|\Delta\right|^2 = (de - \beta h^2)^2 + h^2(\beta c + d)^2 \tag{20}$$

Let us consider the condition of maximizing the absorption efficiency as a function of parameter β, by differentiating η with respect to β and setting it equal to zero. Then we can obtain the following relation as that maximum condition:

$$\beta^2 = \frac{d^2(h^2+e^2)}{h^2(c^2+h^2)} \tag{21}$$

In this case, the maximum of the absorption efficiency takes the form

$$\eta_{\max} = \cfrac{1}{1 + \cfrac{d^2(h^2+e^2)}{\beta h^2 b^2}} \tag{22}$$

It is obvious from (19) that the absorption efficiency becomes zero at the frequency satisfying $b = 0$. That is, from (11) and (14), at the frequency satisfying $KR = Kr + 1$.

2.4 Reflection and transmission waves

Once the motions of a floating body are obtained, the coefficients of reflection and transmission waves due to wave diffraction and radiation by that floating body can be computed. Let us denote the reflection and transmission wave coefficients with C_R and C_T, respectively. Then, in terms of these, the components of symmetric wave ($\mathcal{A}$) and anti-symmetric wave ($\mathcal{B}$) can be given by

$$\mathcal{A} = \frac{1}{2}\big(C_R + C_T\big) = \frac{1}{2}\frac{H_3^+}{\overline{H_3^+}} - i\,KR\,\frac{Y}{\zeta_a}H_3^+ \tag{23}$$

$$\mathcal{B} = \frac{1}{2}\big(C_R - C_T\big) = \frac{1}{2}\frac{H_4^+}{\overline{H_4^+}} - i\,(KR)^2\frac{\Phi}{K\zeta_a}H_4^+ \tag{24}$$

Here H_3^+ denotes the Kochin function in the heave radiation problem and Y the complex amplitude of heave motion. The first terms on the right-hand side of (23) and (24) represent the scattered wave in the diffraction problem; which can be given with the Kochin function in the radiation problem, known as the so-called Bessho-Newman relation.

It is obvious that the symmetric wave $\mathcal{A}$ is not affected by the roll motion at all and its amplitude remains the same and equal to 1/2 irrespective of the wave frequency. On the other hand, the anti-symmetric wave $\mathcal{B}$ is changed by the roll motion. Substituting (12) for $\Phi/K\zeta_a$ into (24) and using the notations of (14) and (16), we have the following:

$$\mathcal{B} = \frac{1}{2}\frac{H_4^+}{\overline{H_4^+}} - i\left(H_4^+\right)^2\frac{d + i\beta h}{\Delta}$$
$$= \frac{1}{2}\frac{H_4^+}{\overline{H_4^+}}\frac{de + \beta h^2 + ih(\beta c - d)}{de - \beta h^2 + ih(\beta c + d)} \tag{25}$$

We can see from (25) that the perfect absorption of the anti-symmetric wave can be realized by setting the numerator of (25) equal to zero; that is

$$\beta = \frac{d(h + i\,e)}{h(c - i\,h)} \tag{26}$$

By considering the complex conjugate $\overline{\beta}$ of (26) and multiplication $\beta\overline{\beta} = |\beta|^2$, we can see that the condition for maximizing the absorption efficiency, (21), is essentially the same as (26).

However, the coefficient β must be of real quantity (and also must be positive). From this requirement, we can obtain the following conditions

$$ec + h^2 = 0\,, \quad \beta = \frac{d}{c} > 0 \tag{27}$$

for the perfect absorption of the anti-symmetric wave component. It is obvious that (25) becomes zero when (27) is satisfied. Furthermore in this case we can show that

$$\frac{d^2(h^2+e^2)}{\beta h^2 b^2} = \frac{d(c-e)}{b^2} = 1 \tag{28}$$

Therefore, we can see from (22) that the maximum of the wave-energy absorption efficiency becomes $\eta_{\max} = 1/2$ when the anti-symmetric wave component is completely absorbed; which is natural from the energy conservation principle.

3. Numerical Results and Discussions

In order to confirm theoretical results, numerical computations have been performed for a rectangular floating body, shown in Fig. 1, with inscribed circle of radius R. The hydrodynamic forces were computed numerically by the free-surface Green function method (boundary element method) based on the potential-flow theory. Obtained results were confirmed to satisfy very accurately various relations proven theoretically, such as the Haskind relation and the energy conservation principle shown as (9).

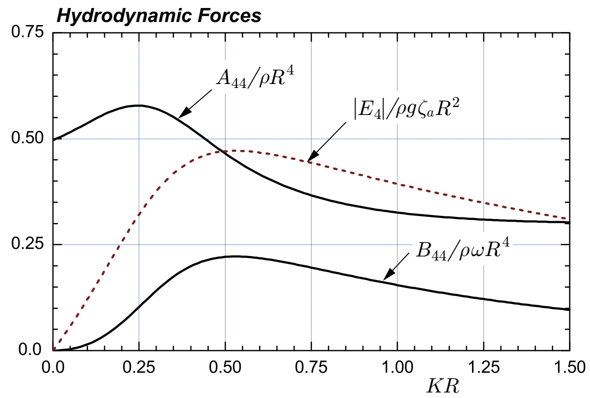

Fig. 2 Hydrodynamic forces in roll motion of a rectangular floating body

Computed results are shown in Fig. 2, for the added moment of inertia A_{44}, the wave damping coefficient B_{44}, and the amplitude of the wave-exciting roll moment $|E_4|$ in nondimensional forms. The abscissa is the nondimensional wavenumber $KR = \omega^2 R/g$.

For computing coupled motions of the floating body and the inner circular cylinder rotating along the interior circular surface without sliding, the values of coefficients related to the inertia and restoring moments should be given. The mass ratio between the floating body M and the smaller circular cylinder inside m is taken as $M/m = 4.0$. The values of gyrational radius κ, metacentric height $\overline{GM}$, and radius of inner circular cylinder r are taken as $\kappa = 0.5$, $\overline{GM} = 0.2$, and $r = 0.1$, respectively; these are given in nondimension in terms of R. Then the moment of inertia of the floating body is given as $I_0 = M\kappa^2$ and the moment of inertia of the smaller cylinder is given with the assumption of constant density by $I_C = \frac{1}{2} m r^2$. Coefficient β, defined in (10) for the damping coefficient of electric-power generator, is tentatively set equal to $\beta = 2.35$, with which the perfect absorption of anti-symmetric wave must be realized at $KR = 0.5954$, as will be shown later.

Computed results are shown in Fig. 3 for the nondimensional motion amplitudes, $|\Phi|/K\zeta_a$ and $|\Theta|/K\zeta_a$, and in Fig. 4 for the wave-energy absorption efficiency, $\eta = P_E/P_W$.

In order to understand the relation of various coefficients defined in (11) and (14) with the wave-energy absorption efficiency and motion characteristics, Fig. 5 is provided. Furthermore, the amplitudes of transmitted and reflected waves ($|C_T|$ and $|C_R|$) and also of the anti-symmetric wave component ($\frac{1}{2}|C_T - C_R|$) are shown in Fig. 6.

First, we can see from Fig. 5 that at $KR = 0.5954$, $ec + h^2 = 0$ and $\beta - d/c = 0$ are satisfied almost simultaneously, which is the condition for the perfect absorption of anti-symmetric wave and the wave-energy absorption efficiency equal to $\eta = 1/2$, as shown in (27). At this particular wavenumber, we can confirm that $\eta = 1/2$ is satisfied in Fig. 4 and $|C_T - C_R| = 0$ and $|C_T| = |C_R|$ are satisfied in Fig. 6. We can also confirm in Fig. 4 that the absorption efficiency becomes exactly zero at a frequency where $b = 0$ (i.e. $KR = Kr + 1$) is satisfied; which is realized in the present case at $KR = 1.11$.

Another thing to be noted in Fig. 3 and Fig. 5 is that the wavenumber where the roll-motion amplitude becomes maximal is related to $P^2 (= a - q) = 0$ and likewise the wavenumber where the swing angle of inner circular cylinder becomes large is related to $S^2 (= d) = 0$. Of course, the actual peak (resonant) frequency is slightly shifted to a lower frequency than that given by $P^2 = 0$ or $S^2 = 0$ due to the effect of damping force. We can see that the efficiency of wave-energy absorption increases around these resonant frequencies of both floating body and inner circular cylinder.

An experiment for measuring coupled motions of a floating body and a smaller circular cylinder rotating inside of the floating body without sliding is now in progress and results will be presented at the Workshop.

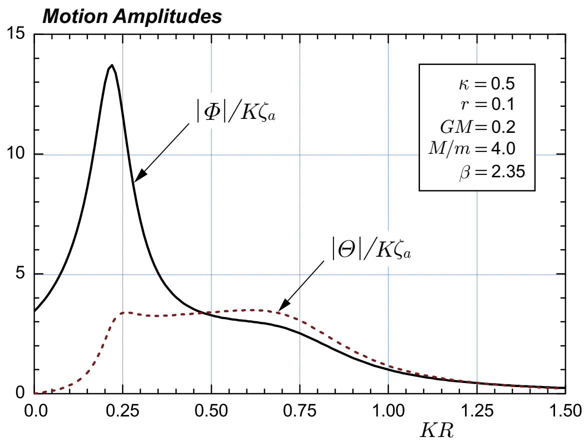

Fig. 3 Amplitudes of the roll motion of a floating body and the swing motion of an inner circular cylinder

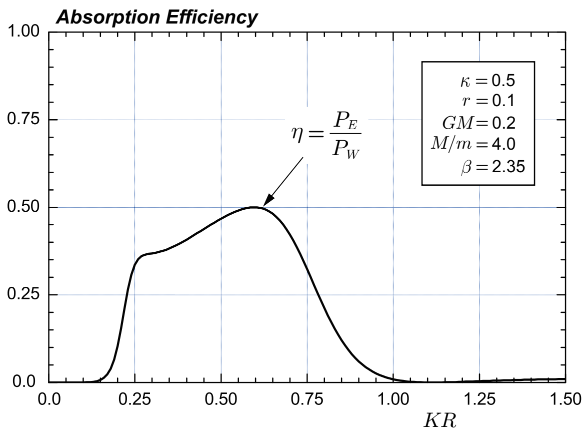

Fig. 4 Efficiency of wave-energy absorption

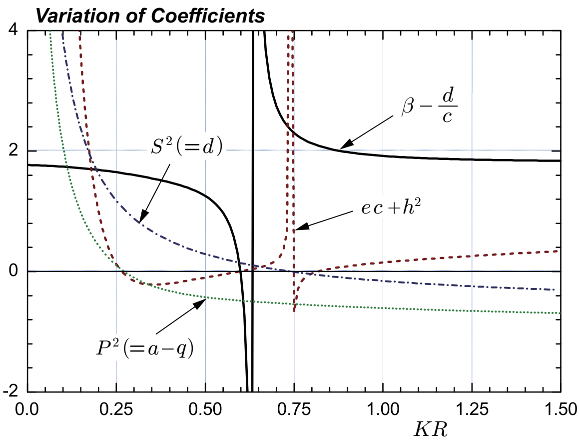

Fig. 5 Variation of coefficients related to motion equations and wave-energy absorption

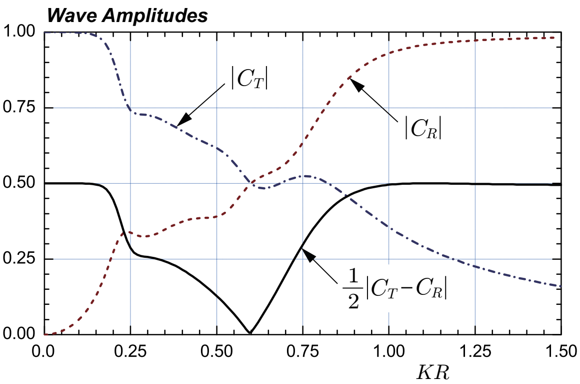

Fig. 6 Coefficients of reflected and transmitted waves and anti-symmetric wave component